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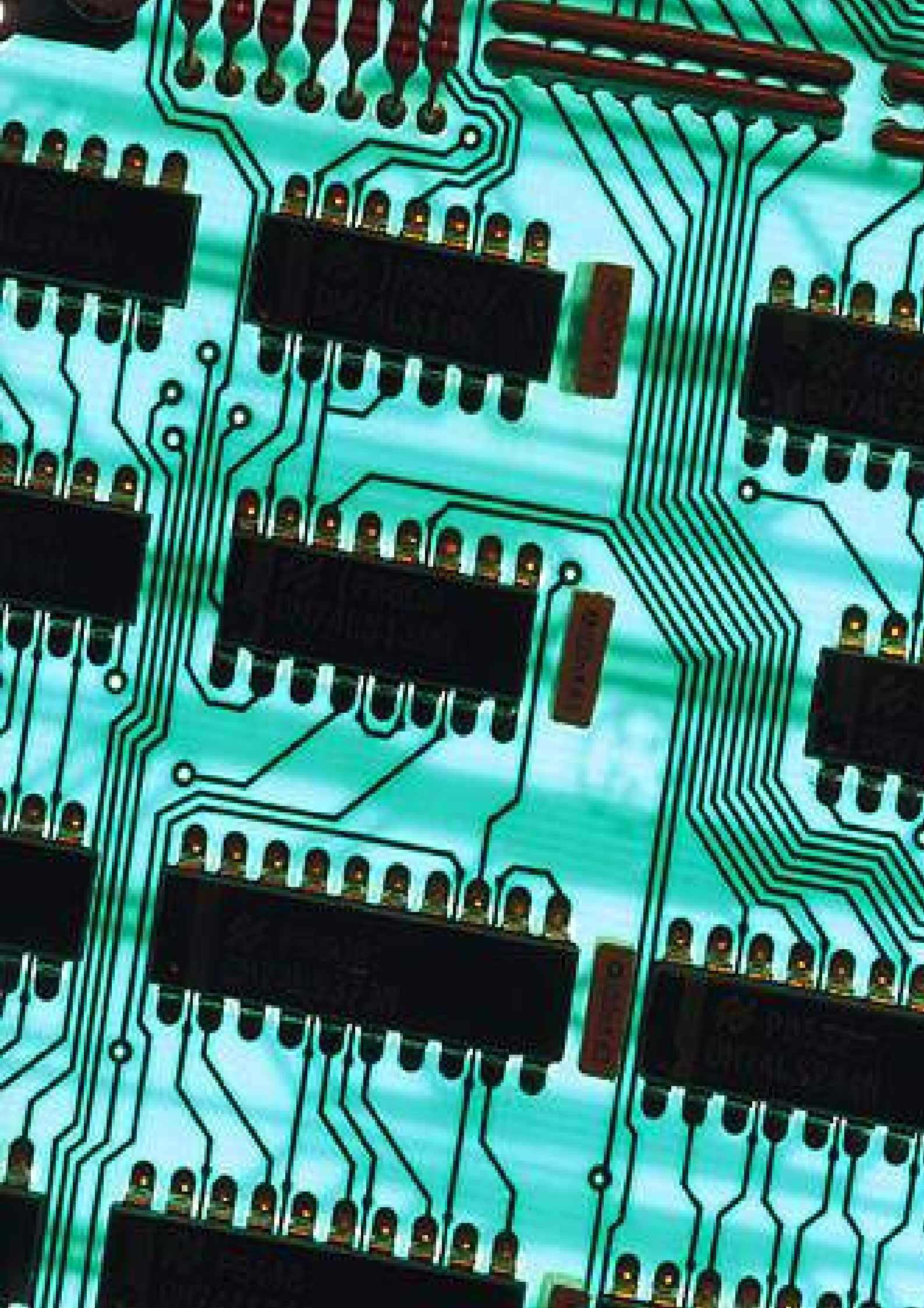
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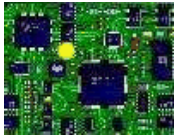


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## ***Theory and Design of Electrical and Electronic Circuits***

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# **Theory and Design of** **Electrical and Electronic Circuits**

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## **Introduction**

Spent the years, the Electrical and Electronic technology has bloomed in white hairs; white technologically for much people and green socially for others.

To who writes to them, it wants with this theoretical and practical book, to teach criteria of design with the experience of more than thirty years. I hope know to take advantage of it because, in truth, I offer its content without interest, affection and love by the fellow.

*Eugenio Máximo Tait*

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## Chap. 01 Generalities

### [Introduction](#)

### [System of units](#)

### [Algebraic and graphical simbology](#)

### [Nomenclature](#)

### [Advice for the designer](#)

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### [Introduction](#)

In this chapter generalizations of the work are explained.

Almost all the designs that appear have been experienced satisfactorily by who speaks to them. But by the writing the equations can have some small errors that will be perfected with time.

The reading of the chapters must be ascending, because they will be occurred the subjects being based on the previous ones.

### [System of units](#)

Except the opposite clarifies itself, all the units are in M. K. S. They are the Volt, Ampere, Ohm, Siemens, Newton, Kilogram, Second, Meter, Weber, Gaussian, etc.

The temperature preferably will treat it in degrees Celsius, or in Kelvin.

All the designs do not have units because incorporating each variable in M. K. S., will be satisfactory its result.

### [Algebraic and graphical simbology](#)

Often, to simplify, we will use certain symbols. For example:

— Parallel of components  $1 / (1/X_1 + 1/X_2 + \dots)$  like  $X_1 // X_2 // \dots$

— Signs like "greater or smaller" ( $\geq \leq$ ), "equal or different" ( $= \neq$ ), etc., they are made of form similar to the conventional one to have a limited typesetter source.

In the parameters (curves of level) of the graphs they will often appear small arrows that indicate the increasing sense.

In the drawn circuits when two lines (conductors) are crossed, there will only be connection between such if they are united with a point. If they are drawn with lines of points it indicates that

this conductor and what he connects is optative.

## Nomenclature

A same nomenclature in all the work will be used. It will be:

— instantaneous (small)	$v$
— continuous or average (great)	$V$
— effective (great)	$V$ or $V_{ef}$
— peak	$V_{pico}$ or $v_p$
— maximum	$V_{max}$
— permissible (limit to the breakage)	$V_{ADM}$

## Advice for the designer

All the designs that become are not for arming them and that works in their beginning, but to only *have an approximated idea* of the components to use. To remember here one of the laws of Murphy: " If you make something and works, it is that it has omitted something by stop ".

The calculations have so much the heuristic form (test and error) like algorithmic (equations) and, therefore, they will be only contingent; that is to say, that one must correct them until reaching the finished result.

So that a component, signal or another thing is despicable front to another one, to choose among them 10 times often is not sufficient. One advises at least 30 times as far as possible. But two cases exist that are possible; and more still, up to 5 times, that is when he is geometric ( $5^2 = 25$ ), that is to say, when the leg of a triangle rectangle respect to the other is of that greater magnitude or. This is when we must simplify a component reactive of another pasive, or to move away to us of pole or zero of a transference.

As far as simple constants of time, it is to say in those transferences of a single pole and that is excited with steps being exponential a their exit, normally 5 constants are taken from time to arrive in the end. But, in truth, this is unreal and little practical. One arrives at 98% just by 3 constants from time and this magnitude will be sufficient.

As far as the calculations of the permissible regimes, adopted or calculated, always he is advisable to sobredetermine the proportions them.

The losses in the condensers are important, for that reason he is advisable to choose of high value of voltage the electrolytic ones and that are of recognized mark (v.g.: Siemens). With the ceramic ones also always there are problems, because they have many losses (Q of less than 10 in many applications) when also they are extremely variable with the temperature (v.g.: 10 [°C] can change in 10 [%] to it or more), thus is advised to use them solely as of it desacopled and, preferably, always to avoid them. Those of poliester are something more stable. Those of mica and air or oil in works of high voltage are always recommendable.

When oscillating or timers are designed that depend on capacitiva or inductive constant of times, he is not prudent to approach periods demarcated over this constant of time, because small variations of her due to the reactive devices (v.g.: time, temperature or bad manufacture, usually changes a little the magnitude of a condenser) it will change to much the awaited result.

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## Chap. 02 Polarization of components

### Bipolar transistor of junction (TBJ)

Theory

Design

Fast design

### Unipolar transistor of junction (JFET)

Theory

Design

### Operational Amplifier of Voltage (AOV)

Theory

Design

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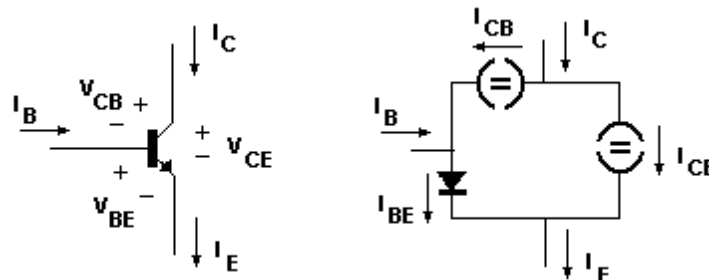
### Bipolar transistor of junction (TBJ)

Theory

Polarizing to the bases-emitter diode in direct and collector-bases on inverse, we have the model approximated for continuous. The static gains of current in common emitter and common bases are defined respectively

$$\beta = h_{21E} = h_{FE} = I_C / I_B \sim h_{21e} = h_{fe} (>> 1 \text{ para TBJ comunes})$$

$$\alpha = h_{21B} = h_{FB} = I_C / I_E \sim h_{21b} = h_{fb} (\sim < 1 \text{ para TBJ comunes})$$



La corriente entre collector y base  $I_{CB}$  es de fuga, y sigue aproximadamente la ley

The current between collector and bases  $I_{CB}$  it is of loss, and it follows approximately the law



$$I_{CB} = I_{CB0} (1 - e^{V_{CB}/V_T}) \sim I_{CB0}$$

where

$$V_T = 0,000172 \cdot (T + 273)$$

$$I_{CB} = I_{CB0(25^\circ\text{C})} \cdot 2^{\Delta T/10}$$

with  $\Delta T$  the temperature jump respect to the atmosphere 25 [°C]. From this it is then

$$\Delta T = T - 25$$

$$\partial I_{CB} / \partial T = \partial I_{CB} / \partial \Delta T \sim 0,07 \cdot I_{CB0(25^\circ\text{C})} \cdot 2^{\Delta T/10}$$

On the other hand, the dependency of the bases-emitter voltage respect to the temperature, to current of constant bases, we know that it is

$$\partial V_{BE} / \partial T \sim -0,002 \text{ [V/}^\circ\text{C]}$$

The existing relation between the previous current of collector and gains will be determined now

$$I_C = I_{CE} + I_{CB} = \alpha I_E + I_{CB}$$

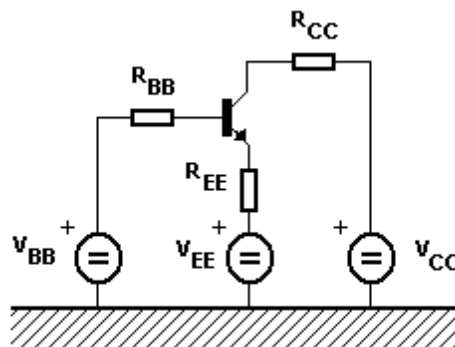
$$I_C = I_{CE} + I_{CB} = \beta I_{BE} + I_{CB} = \beta (I_{BE} + I_{CB}) + I_{CB} \sim \beta (I_{BE} + I_{CB})$$

$$\beta = \alpha / (1 - \alpha)$$

$$\alpha = \beta / (1 + \beta)$$

Next let us study the behavior of the collector current respect to the temperature and the voltages

$$\begin{aligned} \Delta I_C = & (\partial I_C / \partial I_{CB}) \Delta I_{CB} + (\partial I_C / \partial V_{BE}) \Delta V_{BE} + (\partial I_C / \partial V_{CC}) \Delta V_{CC} + \\ & + (\partial I_C / \partial V_{BB}) \Delta V_{BB} + (\partial I_C / \partial V_{EE}) \Delta V_{EE} \end{aligned}$$



of where they are deduced of the previous expressions

$$\Delta I_{CB} = 0,07 \cdot I_{CB0(25^\circ C)} \cdot 2^{\Delta T/10} \Delta T$$

$$\Delta V_{BE} = -0,002 \Delta T$$

$$V_{BB} - V_{EE} = I_B (R_{BB} + R_{EE}) + V_{BE} + I_C R_{EE}$$

$$I_C = [V_{BB} - V_{EE} - V_{BE} + I_B (R_{BB} + R_{EE})] / [R_E + (R_{BB} + R_{EE}) \beta^{-1}]$$

$$S_I = (\partial I_C / \partial I_{CB}) \sim (R_{BB} + R_{EE}) / [R_{EE} + R_{BB} \beta^{-1}]$$

$$S_V = (\partial I_C / \partial V_{BE}) = (\partial I_C / \partial V_{EE}) = -(\partial I_C / \partial V_{BB}) = -1 / (R_E + R_{BB} \beta^{-1})$$

$$(\partial I_C / \partial V_{CC}) = 0$$

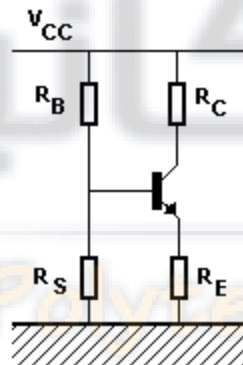
being

$$\Delta I_C = [0,07 \cdot 2^{\Delta T/10} (R_{BB} + R_{EE}) (R_{EE} + R_{BB} \beta^{-1})^{-1} I_{CB0(25^\circ C)} + 0,002 (R_{EE} + R_{BB} \beta^{-1})^{-1}] \Delta T + (R_E + R_{BB} \beta^{-1})^{-1} (\Delta V_{BB} - \Delta V_{EE})$$

## Design

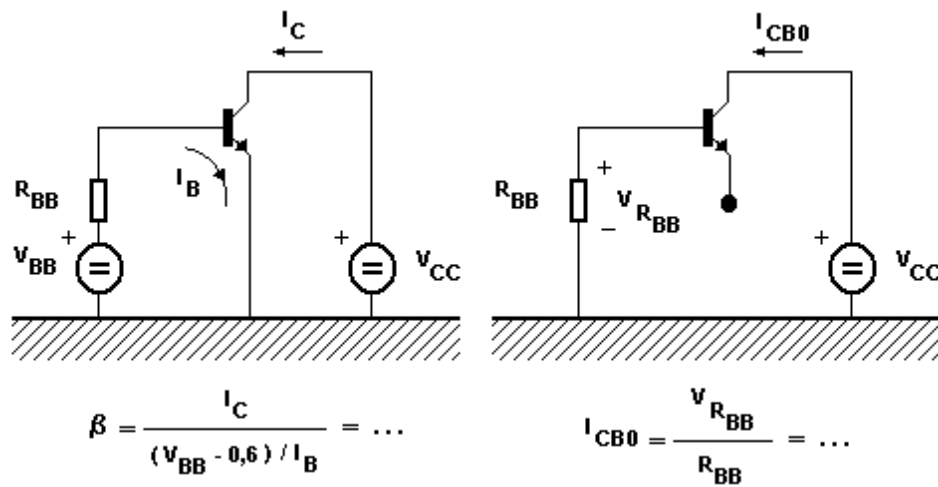
Be the data

$$I_C = \dots V_{CE} = \dots \Delta T = \dots I_{Cmax} = \dots R_C = \dots$$



From manual or the experimentation according to the graphs they are obtained

$$\beta = \dots I_{CB0(25^\circ C)} = \dots V_{BE} = \dots (\sim 0,6 [V] \text{ para TBJ de baja potencia})$$



and they are determined analyzing this circuit

$$R_{BB} = R_B // R_S$$

$$V_{BB} = V_{CC} \cdot R_S (R_B + R_S)^{-1} = V_{CC} \cdot R_{BB} / R_B$$

$$\Delta V_{BB} = \Delta V_{CC} \cdot R_{BB} / R_S = 0$$

$$\Delta V_{EE} = 0$$

$$R_{EE} = R_E$$

$$R_{CC} = R_C$$

and if to simplify calculations we do

$$R_E \gg R_{BB} / \beta$$

us it gives

$$S_I = 1 + R_{BB} / R_E$$

$$S_V = -1 / R_E$$

$$\Delta I_{Cmax} = (S_I \cdot 0.07 \cdot 2^{\Delta T/10} I_{CB0(25^\circ C)} - S_V \cdot 0.002) \cdot \Delta T$$

and if now we suppose by simplicity

$$\Delta I_{Cmax} \gg S_V \cdot 0.002 \cdot \Delta T$$

are

$$R_E = \dots \gg 0.002 \cdot \Delta T / \Delta I_{Cmax}$$

$$R_E [(\Delta I_{Cmax} / 0.07 \cdot 2^{\Delta T/10} I_{CB0(25^\circ C)} \cdot \Delta T) - 1] = \dots > R_{BB} = \dots \ll \beta R_E = \dots$$

being able to take a  $\Delta I_C$  smaller than  $\Delta I_{Cmax}$  if it is desired.

Next, as it is understood that

$$V_{BB} = I_B R_{BB} + V_{BE} + I_E R_E \sim [(I_C \beta^{-1} - I_{CB0(25^\circ C)}) R_{BB} + V_{BE} + I_E R_E = \dots$$

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E \sim I_C (R_C + R_E) + V_{CE} = \dots$$

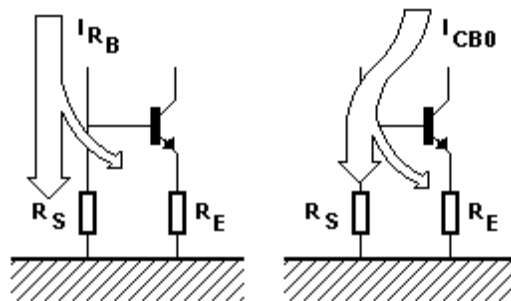
they are finally

$$R_B = R_{BB} V_{CC} / V_{BB} = \dots$$

$$R_S = R_B R_{BB} / R_B - R_{BB} = \dots$$

### Fast design

This design is based on which the variation of the  $I_C$  depends solely on the variation of the  $I_{CB}$ . For this reason one will be to prevent it circulates to the base of the transistor and is amplified. Two criteria exist here: to diminish  $R_S$  or to enlarge the  $R_E$ . Therefore, we will make reasons both; that is to say, that we will do that  $I_S \gg I_B$  and that  $V_{RE} > 1 [V]$  —since for  $I_C$  of the order of miliamperes are resistance  $R_E > 500 [\Omega]$  that they are generally sufficient in all thermal stabilization.



Be the data

$$I_C = \dots \quad V_{CE} = \dots \quad R_C = \dots$$

From manual or the experimentation they are obtained

$$\beta = \dots$$

what will allow to adopt with it

$$I_S = \dots \gg I_C \beta^{-1}$$

$$V_{RE} = \dots > 1 [V]$$

and to calculate

$$V_{CC} = I_C R_C + V_{CE} + V_{RE} = \dots$$

$$R_E = V_{RE} / I_C = \dots$$

$$R_S = (0,6 + V_{RE}) / I_S = \dots$$

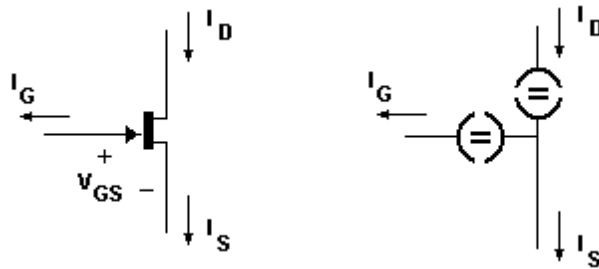
$$R_B = (V_{CC} - 0,6 - V_{RE}) / I_S = \dots$$

## Unipolar transistor of junction (JFET)

### Theory

We raised the equivalent circuit for an inverse polarization between gate and drain, being  $I_G$  the current of lost of the diode that is

$$I_G = I_{G0} (1 - e^{V_{GS}/V_T}) \sim I_{G0} = I_{G0(25^\circ\text{C})} \cdot 2^{\Delta T/10}$$



If now we cleared

$$V_{GS} = V_T \cdot \ln(1 + I_G/I_{G0}) \sim 0,7 \cdot V_T$$

$$\partial V_{GS} / \partial T \sim 0,00012 \text{ [V/}^\circ\text{C]}$$

On the other hand, we know that  $I_D$  it depends on  $V_{GS}$  according to the following equations

$$I_D \sim I_{DSS} [ 2 V_{DS} ( 1 + V_{GS} / V_P ) / V_P - ( V_{GS} / V_P )^2 ] \quad \text{con } V_{DS} < V_P$$

$$I_D \sim I_{DSS} ( 1 + V_{GS} / V_P )^2 \quad \text{con } V_{DS} > V_P$$

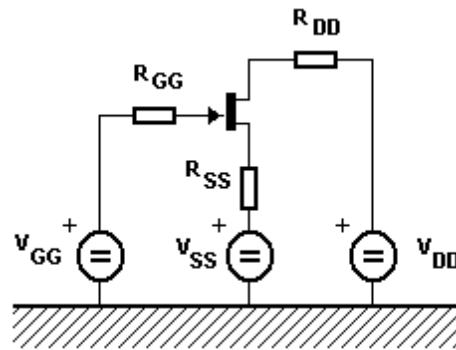
$$I_D = I_G + I_S \sim I_S \quad \text{siempre}$$

being  $V_P$  the denominated voltage of PINCH-OFF or "*strangulation of the channel*" defined in the curves of exit of the transistor, whose module agrees numerically with the voltage of cut in the curves of input of the transistor.

We can then find the variation of the current in the drain

$$\Delta I_D = (\partial I_D / \partial V_{DD}) \Delta V_{DD} + (\partial I_D / \partial V_{SS}) \Delta V_{SS} + (\partial I_D / \partial V_{GG}) \Delta V_{GG} +$$

$$+ (\partial I_D / \partial I_G) \Delta I_G + (\partial I_D / \partial V_{GS}) \Delta V_{GS}$$



of where

$$V_{GG} - V_{SS} = -I_G R_{GG} + V_{GS} + I_D R_{SS}$$

$$I_D = (V_{GG} - V_{SS} - V_{GS} + I_G R_{GG}) / R_{SS}$$

$$\partial I_D / \partial V_{GG} = -\partial I_D / \partial V_{SS} = 1 / R_{SS}$$

$$\partial I_D / \partial T = (\partial I_D / \partial V_{GS}) (\partial V_{GS} / \partial T) + (\partial I_D / \partial I_G) (\partial I_G / \partial T) =$$

$$= (-1/R_{SS}) (0,00012) + (0,7 \cdot I_{G0(25^\circ C)} \cdot 2^{\Delta T/10}) (R_{GG} / R_{SS})$$

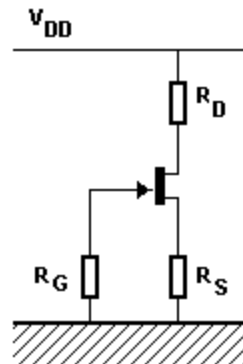
and finally

$$\Delta I_D = \{ [(0,7 \cdot I_{G0(25^\circ C)} \cdot 2^{\Delta T/10} R_{GG} - 0,00012)] \Delta T + \Delta V_{GG} - \Delta V_{SS} \} / R_{SS}$$

## Design

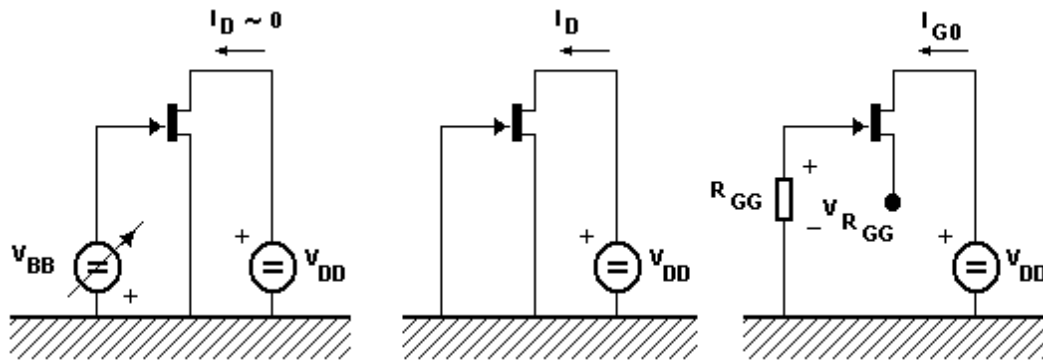
Be the data

$$I_D = \dots V_{DS} = \dots \Delta T = \dots \Delta I_{Dmax} = \dots R_D = \dots$$



From manual or the experimentation according to the graphs they are obtained

$$I_{DSS} = \dots I_{GB0(25^\circ C)} = \dots V_P = \dots$$



and therefore

$$R_S = V_P [1 - (I_D / I_{DSS})^{-1/2}] / I_D = \dots$$

$$R_G = \dots < [(R_S I_{Dmax} / \Delta T) + 0,00012] / 0,7 \cdot I_{G0(25^\circ C)} \cdot 2^{\Delta T/10}$$

$$V_{DD} = I_D (R_D + R_S) + V_{DS} = \dots$$

## Operational Amplifier of Voltage (AOV)

### Theory

Thus it is called by its multiple possibilities of analogical operations, differential to TBJ or JFET can be implemented with entrance, as also all manufacturer respects the following properties:

Power supply ( $2 \cdot V_{CC}$ ) between 18 y 36 [V]

Resistance of input differential ( $R_D$ ) greater than 100 [K $\Omega$ ]

Resistance of input of common way ( $R_C$ ) greater than 1 [M $\Omega$ ]

Resistance of output of common way ( $R_O$ ) minor of 200 [ $\Omega$ ]

Gain differential with output in common way ( $A_0$ ) greater than 1000 [veces]

We can nowadays suppose the following values:  $R_D = R_C = \infty$ ,  $R_O = 0$  (null by the future feedback) and  $A_0 = \infty$ . This last one will give, using it like linear amplifier, exits limited in the power supply  $V_{CC}$  and therefore voltages practically null differentials to input his.

On the other hand, the bad complementariness of the transistors brings problems. We know that voltage-current the direct characteristic of a diode can be considered like the one of a generator of voltage ; for that reason, the different transistors have a voltage differential of offset  $V_{OS}$  of some millivolts. For the TBJ inconvenient other is added; the currents of polarization to the bases are different ( $I_{1B}$  e  $I_{2B}$ ) and they produce with the external resistance also unequal voltages that are added  $V_{OS}$ ; we will call to its difference  $I_{OS}$  and typical the polarizing  $I_B$ .

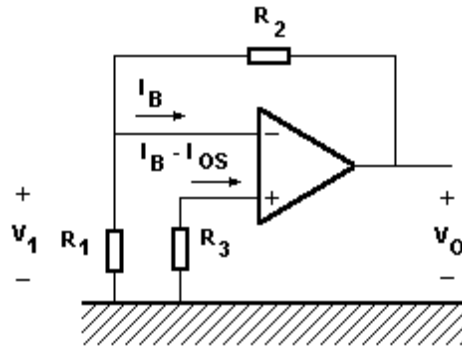
One adds to these problems other two that the manufacturer of the component specifies. They are they it variation of  $V_{OS}$  with respect to temperature  $\alpha_T$  and to the voltage of feeding  $\alpha_V$ .

If we added all these defects in a typical implementation

$$R_C = V_1 / I_B$$



$$V_1 = V_O \cdot (R_1 // R_C) / [R_2 + (R_1 // R_C)]$$



also

$$V_1 = V_{OS} - (I_B - I_{OS}) R_3$$

and therefore

$$V_1 = (V_{OS} - I_B R_2) / (1 + R_2 / R_1)$$

arriving finally at the following general expression for all offset

$$V_O = V_{OS} (1 + R_2 / R_1) + I_{OS} R_3 (1 + R_2 / R_1) + I_B [R_2 - R_3 (1 + R_2 / R_1)] + [\alpha_T \Delta T + \alpha_T \Delta V_{CC}] (1 + R_2 / R_1)$$

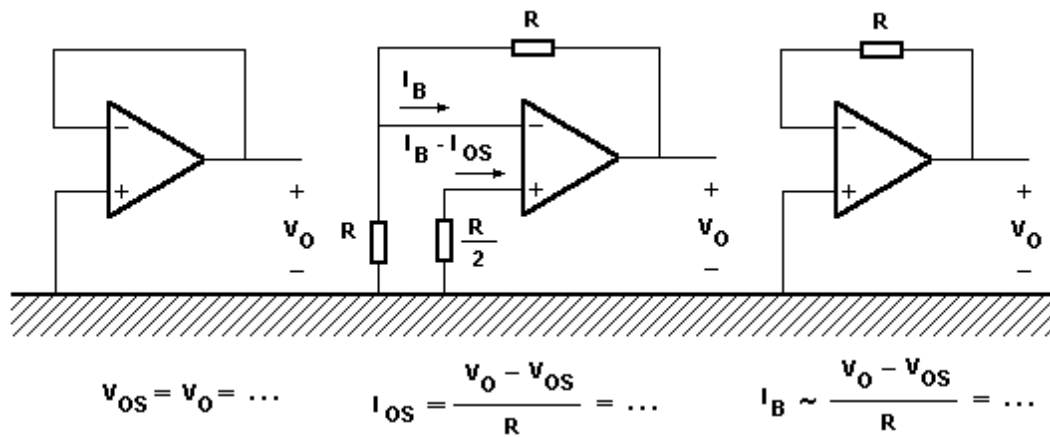
that it is simplified for the AOV with JFET

$$V_O = (V_{OS} + \alpha_T \Delta T + \alpha_T \Delta V_{CC}) (1 + R_2 / R_1)$$

and for the one of TBJ that is designed with  $R_3 = R_1 // R_2$

$$V_O = (V_{OS} + I_{OS} R_3 + \alpha_T \Delta T + \alpha_T \Delta V_{CC}) (1 + R_2 / R_1)$$

If we wanted to experience the values  $V_{OS}$  and  $I_{OS}$  we can use this general expression with the aid of the circuits that are

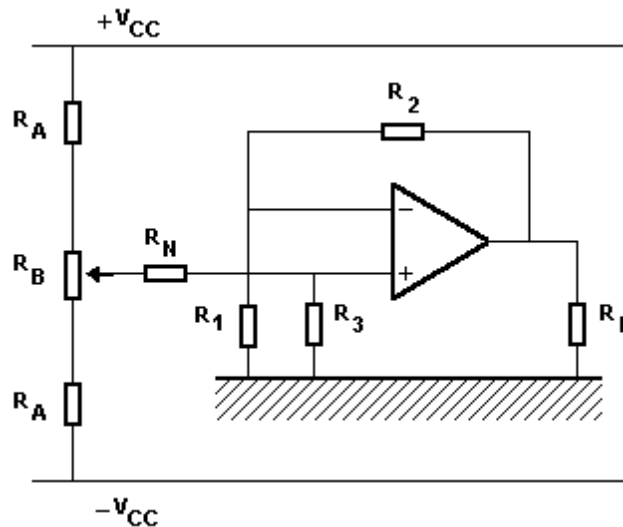


In order to annul the total effect of the offset, we can experimentally connect a pre-set to null voltage of output. This can be made as much in the inverter terminal as in the not-inverter. One advises in these cases, to project the resistives components in such a way that they do not load to the original circuit.

## Diseño

Be the data (with  $A = R_2/R_1$  the amplification or atenuation inverter)

$V_{OS} = \dots$   $I_{OS} = \dots$   $I_B = \dots$   $V_{CC} = \dots$   $A = \dots$   $P_{AOVmax} = \dots$  (normally 0,25 [W])



With the previous considerations we found

$$R_3 = \dots \gg V_{CC} / (2 I_B - I_{OS})$$

$$R_1 = (1 + 1/A) R_3 = \dots$$

$$R_2 = A R_1 = \dots$$

$$R_L = \dots \gg V_{CC}^2 / P_{AOVmax}$$

$$R_N = \dots \gg R_3$$

and with a margin of 50 % in the calculations

$$V_{RB} = 1,5 \cdot (2 R_N / R_3) \cdot (V_{OS} - I_B R_3) = \dots$$

$$V_{RB}^2 / 0,25 < R_B = \dots \ll R_N$$

$$2 R_A = (2 V_{CC} - V_{RB}) / (V_{RB} / R_B) \Rightarrow R_A = R_B [ (V_{CC} / V_{RB}) - 0,5 ] = \dots$$

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## Cap. 03    Dissipators of heat

### General characteristics

#### Continuous regime

#### Design

---

### General characteristics

All semiconductor component tolerates a temperature in its permissible junction  $T_{JADM}$  and power  $P_{ADM}$ . We called thermal impedance  $Z_{JC}$  to that it exists between this point and its capsule, by a thermal resistance  $\theta_{JC}$  and a capacitance  $C_{JC}$  also thermal.

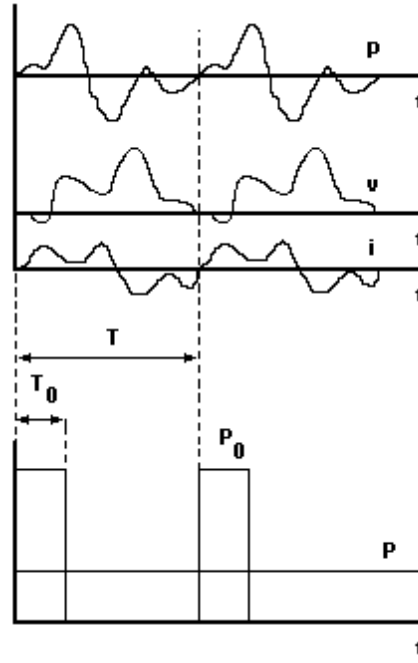
When an instantaneous current circulates around the component «i» and between its terminals there is an instantaneous voltage also «v», we will have then an instantaneous power given like his product « $p = i.v$ », and another average that we denominated simply  $P$  and that is constant throughout all period of change  $T$

$$P = p_{med} = T^{-1} \cdot \int_0^T p \, dt = T^{-1} \cdot \int_0^T i.v \, dt$$

and it can be actually of analytical or geometric way.

Also, this constant  $P$ , can be thought as it shows the following figure in intervals of duration  $T_0$ , and that will be obtained from the following expression

$$T_0 = P_0 / P$$



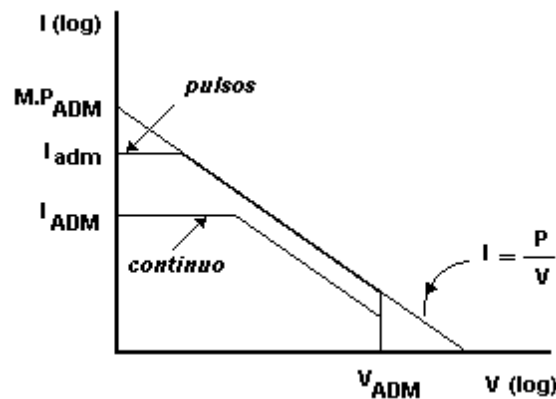
To consider a power repetitive is to remember a harmonic analysis of voltage and current. Therefore, the thermal impedance of the component will have to release this active internal heat

$$P_{ADM} = (T_{JADM} - T_A) / |Z_{JC}| \cos \phi_{JC} = P_{ADM} \theta_{JC} / |Z_{JC}| \cos \phi_{JC}$$

with  $T_A$  the ambient temperature. For the worse case

$$P_{ADM} = P_{ADM} \theta_{JC} / |Z_{JC}| = P_{ADM} \cdot M$$

being M a factor that the manufacturer specifies sometimes according to the following graph



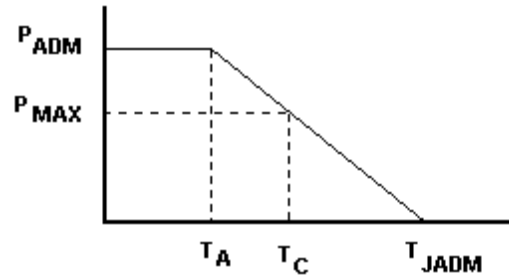
### Continuous regime

When the power is not repetitive, the equations are simplified then the following thing

$$P_{ADM} = (T_{JADM} - T_A) / \theta_{JC}$$

and for a capsule to a temperature greater than the one of the ambient

$$P_{MAX} = (T_{JADM} - T_C) / \theta_{JC}$$



On the other hand, the thermal resistance between the capsule and ambient  $\theta_{CA}$  will be the sum  $\theta_{CD}$  (capsule to the dissipator) plus the  $\theta_{DA}$  (dissipator to the ambient by thermal contacts of compression by the screws). Thus it is finally

$$\theta_{CA} = \theta_{CD} + \theta_{DA}$$

$$\theta_{CA} = (T_C - T_A) / P_{MAX} = (T_C - T_A) (T_{JADM} - T_A) / P_{ADM} (T_{JADM} - T_C)$$

## Design

Be the data

$$P = \dots \quad T_A = \dots \quad (\sim 25 [^{\circ}\text{C}])$$

we obtain from the manual of the component

$$P_{ADM} = \dots \quad T_{JADM} = \dots \quad (\sim 100 [^{\circ}\text{C}] \text{ para el silicio})$$

and we calculated

$$\theta_{JC} = (T_{JADM} - T_A) / P_{ADM} = \dots$$

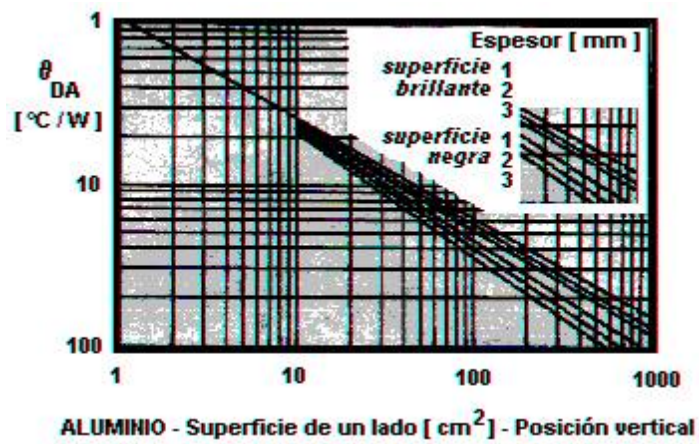
being able to adopt the temperature to that it will be the junction, and there to calculate the size of the dissipator

$$T_J = \dots < T_{JADM}$$

and with it (it can be considered  $\theta_{DA} \sim 1 [^{\circ}\text{C}/\text{W}]$ )

$$\theta_{DA} = \theta_{CA} - \theta_{JC} = \{ [(T_J - T_A) / P] - \theta_{JC} \} - 1 = \dots$$

and with the aid of the abacus following or other, to acquire the dimensions of the dissipator





## Chap. 04 Inductors of small value

[Generalities](#)

[Q- meter](#)

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[Oneloop](#)

[Solenoidal onelayer](#)

[Toroidal onelayer](#)

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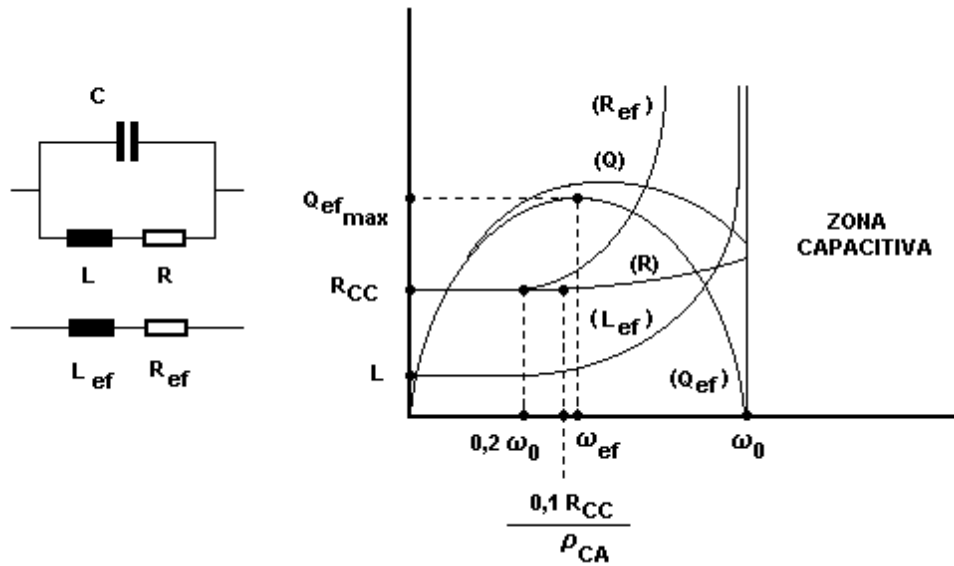
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### [Generalities](#)

We differentiated the terminology resistance, inductance and capacitance, of those of resistor, inductor and capacitor. Second they indicate imperfections given by the combination of first.

The equivalent circuit for an inductor in general is the one of the following figure, where resistance  $R$  is practically the ohmic one of the wire to DC  $R_{CC}$  added to that one takes place by effect to skin  $\rho_{CA} \cdot \omega^2$ , not deigning the one that of losses of heat by the ferromagnetic nucleus; capacitance  $C$  will be it by addition of the loops; and finally inductance  $L$  by geometry and nucleus.

This assembly will determine an inductor in the rank of frequencies until  $\omega_0$  given by effective the  $L_{ef}$  and  $R_{ef}$  until certain frequency of self-oscillation  $\omega_0$  and where one will behave like a condenser.



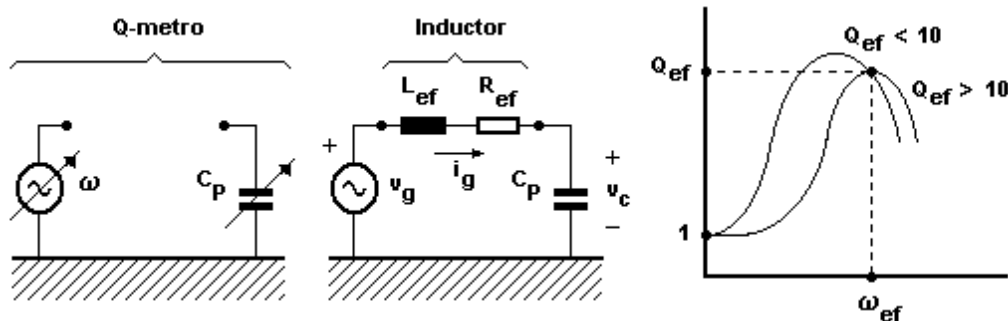
The graphs say

$$\begin{aligned}
 Z &= (R + sL) // (1/sC) = R_{ef} + sL_{ef} \\
 R_{ef} &= R / [(1 - \gamma)^2 + (\omega RC)^2] \sim R / (1 - \gamma)^2 \\
 L_{ef} &= [L(1 - \gamma) - R^2C] / [(1 - \gamma)^2 + (\omega RC)^2] \sim L / (1 - \gamma) \\
 \gamma &= (\omega / \omega_0)^2 \\
 \omega_0 &= (LC)^{-1/2} \\
 Q &= \omega L / R = L(\rho_{CA}\omega^2 + R_{CC}/\omega) \\
 Q_{ef} &= \omega L_{ef} / R_{ef} = Q(1 - \gamma)
 \end{aligned}$$

### Q- meter

In order to measure the components of the inductor the use of the Q-meter is common. This factor of reactive merit is the relation between the powers reactive and activates of the device, and for syntonies series or parallel its magnitude agrees with the overcurrent or overvoltage, respectively, in its resistive component.

In the following figure is its basic implementation where the  $V_g$  amplitude is always the same one for any frequency, and where also the frequency will be able to be read, to the capacitance pattern  $C_p$  and the factor of effective merit  $Q_{ef}$  (obtained of the overvalue by the voltage ratio between the one of capacitor  $C_p$  and the one of the generator  $v_g$ ).



The measurement method is based on which generally the measured  $Q_{ef}$  to one  $\omega_{ef}$  anyone is always very great :  $Q_{ef} \gg 1$ , and therefore in these conditions one is fulfilled

$$V_C = I_{gmax} / \omega_{ef} C_P = V_g / R_{ef} \omega_{ef} C_P = V_g / Q_{efmax}$$

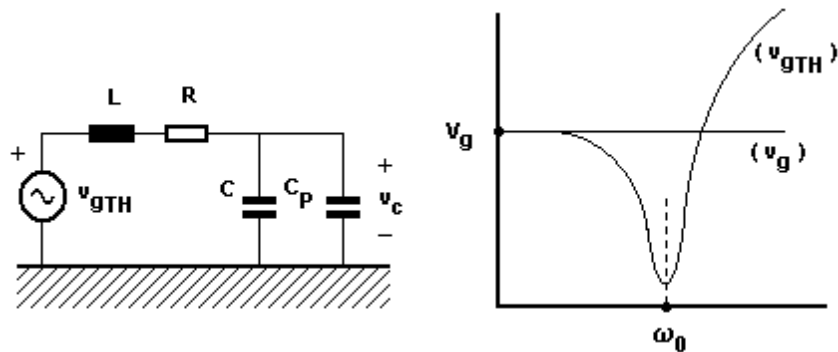
and if we applied Thevenin

$$V_{gTH} = V_g (R + sL) / (R + sL) // (1/sC) = K (s^2 + s. 2 \xi \omega_0 + \omega_0^2)$$

$$K = V_g L C$$

$$\omega_0 = (LC)^{-1/2}$$

$$\xi = R / 2 (L / C)^{1/2}$$



that not to affect the calculations one will be due to work far from the capacitive zone (or resonant), it is to say with the condition

$$\omega \ll \omega_0$$

then, varying  $\omega$  and  $C_P$  we arrived at a resonance anyone detecting a maximum  $V_C$

$$\omega_{ef1} = [L (C + C_{p1})]^{-1/2} = \dots$$

$$C_{p1} = \dots$$

$$Q_{ef1max} = \dots$$

and if we repeated for  $n$  times ( $n > 1$ )

$$\omega_{ef2} = n \omega_{ef1} = [L (C + C_{p2})]^{-1/2} = \dots$$

$$C_{p2} = \dots$$

$$Q_{ef2max} = \dots$$

we will be able then to find

$$C = (n^2 C_{p2} - C_{p1}) (1 - n^2)^{-1} = \dots$$

$$L = [\omega_{ef1}^2 (C + C_{p1})]^{-1} = \dots$$

and now

$$\begin{aligned} L_{ef1} &= (1 - \omega_{ef1}^2 L C)^{-1} = \dots \\ L_{ef2} &= (1 - \omega_{ef2}^2 L C)^{-1} = \dots \\ R_{ef1} &= \omega_{ef1} L_{ef1} / Q_{ef1max} = \dots \\ R_{ef2} &= \omega_{ef2} L_{ef2} / Q_{ef2max} = \dots \end{aligned}$$

and as it is

$$R = R_{CC} + \rho_{CA} \omega^2 = R_{ef} (1 - \omega^2 L C)^2$$

finally

$$\begin{aligned} \rho_{CA} &= [R_{ef1} (1 - \omega_{ef1}^2 L C)^2 - R_{ef2} (1 - \omega_{ef2}^2 L C)^2] / \omega_{ef1}^2 (1 - n^2) = \dots \\ R_{CC} &= R_{ef1} (1 - \omega_{ef1}^2 L C)^2 - \rho_{CA} \omega_{ef1}^2 = \dots \end{aligned}$$

## Design of inductors

### Oneloop

Be the data

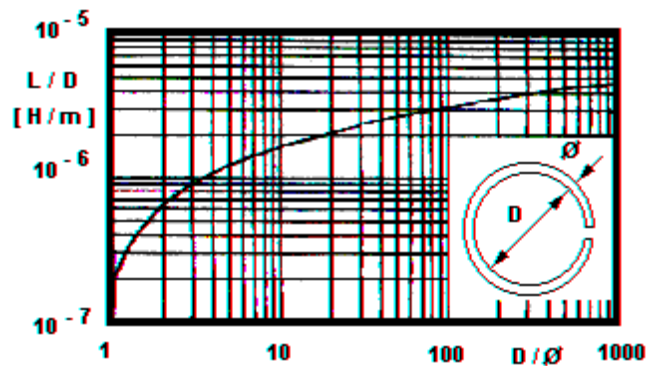
$$L = \dots$$

We adopted a diameter of the inductor

$$D = \dots$$

and from the abacus we obtain his wire

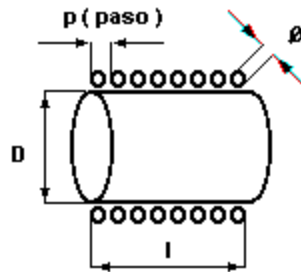
$$\varnothing = (\varnothing/D) D = \dots$$



### Solenoidal onelayer

Be the data

$$L_{ef} = L = \dots f_{max} = \dots f_{min} = \dots Q_{efmin} = \dots$$



We adopted a format of the inductor

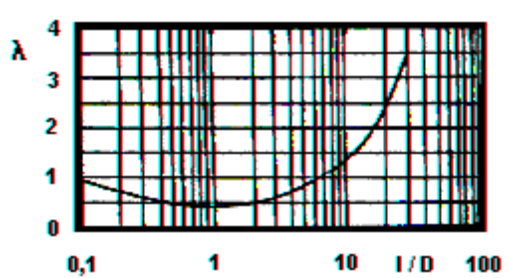
$$0,3 < (l/D) = \dots < 4$$

$$D = \dots$$

$$l = (l/D) D = \dots$$

and from the abacus we obtain distributed capacitance C

$$\lambda = 10^6 C / D = \dots$$



if now we remember the explained thing previously

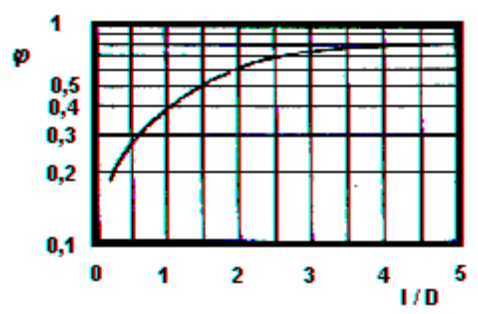
$$\omega_{max} < 0,2 \omega_0 = 0,2 (LC)^{-1/2}$$

we can to verify the inductive zone

$$10^{-3} / \lambda L f_{max}^2 = \dots > D$$

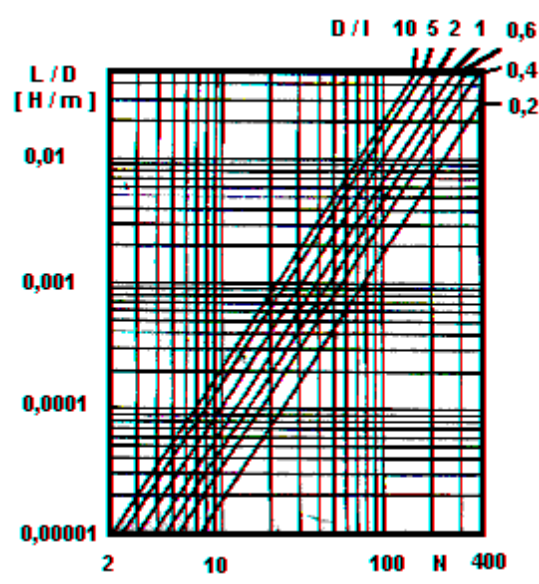
and the reactive factor

$$7,5 \cdot D \cdot \varphi \cdot f_{min}^{1/2} = \dots > Q_{efmin}$$



From the equation of Wheeler expressed in the abacus, is the amount of together loops ( $\text{Ø/paso} \sim 1$ , it is to say enameled wire)

$$N = \dots$$



and of there the wire

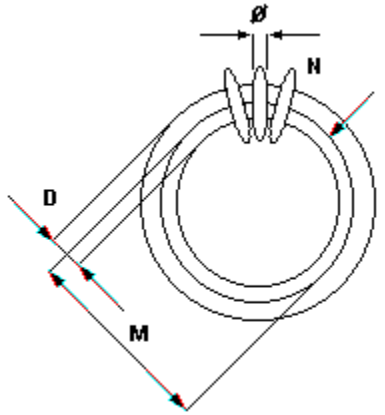
$$\text{Ø} = (\text{Ø/paso}) l / N \sim l / N = \dots$$

This design has been made for  $\omega_{\max} < 0,2 \omega_0$ , but it can be modified for greater values of frequency, with the exception of which the equation of the  $Q_{ef}$  would not be fulfilled satisfactorily.

### Toroidal onelayer

Be tha data

$$L = \dots$$



We adopted a format of the inductor

$$M = \dots$$

$$D = \dots$$

being for together loops ( $\text{Ø}/\text{paso} \sim 1$ , it is to say enameled wire)

$$I \sim \pi M = \dots$$

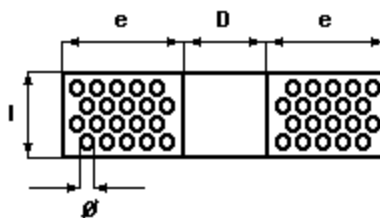
$$N = 1260 \cdot \{ L / [ M - (M^2 - D^2)^{1/2} ]^{-1} \}^{1/2} = \dots$$

$$\text{Ø} \sim \pi M / N = \dots$$

### Solenoidal multilayer

Be tha data

$$L = \dots$$



We adopted a format of the inductor

$$D = \dots > I = \dots$$

$$0,1 \cdot l < e = \dots < 5 \cdot l$$

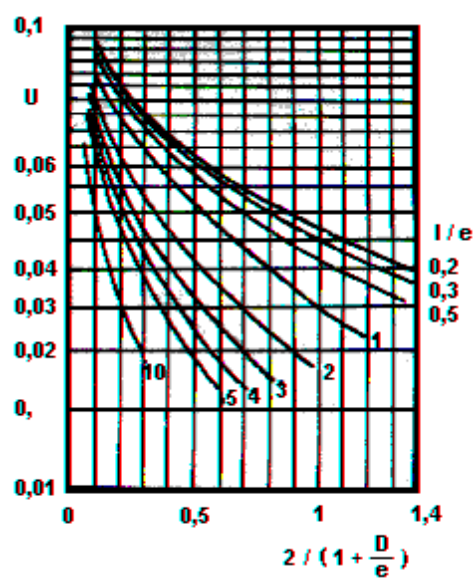
and of the abacus

$$U = \dots$$

$$N = 225 \cdot [ L / (D + e) U ]^{1/2} = \dots$$

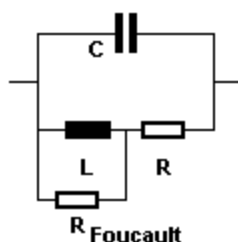
$$\text{Ø} \sim (e \cdot l / 4 \cdot N)^{1/2} = \dots$$





### Design of inductors with nucleus of ferrite

To all the inductors with nucleus of air when introducing to them ferrite its  $L_{ef}$  increases, but its  $Q_{ef}$  will diminish by the losses of Foucault.



Thus, for all the seen cases, when putting to them a magnetic nucleus the final value is

$$L_{FINAL} = \mu_{ref} \cdot L$$

$$\mu_{ref} > 1$$

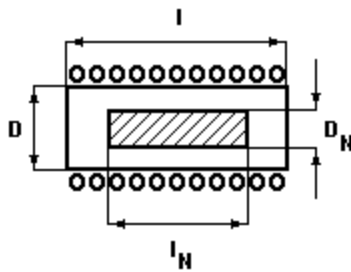
where  $\mu_{ref}$  is *permeability relative effective* (or *toroidal permeability*, that for the air it is  $\mu_{ref} = 1$ ) that it changes with the position of the nucleus within the coil, like also with the material implemented in its manufacture.

We said that commonly to  $\mu_{ref}$  is specified it in the leaves of data like *toroidal permeability*. This is thus because in geometry *toro* the nucleus is not run nor has air.

In most of the designs, due to the great variety of existing ferrite materials and of which it is not had catalogues, it is the most usual experimentation to obtain its characteristics. For this the inductance is measurement with and without nucleus, and  $\mu_{ref}$  of the previous equation is obtained.

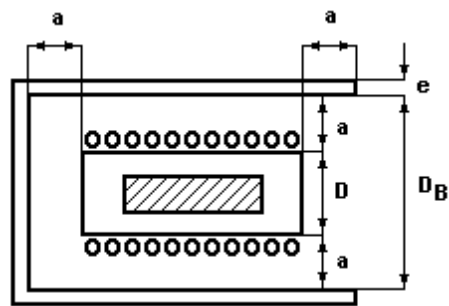
It can resort to the following approximated equation to obtain the final inductance

$$\mu_{efFINAL} \sim \mu_{ref} \cdot (D_N/D)^2 (I_N/I)^{1/3}$$



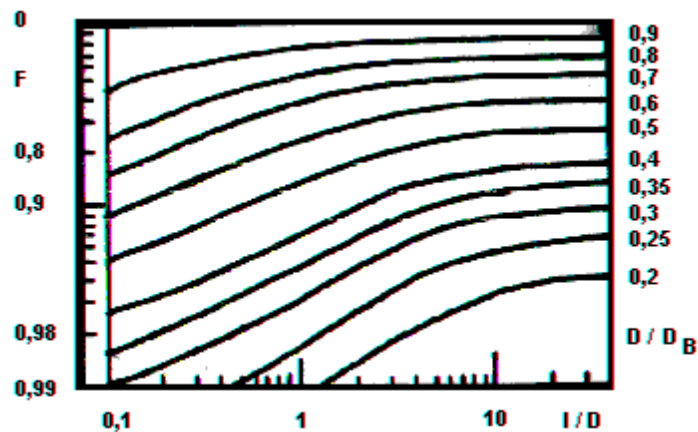
### Shield to solenoidal multilayer inductors

When a shield to an inductance with or without ferrite, they will appear second losses by Foucault due to the undesirable currents that will circulate around the body of this shield — electrically it is equivalent this to another resistance in parallel.



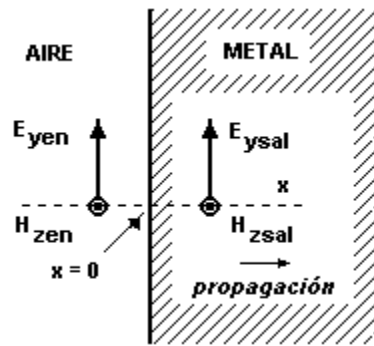
For the case that we are seeing the final *total inductance* will be given by

$$L_{\text{FINALtotal}} = F \cdot L_{\text{FINAL}} = F \cdot \mu_{\text{ref}} \cdot L$$



In order to adopt the thickness of the shield present is due to have the frequency of work and, therefore, the *penetration*  $\delta$  that it has the external electromagnetic radiation. In order to find this value we reasoned of the way that follows. We suppose that the wave front has the polarized form of its electric field

$$E_{\text{yen}} = E_{\text{pico}} e^{j(\omega t - \beta x)}$$



and considering two of the equations of Maxwell in the vacuum (~ air)

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

we obtain

$$-\frac{\partial H_{zsal}}{\partial x} = \sigma E_{ysal} + \epsilon \frac{\partial E_{ysal}}{\partial t}$$

$$\frac{\partial E_{ysal}}{\partial x} = -\mu \frac{\partial H_{zsal}}{\partial t}$$

and therefore

$$\frac{\partial}{\partial t} \left( \frac{\partial H_{zsal}}{\partial x} \right) / \frac{\partial x} = -\sigma \frac{\partial E_{ysal}}{\partial t} + \epsilon \frac{\partial^2 E_{ysal}}{\partial t^2} = -\mu^{-1} \frac{\partial^2 E_{ysal}}{\partial x^2}$$

$$\frac{\partial^2 E_{ysal}}{\partial x^2} - \gamma^2 E_{ysal} = 0$$

being

$$\gamma = [\mu \omega (j\sigma - \omega\epsilon)]^{1/2} \sim (j\sigma\omega\mu)^{1/2} = (1+j)(\sigma\omega\mu/2)^{1/2}$$

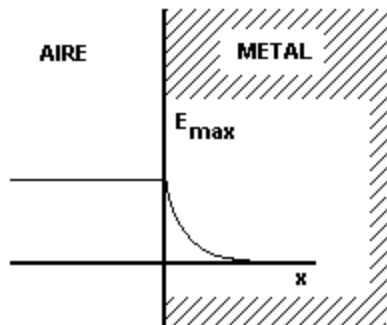
$\sigma$  = conductivity

$\mu = \mu_0 \mu_r$  = magnetic permeability (of the air X the relative one of the material to the air)

$\epsilon = \epsilon_0 \epsilon_r$  = electric impermeability (of the air X the relative one of the material to the air)

and it determines the following equation that satisfies to the wave

$$E_{ysal} = E_{ysalpico(0)} e^{-\gamma x} = E_{yenpico(0)} e^{-\gamma x} = E_{yenpico(0)} e^{x(\sigma\omega\mu/2)^{1/2}} e^{jx(\sigma\omega\mu/2)^{1/2}}$$



Next, without considering the introduced phase

$$\int_0^\infty E_{ysal} \partial x = E_{yenpico(0)} / \gamma$$

$$\int_0^{1/\gamma} E_{ysal} \partial x \sim 0,63 E_{yenpico(0)} / \gamma$$

and as 63 % are a reasonable percentage, he is usual to define to the *penetration*  $\delta$  like this magnitude (to remember that to 98 % they are  $\sim 3\delta$ ) where one assumes concentrated the interferente energy

$$\delta = (2 / \sigma \omega \mu)^{1/2}$$

being typical values for copper and aluminum

$$\delta_{Cu} = 6600 (f)^{1/2}$$

$$\delta_{Al} = 8300 (f)^{1/2}$$

## Design

Be the data

$f = \dots$  (or better the minimum value of work)

$$L_{FINALtotal} = \dots \quad L_{FINAL} = \dots \quad l = \dots \quad D = \dots$$

therefore of the abacus

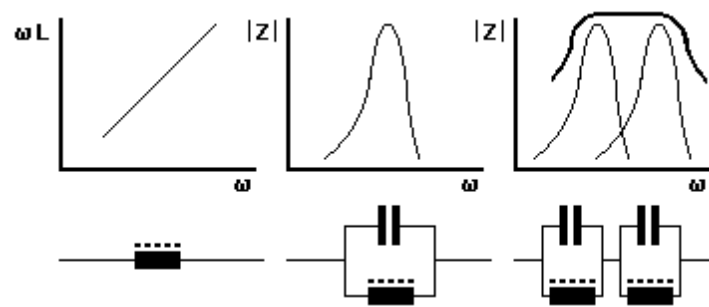
$$D_B = (D_B/D) \cdot D = \dots$$

and if it is adopted, for example aluminum, we obtain necessary the minimum thickness

$$e = \dots > 8300 / (f)^{1/2}$$

## Choke coil of radio frequency

The intuctors thus designed offer a great inductive reactance with respect to the rest of the circuit. Also usually they make like syntonies taking advantage of the own distributed capacitance, although at the moment it has been let implement this position. In the following figures are these three possible effects.



## Chap. 05 Transformers of small value

### Generalities

#### Design of transformers

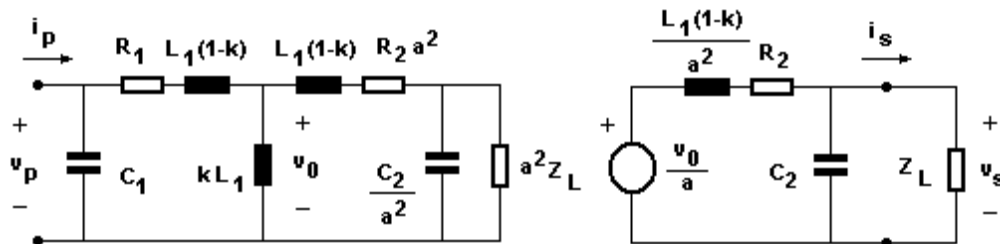
#### Solenoidal onelayer

#### Solenoidal multilayer

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### Generalities

First we see the equivalent circuit of a small transformer, where the capacitance between both windings it is not important

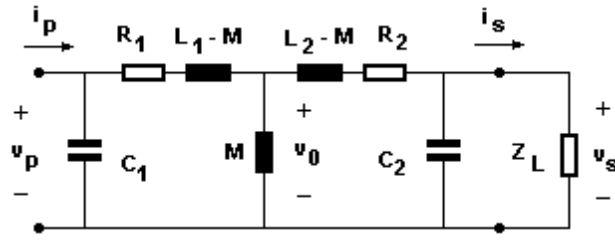


The number «a» denominates *transformation relation* and is also equivalent to call it as *effective relation of loops*. The «k» is the *coefficient of coupling* between the windings primary and secondary, that is a constant magnitude with the frequency because it depends on the geometric conditions of the device. The inductance in derivation  $kL_1$  is the magnetic coil. Generally this circuit for the analysis is not used since he is complex, but that considers it according to the rank of work frequencies. Thus, we can distinguish three types of transformers, that is to say:

- radiofrequency ( $k < 1$ )
  - nucleus of air ( $k \ll 1$ )
  - nucleus of ferrite ( $k < 1$ )
- audiofrequency ( $k \sim 1$ )
- line ( $k = 1$ )

In this chapter we will analyze that of radiofrequencies. We will see as this one is come off the previous studies. The continuous aislación of simplifying has been omitted —if he were necessary

this, could think to it connected it to a second ideal transformer of relation 1:1.



This model of circuit is from the analysis of the transformer

$$\begin{cases} v_p = i_p Z_{11} + i_s Z_{12} \\ v_s = i_p Z_{21} + i_s Z_{22} \end{cases}$$

$$\begin{aligned} Z_{11}(i_s=0) &= v_p / i_p = sL_1 \\ Z_{22}(i_p=0) &= v_s / i_s = -sL_2 \\ Z_{21}(i_s=0) &= v_s / i_p = sM \\ Z_{12}(i_p=0) &= v_p / i_s = -sM \end{aligned}$$

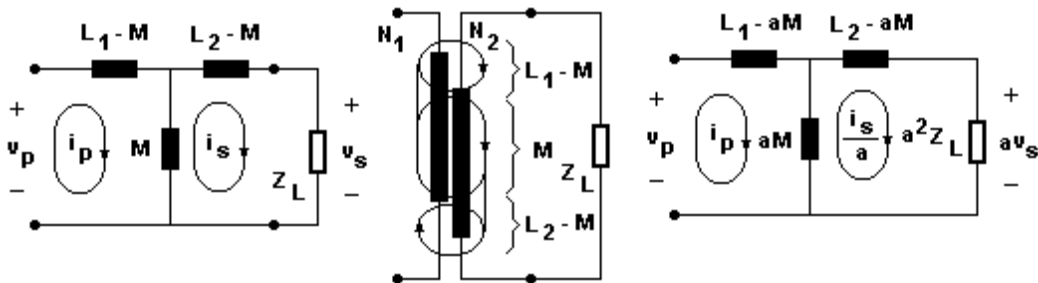
$$M = k (L_1 L_2)^{1/2}$$

where the negative signs are of the convention of the salient sense of the current  $i_s$ . Then

$$\begin{cases} v_p = i_p sL_1 - i_s sM = i_p s(L_1 - M) + (i_p - i_s) sM \\ v_s = i_p sM - i_s sL_2 = (i_p - i_s) sM - i_s s(L_2 - M) = i_s Z_L \end{cases}$$

equations that show the following circuit of meshes that, if we want to reflect it to the primary one, then they modify the previous operations for a transformation operator that we denominate «a»

$$\begin{cases} v_p = i_p s(L_1 - aM) + (i_p - i_s/a) saM \\ av_s = (i_p - i_s/a) saM - (i_s/a) \cdot s(a^2 L_2 - aM) = (i_s/a) \cdot a^2 Z_L \end{cases}$$



and of where

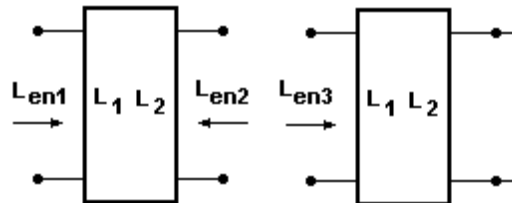
$$\begin{aligned}
L_1 &= N_1^2 S_1 \mu_{ef1} / l_1 \\
L_2 &= N_2^2 S_2 \mu_{ef2} / l_2 \\
L_1 / L_2 &= a^2 \\
a &= n ( N_1^2 S_1 \mu_{ef1} l_1 / N_2^2 S_2 \mu_{ef2} l_1 )^{1/2} \\
n &= N_1 / N_2
\end{aligned}$$

and consequently

$$\begin{aligned}
L_1 - aM &= L_1 (1 - k) \\
L_2 - aM &= L_1 (1 - k) \\
aM &= L_1 k
\end{aligned}$$

A quick form to obtain the components could be, among other, opening up and shortcircuiting the transformer

$$\begin{aligned}
1^0) \quad Z_L &= \infty \\
L_{en1} &= (L_1 - M) + M = \dots \\
L_{en2} &= (L_2 - M) + M = \dots \\
2^0) \quad Z_L &= 0 \\
L_{en3} &= (L_1 - M) + [M // (L_2 - M)] = L_1 - M^2 / L_2 = \dots \\
3^0) \quad L_1 &= L_{en1} = \dots \\
L_2 &= L_{en2} = \dots \\
k &= (1 - L_{en3} / L_{en1})^{1/2} = \dots \\
M &= [(L_{en1} - L_{en3}) L_{en2}]^{1/2} = \dots
\end{aligned}$$



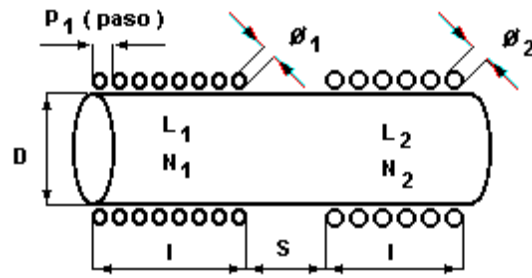
## Design of transformers

### Solenoidal onelayer

Be the data

$$k = \dots$$





We calculate the inductances of the primary and secondary as it has been seen in the chapter of design of *solenoids onelayer*

$$L_1 = \dots$$

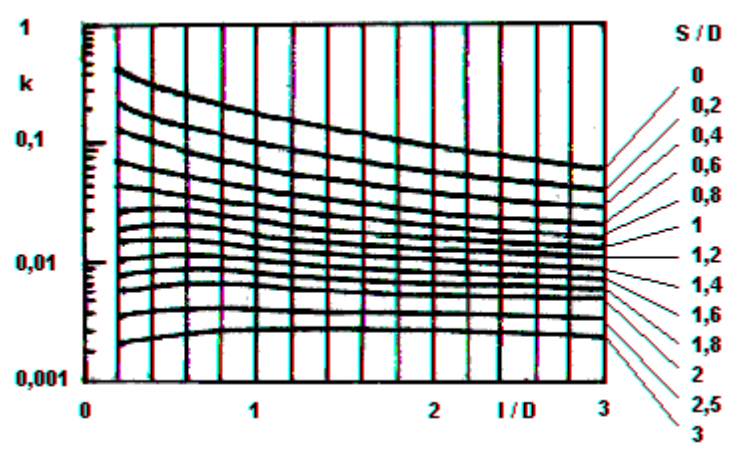
$$L_2 = \dots$$

$$I = \dots$$

$$D = \dots$$

and now here of the abacus

$$S = (S/D) D = \dots$$



### Solenoidal multilayer

Be the data

$$k = \dots$$

We calculate the inductances of the primary and secondary as it has been seen in the chapter of design of *inductors solenoidales multilayer*

$$L_1 = \dots$$

$$L_2 = \dots$$

$$N_2 = \dots$$

## Chap. 06 Inductors and Transformadores of great value

[Equivalent circuit of a transformer](#)

[Equivalent circuit of an inductor](#)

[Measurement of the characteristics](#)

[Transformer of alimentation](#)

Design

[Transformer of audiofrequency](#)

[Transformer of pulses](#)

Design

[Inductors of filter with continuous component](#)

Diseño

[Inductors of filter without continuous component](#)

Design

[Autotransformer](#)

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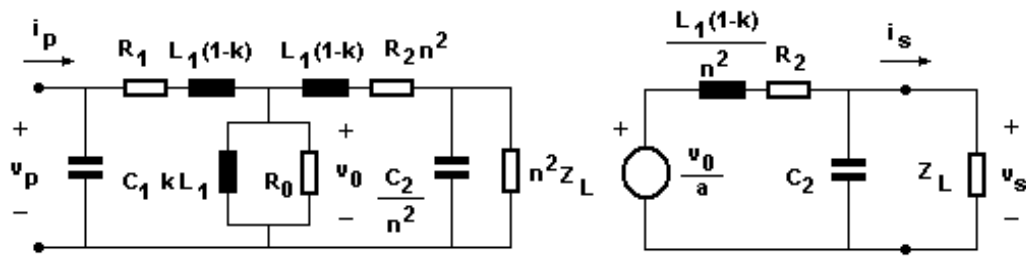
### [Equivalent circuit of a transformer](#)

It has been spoken in the chapter that deals with *transformer of small value* on the equivalent circuit, and that now we reproduce for low frequencies and enlarging it

$a = n = N_1 / N_2$	<i>relation of transformation or turns</i>
$nM$	<i>magnetic inductance</i>
$M = k (L_1 L_2)^{1/2}$	<i>mutual inductance between primary and secondary</i>
$k \sim 1$	<i>coupling coefficient</i>
$L_1$	<i>inductance of the winding of the primary (secondary open)</i>
$L_2$	<i>inductance of the winding of the secondary (primary open)</i>
$L_1 (1-k)$	<i>inductance of dispersion of the primary</i>
$L_2 (1-k) / n^2$	<i>inductance of reflected dispersion of the secondary</i>
$R_1$	<i>resistance of the copper of the wire of the primary</i>
$R_2$	<i>resistance of the copper of the wire of the secondary</i>
$R_0$	<i>resistance of losses for Foucault and hysteresis</i>
$C_1$	<i>distributed capacitance of the winding of the primary</i>
$C_2$	<i>distributed capacitance of the winding of the secondary</i>

$Z_L$

load impedance



and their geometric components

$S$

section of the nucleus

$l_A$

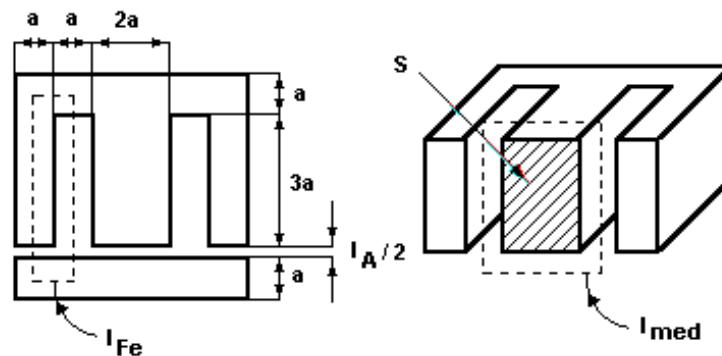
longitude of the air

$l_{Fe}$

longitude of the iron

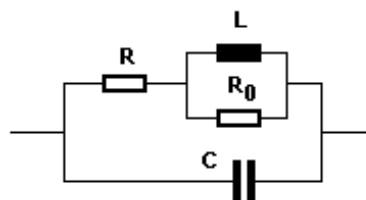
$l_{med}$

longitude of the half spire



### Equivalent circuit of a inductor

If to the previous circuit we don't put him load, we will have the circuit of an inductor anyone with magnetic nucleus. The figure following sample their simplification



where  $L = L_1$ ,  $R = R_1$  and  $C = C_1$ .

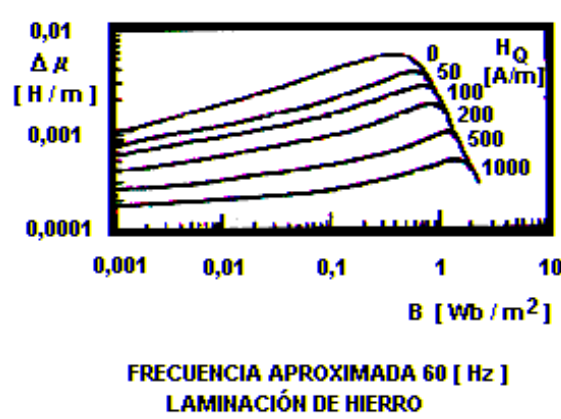
It is of supreme importance to know that the value of the inductance varies with the continuous current (or in its defect with the average value of a pulses) of polarization. This is because the variation of the permeability, denominated *incremental permeability*  $\Delta\mu$ , changes according to the work point in the hysteresis curve. If we call as *effective* their value  $\Delta\mu_{ef}$ , for a

section of the nucleus  $S$  and a longitude of the magnetic circuit  $l_{Fe}$  (remember you that the total one will consider the worthless of the air  $l_a$ ), we will have that

$$L = \Delta\mu_{ef} \cdot N^2 S / l_{Fe}$$

$$\Delta\mu_{ef} = \mu_{ef} \quad \text{sin polarización}$$

Now we see an abacus that shows their magnitude for intertwined foils and 60 [Hz] (also for 50 [Hz] without more inconveniences)

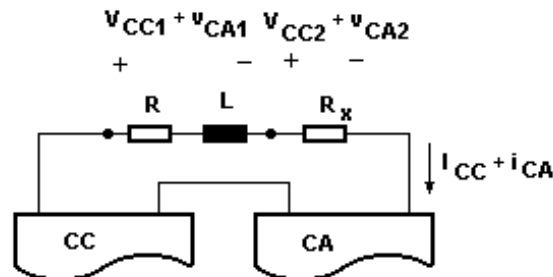


### Measurement of the characteristics

Subsequently we will see a way to measure  $L$ ,  $\Delta\mu_{ef}$  y  $\mu_{ef}$ .

With the help of a power supply DC and a transformer CA the circuit that is shown, where they are injected to the inductor alternating and continuous polarized limited by a resistance experimental  $R_x$ . Then we write down the data obtained in continuous and effective

$$V_{CC1} = \dots \quad V_{CC2} = \dots \quad V_{CA1} = \dots \quad V_{CA2} = \dots$$



and we determine

$$R = V_{CC1} / I_{CC} = V_{CC1} R_x / V_{CC2} = \dots$$

$$L = \omega^{-1} (|Z|^2 - R^2)^{1/2} = \omega^{-1} [(V_{CA1} R_x / V_{CA2})^2 - R^2]^{1/2} = \dots$$

If we measure the dimensions of the inductor (or transformer) we also obtain for the previous equation the permeability effective dynamics

$$\Delta\mu_{ef} = L I_{Fe} / N^2 S = \dots$$

and that effective one without polarization (we disconnect the source of power DC) repeating the operation

$$V_{CA1} = \dots \quad V_{CA2} = \dots \quad I_{CC} = 0 \quad R = \dots \text{ (with a ohmeter)}$$

and with it

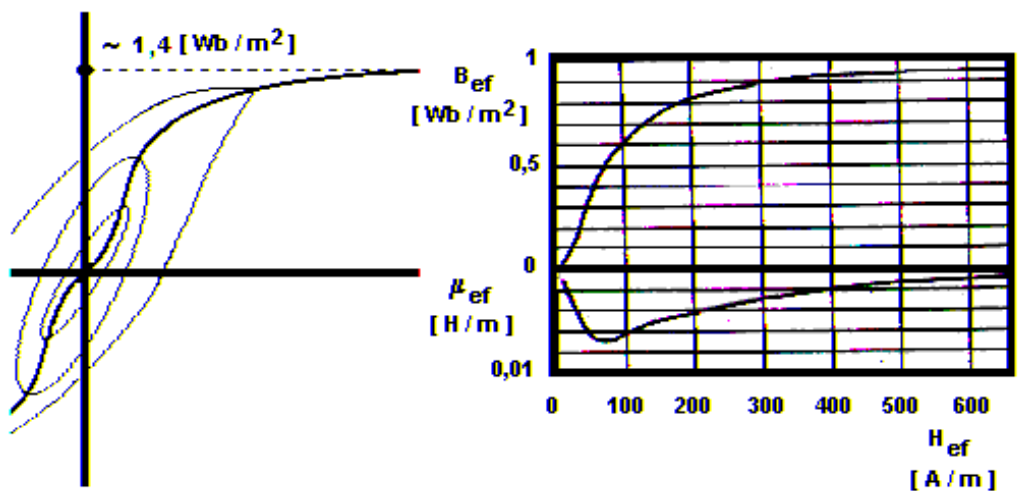
$$L = \omega^{-1} ( |Z|^2 - R^2 )^{1/2} = \omega^{-1} [ ( V_{CA1} R_x / V_{CA2} )^2 - R^2 ]^{1/2} = \dots$$

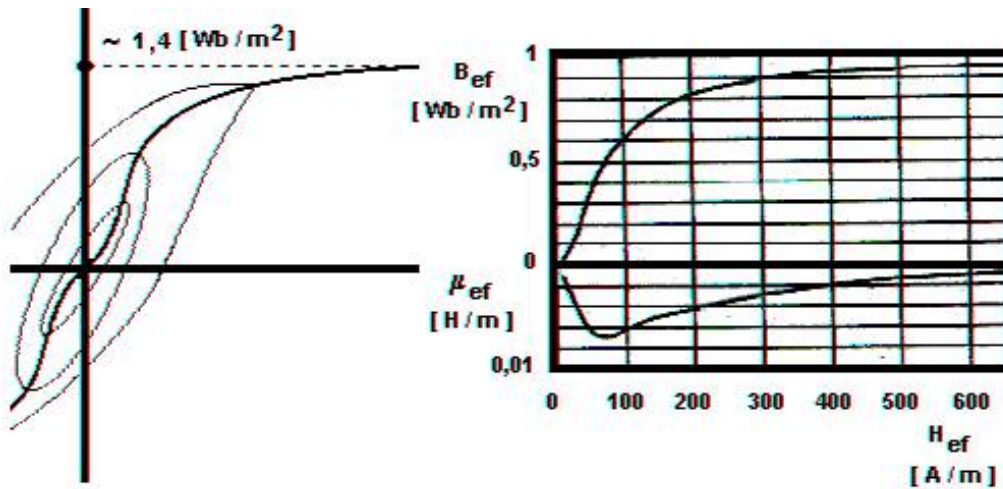
$$\mu_{ef} = L I_{Fe} / N^2 S = \dots$$

### Transformer of alimentention

For projects of up to 500 [VA] can reject at  $R_0$  in front of the magnetic nM and, like one works in frequencies of line of 50 or 60 [Hz], that is to say low, it is also possible to simplify the undesirable capacitances of the windings  $C_1$  and  $C_2$  because they will present high reactances.

As it is known, the characteristic of hysteresis of a magnetic material is asymmetric as it is shown approximately in the following figures. The same one, but not of magnitudes continuous DC but you alternate CA it will coincide with the one denominated *curve of normal magnetization*, since between the value pick and the effective one the value in way 0,707 only exists.





For this transformer this way considered it is desirable whenever it transmits a sine wave the purest thing possible. This determines to attack to the nucleus by means of an induction  $B$  sine wave although the magnetic current for the winding is not it; besides this the saturation magnitude will be the limit of the applied voltage.

In other terms, when applying an entrance of voltage in the primary one it will be, practically, the same one that will appear in the magnetic inductance because we reject the dispersion and fall in the primary winding. This way

$$v_1 \sim v_0 = V_{0\text{pico}} \cos \omega t$$

therefore

$$B = \phi / S = (N_1^{-1} \int v_0 dt) / S = V_{0\text{pico}} (\omega S N_1)^{-1} \sin \omega t = B_{\text{pico}} \sin \omega t$$

$$v_0 / n = N_2 \cdot \partial \phi / \partial t = N_2 \cdot \partial B S / \partial t = B_{\text{pico}} \omega S N_1 \cos \omega t = V_{0\text{pico}} n^{-1} \cos \omega t$$

where the lineal dependence of input can be observed to output, that is to say, without the permeability is in the equations.

Subsequently obtain the law of Hopkinson. She tells us that for a magnetic circuit as the one that are studying, that is to say where the section  $S_{Fe}$  of the iron is practically the same one that that of the air  $S_A$  (remembers you that this last one is considerably bigger for the dispersion of the lines of force), it is completed for a current «i» circulating instantaneous that

$$N_1 i = H_{Fe} l_{Fe} + H_A l_A = B (l_{Fe} / \mu_{Fe} + l_A / \mu_A) \sim B (l_{Fe} / \mu_{Fe} + l_A / \mu_0)$$

$$\phi = B S = N_1 i / \text{Reluctancia} = N_1 i / [S^{-1} (l_{Fe} / \mu_{Fe} + l_A / \mu_A)] = N_1 i S \mu_{Fe} l_{Fe}^{-1}$$

being

$$\mu_0 \quad \text{permeability of the vacuum} (4\pi \cdot 10^{-7} [\text{H/m}])$$

$$\begin{aligned}\mu_r & \text{relative permeability of the means} \\ \mu &= \mu_0 \mu_r \text{ permeability of the means} \\ \mu_{ef} &= \mu_0 [\mu_{rFe}^{-1} + (l_{Fe}/l_A)^{-1}]^{-1} \text{ effective permeability of the means}\end{aligned}$$

and therefore we are under conditions of determining the inductances

$$\begin{aligned}L_1 &= N_1 \phi / i = (N_1 / i) (N_1 i S \mu_{Fe} l_{Fe}^{-1}) = N_1^2 S \mu_{Fe} l_{Fe}^{-1} \\ L_2 &= N_2^2 S \mu_{Fe} l_{Fe}^{-1}\end{aligned}$$

and also

$$L_1 = n^2 L_2$$

On the other hand, according to the consideration of a coupling  $k \sim 1$  they are the dispersion inductances and magnetic

$$\begin{aligned}L_1 (1 - k) &\sim 0 \\ nM &= L_1 k \sim L_1\end{aligned}$$

## Designe

Be the effective data and line frequency

$$V_p = \dots \quad V_s = \dots \quad I_s = \dots \quad f = \dots$$

Of the experience we estimate a section of the nucleus

$$S = \dots > 0,00013 (I_s V_s)^{-1/2}$$

and of there we choose a lamination (the square that is shown can change a little according to the maker)

$$a = \dots \quad A = 3a = \dots \quad l_{Fe} = l_{med} = 12a = \dots$$

### CUADRO DE LAMINACIONES DE HIERRO-SILICIO

Nº LAMINACIÓN	a [mm]	PESO APROXIMADO [Kg] SECCIÓN S CUADRADA
63	3	0,02
37	4,75	0,04
25	6,5	0,1
62	8	0,23



75	9,5	0,3
77	11	0,5
111	12,7	0,7
112	14,3	1
46	15	1,1
125	16	1,34
100	16,5	1,65
155	19	2,36
60	20	2,65
42	21	3,1
150	22,5	3,3
600	25	5,1
500	32	10,5
850	41	34
102	51	44

For not saturating to the nucleus we consider the previous studies

$$V_{p_{\text{pico}}} < N_1 S B_{\text{pico}} \omega$$

$$B_{\text{pico}} < 1 \text{ [ Wb/m}^2 \text{ ]}$$

being

$$N_1 = 0,0025 V_p / S B_{\text{pico}} f = \dots$$

$$N_2 = N_1 V_s / V_p = \dots$$

$$I_p = I_p N_2 / N_1 = \dots$$

As the section of the drivers it is supposed to circulate

$$s = \pi \varnothing^2 / 4$$

and being usual to choose a current density for windings of 3 [A/mm<sup>2</sup>]

$$J = \dots < 3 \cdot 10^6 \text{ [A/mm}^2 \text{]}$$

what will allow us to obtain

$$\varnothing_1 = 1,13 ( I_p / J )^{1/2} = \dots$$

$$\varnothing_2 = 1,13 ( I_p / J )^{1/2} = \dots$$

Subsequently we verify the useless fallen ohmics in the windings

$$R_1 = \rho l_{\text{med}} N_1 / s_1 \sim 22 \cdot 10^{-9} l_{\text{med}} N_1 / \varnothing_1^2 = \dots \ll V_p / I_p$$

$$R_2 \sim 22 \cdot 10^{-9} I_{med} N_2 / \varnothing_2^2 = \dots \ll V_s / I_s$$

and also that the coil enters in the window «A» (according to the following empiric equation for makings to machine, that is to say it doesn't stop manual coils)

$$N_1 s_1 + N_2 s_2 \sim 0,78 (N_1 \varnothing_1^2 + N_2 \varnothing_2^2) = \dots < 0,25 \text{ A}$$

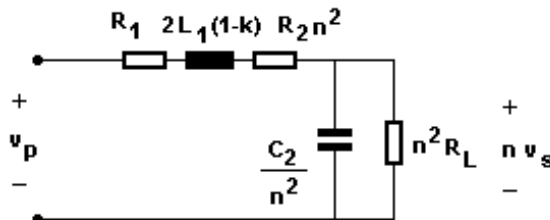
### Transformer of audiofrequency

It is here to manufacture a transformer that allows to pass the audible spectrum. In this component, being similar to that studied to possess magnetic nucleus, the capacitances of the primary and secondary should not be rejected. This reason makes that we cannot reject the dispersion inductances because they will oscillate with the capacitances; that is to say in other words that the coupling coefficient will be considered.

However we can simplify the capacitance of the primary one if we excite with a generator of voltage since if we make it with current it will add us a pole. For this reason the impedance of the generating  $Z_g$  will be necessarily very smaller to the reactance of  $C_1$  in the worst case, that is to say, to the maximum frequency of sharp of audio

$$|Z_g| \ll 1 / \omega_{max} C_1$$

We are under these conditions of analyzing, for a load pure  $Z_L = R_L$  in the audible spectrum, the transfer of the primary system to secondary. We will make it in two parts, a first one for serious and then another for high audible frequency.



Then, like for low frequencies they don't affect the capacitance of the secondary one and therefore neither the dispersion inductances; this way, rejecting the magnetic inductance and the losses in the iron, it is reflected in low frequencies

$$T_{(graves)} = n v_s / v_p \sim \{ 1 + [ (R_1 + R_2 n^2) / R_L n^2 ] \}^{-1} / (s + \omega_{min})$$

$$\omega_{min} = [ L_1 / [ R_1 // (R_2 + R_L) n^2 ] ]^{-1}$$

and the high frequency

$$T_{(agudos)} = [ n^2 / (2 L_1 (1-k) C_2) ]^{-1} / (s^2 + 2 s \xi \omega_{max} + \omega_{max}^2)$$

$$\omega_{max} = \{ [ n^2 / (2 L_1 (1-k) C_2) ]^{-1} \cdot [ 1 + [ (R_1 + R_2 n^2) / R_L n^2 ] ] \}^{1/2}$$

$$\xi = \{ (R_L C_2)^{-1} + [ (R_1 + R_2 n^2) / [2 L_1 (1-k)] ] \} / 2 \omega_{max}$$

and if we simplified the capacitance  $C_2$  we would not have synergy

$$T_{(agudos)} = [n^2 R_L / 2 L_1 (1-k)]^{-1} / (s + \omega_{max})$$

$$\omega_{max} = [R_1 + (R_2 + R_L) n^2] / 2 L_1 (1-k)$$

### Transformer of pulses

This transformer is dedicated to transfer rectangular waves the purest possible. It is convenient for this to be able to reject the capacitance of the primary one exciting with voltage and putting a load purely resistive. The inconvenience is generally due to the low coupling coefficient that impedes, usually, to reject the magnetic inductance.

If we can make a design that has the previous principles, and we add him the following

$$|R_2 n^2 + s L_1 (1-k)| \ll |R_2 n^2 // (n^2 / s C_2)|$$

then it can be demonstrated that for an entrance step «V» in the primary one they are

$$T = n v_s / v_p \sim [n^2 / C_2 L_1 (1-k)] / [(s + \beta)^2 + \omega_0^2]$$

$$\omega_0 = [n^2 / \alpha C_2 L_1 (1-k)] - \beta^2$$

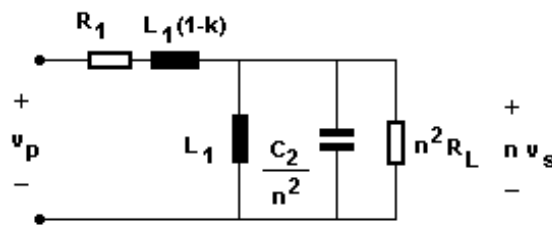
$$\alpha = R_L n^2 / (R_1 + R_L n^2)$$

$$\beta = 0,5 \{ [R_1 / L_1 (1-k)] + (1 / R_L C_2) \}$$

$$n v_s = T V / s \rightarrow \text{antitransformer} \rightarrow V \propto \{ 1 + e^{-\beta t} \cdot \text{sen}(\omega_0 t + \phi) / k \omega_0 \}$$

$$k = [\alpha n^2 / C_2 L_1 (1-k)]^{1/2}$$

$$\phi = \text{arc tag}(\omega_0 / \beta)$$



that it is simplified for worthless dispersion inductance and output capacitance

$$T = n v_s / v_p \sim \alpha \cdot s / [s + (R_1 \alpha / L_1)]$$

$$n v_s = V \propto [1 - (R_1 \alpha / L_1) t]$$

This analysis has been made with the purpose of superimposing the effects of the answer from the transformer to the high and low frequencies for a rectangular excitement; that is to say, respectively, to the *flanks* and *roofs* of the pulses. For this reason we have the series of following equations of design finally

$m = 1, 2, 3, \dots$  (order of the considered pick)

$$\delta = \beta \cdot T_0 / 2\pi$$

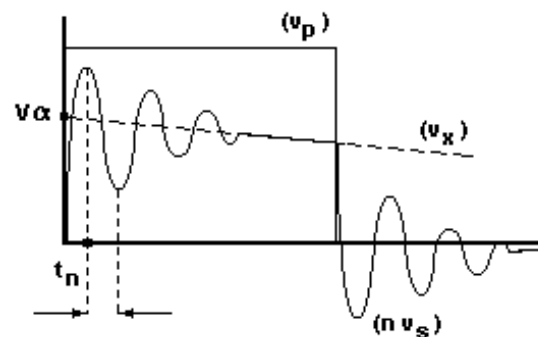
$$T_0 = 2\pi / \omega_0 \sim 2\pi [\alpha L_1(1-k) C_2 / n^2]^{1/2}$$

$$t_m = m \cdot T_0 / 2 (1 - \delta^2)^{1/2}$$

$t_c \sim 0,53 \cdot T_0$  (time of ascent of the  $v_s$ , defined among the 10 % and 90 % of  $V\alpha$ )

$$v_x \sim V\alpha [1 - (R_1 \alpha / L_1) t]$$

$$\varepsilon_v = 1 - (v_{x(\tau)} / V\alpha) = R_1 \tau / L_1 \text{ (slope error)}$$



## Design

Be tha data

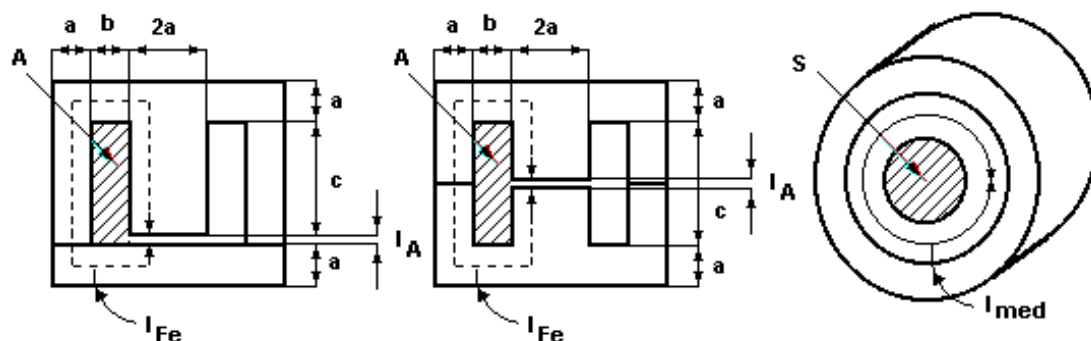
$$\varepsilon_{vmax} = \dots \quad \tau = \dots \quad V = \dots \quad R_L = \dots \quad n = \dots$$

We choose a recipient and they are obtained of their leaves of data

$$a = \dots \quad b = \dots \quad c = \dots \quad l_{Fe} = 2(2a + b + c) - l_A = \dots \quad l_{med} = \pi b = \dots$$

$$A = a b = \dots \quad S = \pi a^2 = \dots \quad l_A = \dots \quad B_{SAT} = \dots$$

$\mu_T = \dots$  (relative permeability commonly denominated as toroid)



For that seen previously

$$\mu_{ef} = \mu_0 (\mu_T^{-1} + l_{Fe}/l_A)^{-1} = 4\pi \cdot 10^{-7} (\mu_T^{-1} + l_{Fe}/l_A)^{-1} = \dots$$

$$B_{pico} = \dots < B_{SAT}$$

$$N_1 = V \tau / 2\pi S B_{pico} = \dots$$

$$N_2 = N_1 / n = \dots$$

$$L_1 = N_1^2 S \mu_{ef} / l_{Fe} = \dots$$

$$R_{1max} = L_1 \varepsilon_{vmax} / \tau = \dots$$

$$R_{2max} = \dots \ll R_L$$

keeping in mind the specific resistivity is obtained

$$\varnothing_1 = \dots > 0,00015 (I_{med} N_1 / R_{1max})^{1/2}$$

$$\varnothing_2 = \dots > 0,00015 (I_{med} N_2 / R_{2max})^{1/2}$$

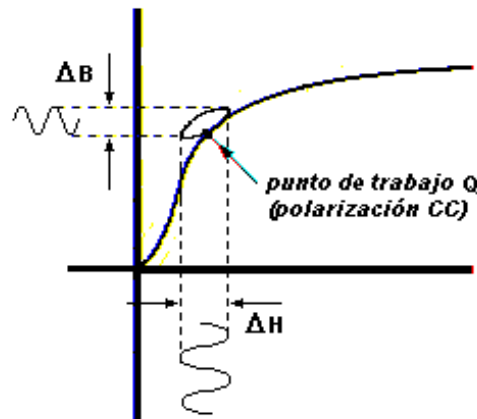
and with it is verified it that they enter in the window

$$N_1 s_1 + N_2 s_2 \sim 0,78 (N_1 \varnothing_1^2 + N_2 \varnothing_2^2) = \dots < 0,25 A$$

The total and final determination of the wave of having left one will only be able to obtain with the data of the coupling coefficient and the distributed capacitance that, as it doesn't have methods for their determination, the transformer will be experienced once armed.

### Inductors of filter with continuous component

The magnetization curve that polarizes in DC to an inductor with magnetic nucleus, their beginning of the magnetism induced remainder will depend  $B_{REM}$  (practically worthless) that has it stops then to follow the curve of normal magnetization. With this it wants to be ahead the fact that it is very critical the determination of the work point. Above this polarization the alternating CA is included determining a hysteresis in the *incremental permeability*  $\Delta\mu$  that its effectiveness of the work point will depend.



In the abacus that was shown previously they were shown for nucleus some values of the incremental permeability. Of this the effective inductance that we will have will be

$$L = N^2 S \Delta\mu_{\text{ef}} / l_{\text{Fe}}$$

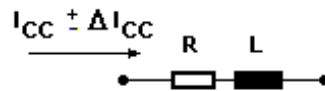
$$\Delta\mu_{\text{ef}} = \mu_0 / [ \Delta\mu_{\text{rFe}}^{-1} + ( l_A / l_{\text{Fe}} ) ] = [ \Delta\mu_{\text{Fe}}^{-1} + ( l_A / \mu_0 l_{\text{Fe}} ) ]^{-1}$$

where  $\Delta\mu_{\text{ef}}$  it is the *effective incremental permeability* of the iron.

## Design

Be the data

$$I_{\text{CC}} = \dots \gg \Delta I_{\text{CC}} = \dots \quad L = \dots \quad R_{\text{max}} = \dots \quad f \sim 50 \text{ [Hz]}$$



We already adopt a lamination of the square presented when designing a transformer

$$a = \dots \quad l_{\text{Fe}} \sim l_{\text{med}} \sim 12 a = \dots \quad S = 4 a^2 = \dots \quad A = 3 a^2 = \dots$$

$$V_{\text{Fe}} = S l_{\text{Fe}} = \dots$$

choosing

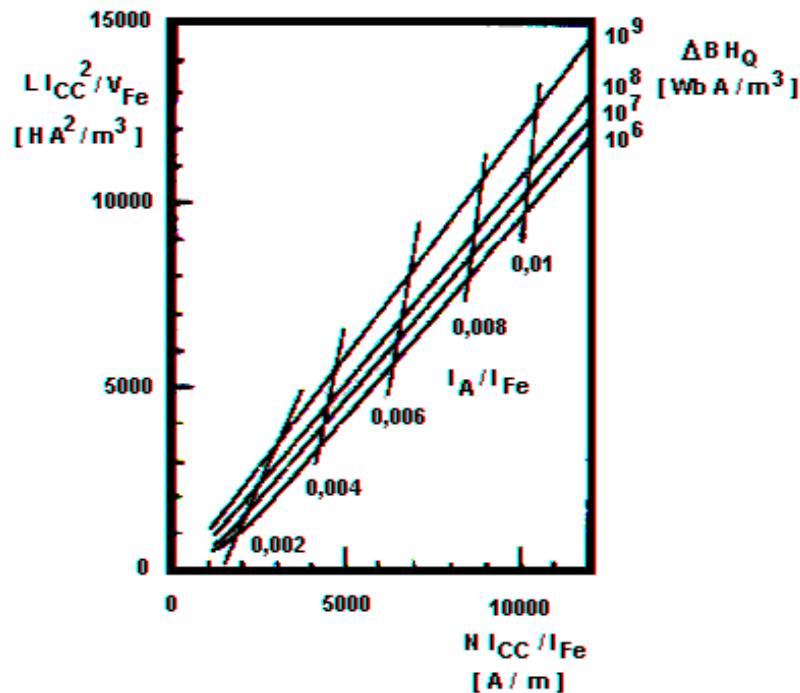
$$l_A = \dots \ll l_{\text{Fe}}$$

We determine now

$$\Delta B H_Q = \Delta H \Delta\mu_{\text{Fe}} H_Q = ( N I_{\text{CC}} \Delta\mu_{\text{Fe}} / l_{\text{Fe}} ) \cdot ( N I_{\text{CC}} / l_{\text{Fe}} ) = I_{\text{CC}} \Delta I_{\text{CC}} L / l_{\text{Fe}} S = \dots$$

so that, of the curves of following Hanna we obtain

$$N = \dots$$



and in function of the specific resistivity

$$\varnothing = \dots > 0,00015 (I_{med} N / R_{max})^{1/2}$$

verifying that the design enters in the window according to the following practical expression

$$N s \sim 0,78 N \varnothing^2 = \dots < 0,25 A$$

### Inductors of filter without continuous component

In approximate form we can design an inductance if we keep in mind the the graphs views and the square of laminations for the iron. This way with it, of the equations

$$L = (N^2 S / I_{Fe}) \cdot (B_{ef} / H_{ef})$$

where  $B_{ef}$  yand  $H_{ef}$  they are the effective values of  $\mu_{ef}$ .

### Design

Be tha data

$$L = \dots I_{max} = \dots (eficaz) f = \dots$$

We already adopt a lamination of the square presented when designing a transformer

$$a = \dots \quad l_{Fe} \sim l_{med} \sim 12 a = \dots \quad S = 4 a^2 = \dots \quad A = 3 a^2 = \dots$$

and we choose a work point in the abacus of the *curve of normal magnetization* of effective values seen in the section previous of *Transformer of alimentation*, where it will be chosen to be far from the saturation of the nucleus and also preferably in the lineal area, this way if the  $l_{max}$  diminished it will also make lineally it the  $\mu_{ef}$  in a proportional way.

$$\mu_{ef} = \dots$$

$$N = (l_{Fe} L / \mu_{ef} S)^{1/2} = \dots$$

For not exceeding in heat to the winding we adopt a density of current of 3 [A/mm<sup>2</sup>]

$$\varnothing = \dots > 0,00065 l_{max}^{1/2}$$

and we verify that this diameter can enter in the window, and that the resistance of the same one doesn't alter the quality of the inductor

$$0,78 N \varnothing^2 = \dots < 0,25 A$$

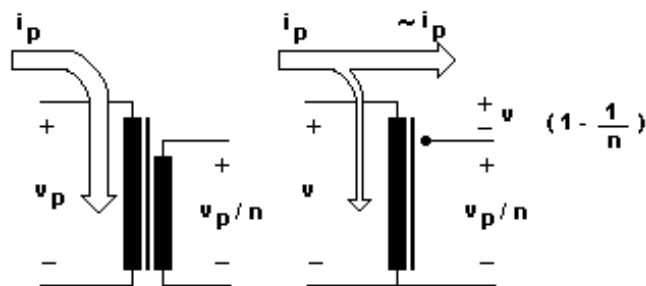
$$22 \cdot 10^{-9} l_{med} N / \varnothing^2 = \dots \ll \omega L$$

## Autotransformer

The physical dimensions of an autotransformer are always much smaller that those of a transformer for the same transferred power. This is due to that in the first one the exposed winding only increases to the increment or deficit of voltage, and then the magnetic inductance continues being low

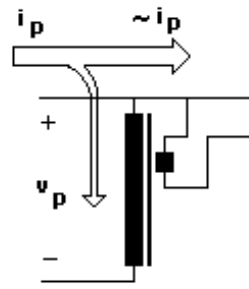
$$\text{POWER IN A TRANSFORMER} = V_p I_p$$

$$\text{POWER IN A AUTOTRANSFORMER} \sim V_p I_p \left| 1 - n^{-1} \right|$$



The calculation and design of this component will follow the steps explained for the design of the transformer, where it will talk to the difference of coils to the same approach that if was a typical secondary.





## Chap. 07 Power supply without stabilizing

### Generalities

#### Power supply of half wave with filter RC

#### Design

#### Abacous of Shade

#### Power supply of complete wave with filter RLC

#### Design

#### Connection of diodes in series

#### Design

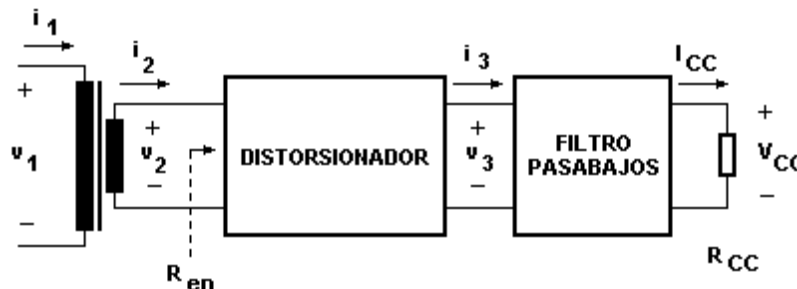
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### Generalities

Those will be studied up to 500 [VA] due to the simplification of their equivalent circuit.

All supply of power follows the outline of the following figure, where the distorting generates harmonic AC and a component continuous DC as alimentation.

The purity of all source is given by two merits: the ability of the filter to attenuate meetly to all the possible harmonics, and the low resistance of output of the same —*regulation* or *stabilization*.



#### Power supply of half wave with filter RC

The distorting, implemented here with a simple diode, will allow him to circulate for him the continuous current of the deformation and its harmonic content

$$i_3 = i_C + i_{CC} = i_C + I_{CC} + i_Z$$

where the harmonic are given for  $i_C + i_Z$  and to  $i_Z$  it denominates *ripple*. This way, the voltage of

continuous of instantaneous output will be worth

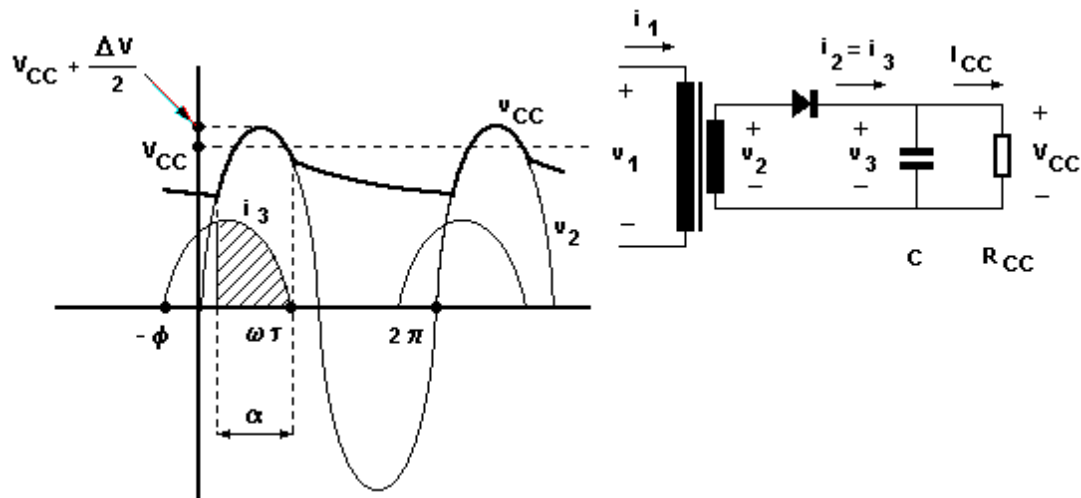
$$v_{CC} = V_{CC} + v_Z = I_{CC} R_{CC} + i_Z R_{CC} = V_{CC} \pm (\Delta V / 2)$$

We define then to the *ripple from the source* to the relationship

$$Z = v_Z / V_{CC}$$

Let us analyze the wave forms that we have. For an entrance sine wave

$$v_2 = E_{2pico} \text{ sen } \omega t$$



it will drive the diode (ideal) when it is completed that

$$v_d = v_2 - v_{CC} > 0$$

and if we reject their fall they are

$$i_2 \sim v_2 Y = I_{2pico} \text{ sen } (\omega t + \phi)$$

$$I_{2pico} = E_{2pico} [ G_{CC}^2 + (\omega C)^2 ]^{1/2}$$

$$\phi = \text{arc tag } \omega C R_{CC}$$

In the disconnection of the diode

$$v_d = v_2 - v_{CC} = 0$$

with the condenser loaded to the value

$$v_{CC}(\tau) = v_2(\tau) = V_{2pico} \text{ sen } \omega \tau$$

that then it will begin to be discharged

$$V_{CC}(t-\tau) = V_{CC}(\tau) e^{-(t-\tau)/C R_{CC}} = V_{2pico} e^{-(t-\tau)/C R_{CC}} \cdot \sin \omega \tau$$

We can have an analytic idea of the ripple if we approach

$$v_z \sim (\Delta V / 2) - (\Delta V \cdot \omega t / 2 \pi)$$

because while the diode doesn't drive it is the condenser who feeds the load

$$\Delta V / \Delta t = I_{CC} / C$$

$$\Delta V = I_{CC} \pi / \omega C$$

and in consequence

$$\begin{aligned} z = v_z / V_{CC} &= [(\int_0^\pi v_z^2 d\omega t) / \pi]^{1/2} / V_{CC} \sim \Delta V / 3,46 V_{CC} = \\ &= \pi / 3,46 \omega C R_{CC} \sim 1 / 7 f C R_{CC} \end{aligned}$$

## Design

Be tha data

$$V_1 = \dots \quad f = \dots \quad V_{CC} = \dots \quad I_{CCmax} = \dots \quad I_{CCmin} = \dots > 0 \quad Z_{max} = \dots$$

We suppose that the design of the transformer possesses proportional inductances of the primary and secondary, that is to say that  $R_1/n^2 \sim R_2$ . This approach that is not for anything far from the reality, will simplify us enough the project.

We avoid in the first place to dissipate energy unsuccessfully in the transformer and we choose

$$R_1/n^2 + R_2 \sim 2 R_2 \ll V_{CC} / I_{CCmax}$$

$$R_2 = \dots \ll 2 V_{CC} / I_{CCmax}$$

and we obtain with it

$$R_S = R_1 / n^2 + R_2 + 0,6 / I_{CCmax} \sim 2 R_2 + 0,6 / I_{CCmax} = \dots$$

$$R_S / R_{CC} \sim (R_{CCmax} + R_{CCmin}) / 2 = [(V_{CC} / I_{CCmin}) - (V_{CC} / I_{CCmin})] / 2 = \dots$$

being able to also choose as magnitude  $R_S / R_{CC}$  for the most convenient case.

Then of the curves of Shade for half wave have

$$C = \dots \quad V_{2pico} = \dots \quad I_{3ef} = \dots \quad I_{3pico} = \dots$$

The data for the election of the diode rectifier they will be (it is always convenient to enlarge them a little)

$$\begin{aligned}
I_{\text{RMS}} &= I_{3\text{ef}} = \dots \\
I_{\text{AVERAGE}} &= I_{\text{CCmax}} = \dots \\
I_{\text{PEAK REPETITIVE}} &= I_{3\text{pico}} = \dots \\
I_{\text{PEAK TRANSITORY}} &= V_{2\text{pico}} / (R_1 / n^2 + R_2) \sim V_{2\text{pico}} / 2 R_2 = \dots \\
V_{\text{PEAK REVERSE}} &= V_{\text{CC}} + V_{2\text{pico}} \sim 2 V_{\text{CC}} = \dots
\end{aligned}$$

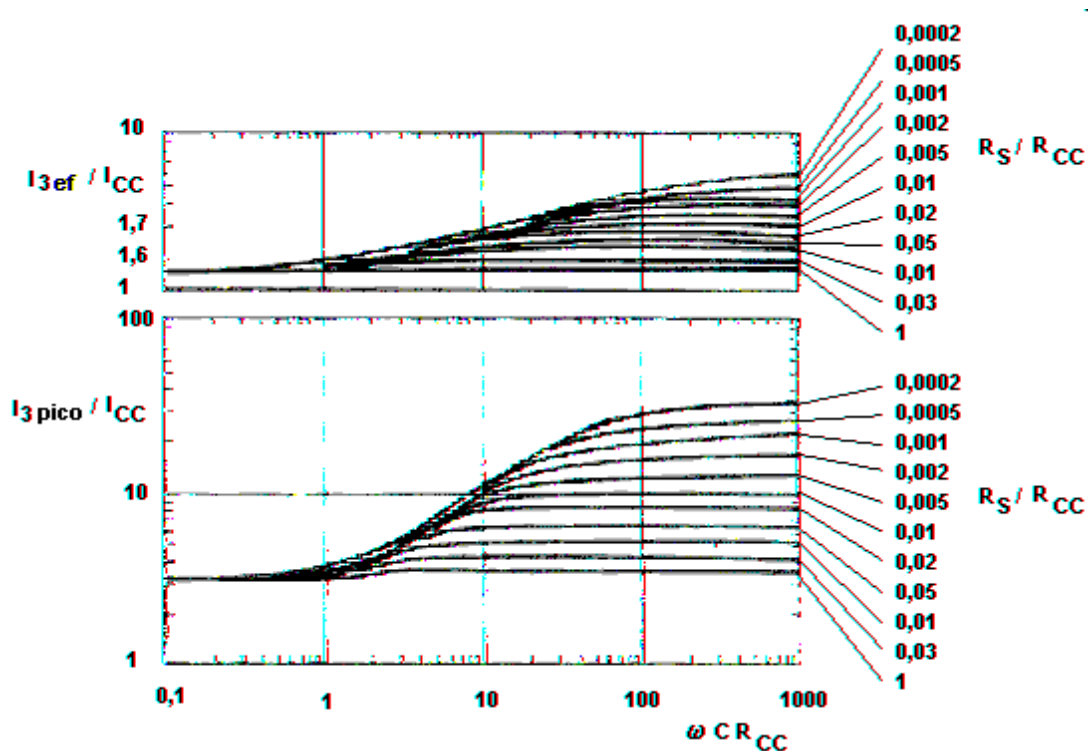
y al fabricante del transformador

VOLTAGE OF PRIMARY	$V_1 = \dots$ (already determined precedently)
FREQUENCY	$f = \dots$ (already determined precedently)
RELATIONSHIP OF SPIRE	$n = V_1 / V_2 \sim 1,41 V_1 / V_{2\text{pico}} = \dots$
RESISTENCE OF SECONDARY	$R_2 = \dots$ (already determined precedently)
RESISTENCE OF PRIMARY	$R_1 = R_2 n^2 = \dots$
APPARENT POWER	$S_1 \sim (V_{\text{CC}} + 0,6) I_{\text{CCmax}} = \dots$

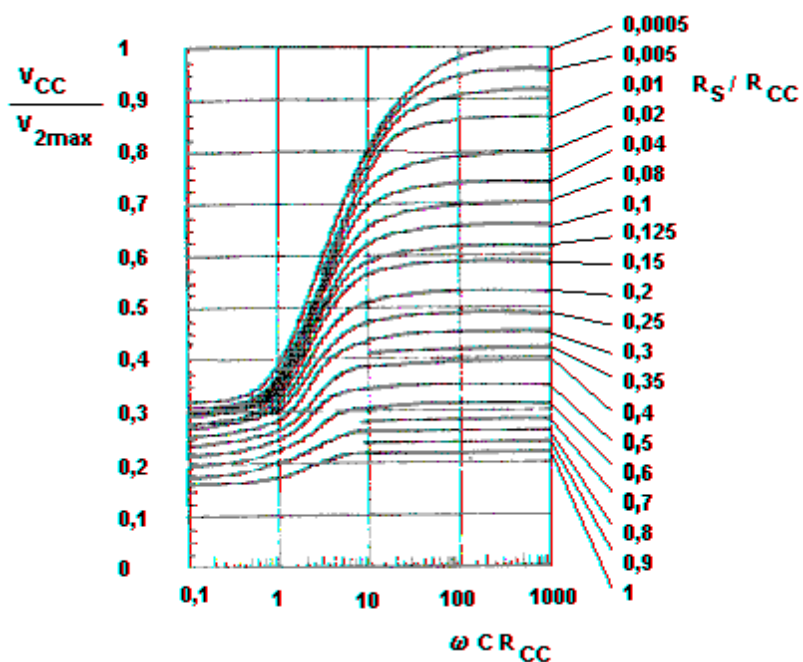
### Abacous of Shade

For further accuracy in the topics that we are seeing and they will continue, we have the experimental curves of Shade that, carried out with valves hole diode, they allow anyway to approach results for the semiconductors. Subsequently those are shown that will use they —exist more than the reader will be able to find in any other bibliography. The first relate the currents for the rectifier  $i_3$  with the continuous one for the load  $I_{\text{CC}}$  (that is the average), where the resistance series  $R_S$  is the sum of all the effective ones: that of the primary reflected to the secondary, the of secondary and the that has the rectifier (diode or diodes) in their conduction static average

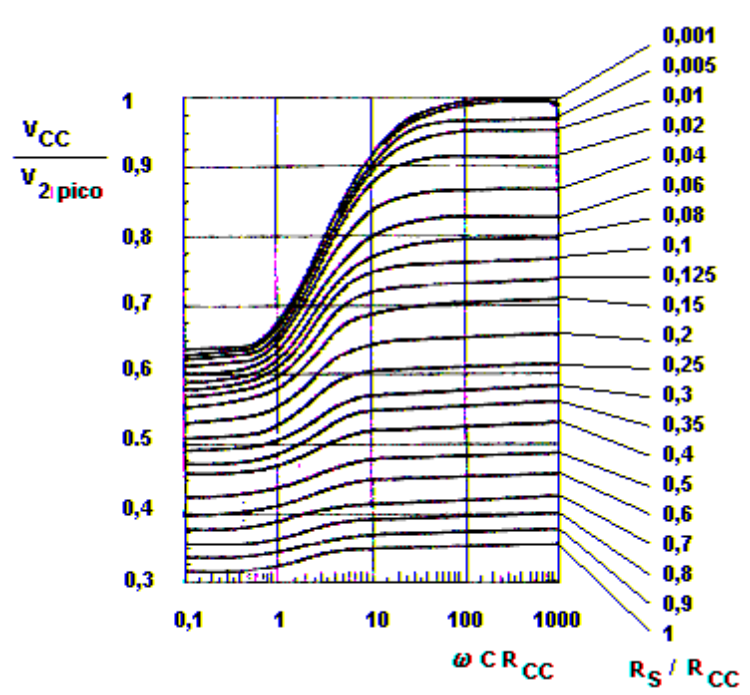
$$\begin{aligned}
R_S &= R_1 / n^2 + R_2 + R_{\text{RECTIFIER}} \\
R_{\text{RECTIFIER (1 diode)}} &\sim 0,6 \text{ [V]} / I_{\text{CC}} \\
R_{\text{RECTIFIER (2 diodes in bridge transformer)}} &\sim 0,6 \text{ [V]} / I_{\text{CC}} \\
R_{\text{RECTIFIER (4 diodes in bridge rectifier)}} &\sim 2 \cdot 0,6 \text{ [V]} / I_{\text{CC}}
\end{aligned}$$



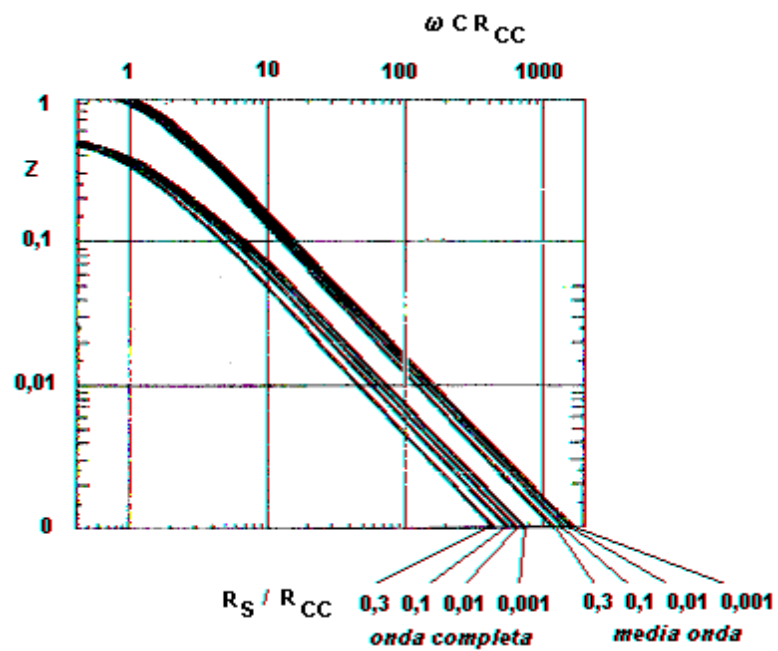
the second express the efficiency of detection  $hd$ , as the relationship among the continuous voltage that we can obtain to respect the value peak of the input sign. First we have the case of a rectifier of half wave with a filter capacitive and then we also have it for that of complete wave with filter capacitive



and then we also have it for that of complete wave with filter capacitive



The third curve of Shade that here present it shows us the ripple percentage

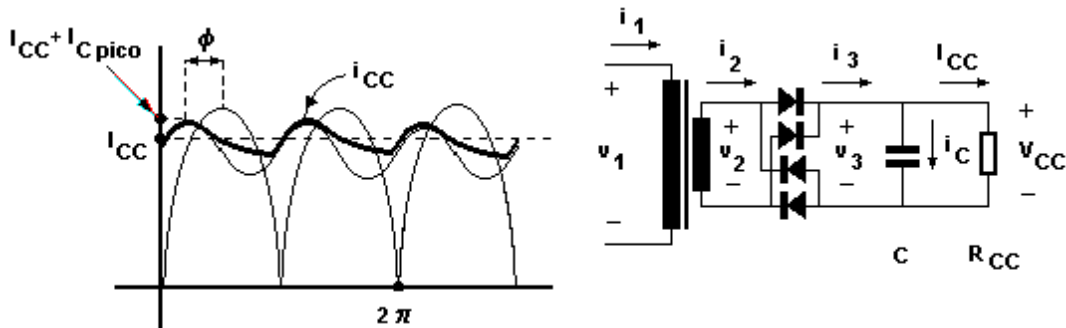


[Power supply of complete wave with filter RLC](#)

This source is used when we want a smaller ripple, bigger voltage stabilization and to avoid abrupt current peaks for the rectifier, overalls in the beginning. For this last reason it is necessary to choose an inductance bigger than a *critical value*  $L_C$  that subsequently will analyze.

Let us suppose that it is

$$v_2 = V_{2pico} \text{ sen } \omega t$$



and seeing the graphs observes that the current for the diodes is a continuous one more an alternating sine wave that takes a desfase  $\phi$ . If we estimate very low the ripple to the exit, since it is what we look for and we should achieve, it is

$$Z = V_{Zsal} / V_{CC} = I_{Zsal} R_{CC} / I_{CC} R_{CC} = I_{Zsal} / I_{CC} \ll 1$$

and rejecting then

$$i_3 \sim I_{CC} + i_c \rightarrow I_{3pico} \text{ sen } (2\omega t + \phi)$$

$$I_{3pico} = (I_{CC}^2 + I_{cpico}^2)^{1/2} = [(V_{CC} / R_{CC})^2 + (2\omega C V_{Zsalpico})^2]^{1/2}$$

$$\phi = \text{arc tag } (I_{cpico} / I_{CC}) = \text{arc tag } (2\omega C V_{Zsalpico} R_{CC} / V_{CC})$$

On the other hand, as  $v_3$  it coincides with the form of  $v_2$ , for Fourier we have

$$v_2(n \cdot 2\pi / 0,5 T) = (2 / T) \cdot \int_0^{T/2} v_2 e^{-j(n \cdot 2\pi / 0,5 T)t} dt = 2 V_{2pico} / \pi (1 - 4n^2)$$

$$v_2 = (2 V_{2pico} / \pi) - (4 V_{2pico} / 3\pi) \cos 2\omega t - (4 V_{2pico} / 15\pi) \cos 4\omega t \dots \sim$$

$$\sim (2 V_{2pico} / \pi) - (4 V_{2pico} / 3\pi) \cos 2\omega t = V_{CC} - V_{Zsalpico} \cos 2\omega t$$

With the purpose of that the ripple circulates for the capacitor and not for the load we make

$$R_{CC} \gg 1 / 2\omega C$$

and also so that all the alternating is on the inductor achieving with it low magnitudes in the load

$$2\omega L \gg 1 / 2\omega C$$

it will allow to analyze



$$I_{Cpico} = V_{Zsalpico} 2\omega C \sim V_{Zentpico} / 2\omega L$$

$$I_{CC} = V_{CC} / R_{CC}$$

and having present that the inductance will always possess a magnitude above a critical value  $L_C$  so that it doesn't allow current pulses on her (and therefore also in the rectifier)

$$I_{Cpico}(L_C) = V_{Zentpico} / 2\omega L = V_{CC} / R_{CC}$$

it is

$$L_C = V_{Zentpico} R_{CC} / 2\omega V_{CC} = 2 R_{CC} / 6\omega \sim 0,053 R_{CC} / f$$

If it is interested in finding the ripple, let us have present that the alternating is attenuated by the divider reactive LC according to the transmission

$$V_{Zsal} / V_{Zent} \sim (1 / 2\omega C) / 2\omega L = 1 / 4\omega^2 LC$$

being finally of the previous equations (the curves of Shade show this same effect)

$$Z = V_{Zsal} / V_{CC} \sim 0,707 V_{Zsalpico} / V_{CC} = 0,707 V_{Zentpico} / 4\omega^2 LC V_{CC} \sim 0,003 / f^2 LC$$

Until here it has not been considered the resistance of the inductor  $R_L$ , which will affect to the voltage of the load according to the simple attenuation

$$V_{CCfinal} = V_{CC} R_{CC} / (R_L + R_{CC}) \sim 2 V_{2pico} / \pi (1 + R_L G_{CC})$$

## Design

Be tha data

$$V_1 = \dots \quad f = \dots \quad V_{CC} = \dots \quad I_{CCmax} = \dots \quad I_{CCmin} = \dots > 0 \quad Z_{max} = \dots$$

We choose a bigger inductance that the critic in the worst case

$$L = \dots > 0,053 V_{CC} / f I_{CCmin}$$

Of the ripple definition

$$Z = V_{Zsalpico} / V_{CC}$$

and as we saw

$$V_{Zentpico} = 4 V_{2pico} / 3\pi \sim 4 V_{CC} / 3\pi \sim 0,424 V_{CC}$$

it is appropriate with these values to obtain to the condenser for the previous equation

$$C = \dots > V_{Zentpico} / V_{Zsalpico} 4\omega^2 L \sim 0,424 V_{CC} / Z_{max} V_{CC} 4\omega^2 L = 0,0027 / f^2 L$$

Subsequently we can obtain the relationship of spires

$$n = V_1 / V_2 \sim 1,41 V_1 / V_{2pico} = 1,41 V_1 / (\pi V_{CC} / 2) \sim 0,897 V_1 / V_{CC} = \dots$$

Now, for not dissipating useless powers in the transformer and winding of the inductor, they are made

$$R_1 = \dots \ll n^2 V_{CC} / I_{CCmax}$$

$$R_2 = \dots \ll V_{CC} / I_{CCmax}$$

$$R_L = \dots \ll V_{CC} / I_{CCmax}$$

It will be consequently the data for the production of the transformer

VOLTAGE OF PRIMARY	$V_1 = \dots$ (already determined precedently)
FREQUENCY	$f = \dots$ (already determined precedently)
RELATIONSHIP OF SPIRE	$n = \dots$
RESISTENCE OF SECUNDARY	$R_2 = \dots$ (already determined precedently)
RESISTENCE OF PRIMARY	$R_1 = \dots$
APPARENT POWER	$S_1 \sim V_{CC} I_{CCmax} = \dots$

those of the inductor

INDUCTANCE	$L = \dots$ (already determined precedently)
RESISTENCE	$R_L = \dots$ (already determined precedently)

and those of the bridge rectifier

$$I_{RMS} = [ I_{CCmax}^2 + ( Z_{max} V_{CC} 2\omega C )^2 ]^{1/2} \sim [ I_{CCmax}^2 + 158 ( Z_{max} V_{CC} f C )^2 ]^{1/2} = \dots$$

$$I_{AVERAGE} = I_{CCmax} = \dots$$

$$I_{PEAK REPETITIVE} = I_{CCmax} + Z_{max} V_{CC} 2\omega C^{1/2} \sim I_{CCmax} + 1,78 Z_{max} V_{CC} f C = \dots$$

$$I_{PEAK TRANSITORY} \sim V_{2pico} / ( R_1 / n^2 + R_2 ) \sim 1,57 V_{CC} / 2 R_2 = \dots$$

$$V_{PEAK REVERSE} \sim ( V_{CC} + V_{2pico} ) / 2 \sim 1,3 V_{CC} = \dots$$

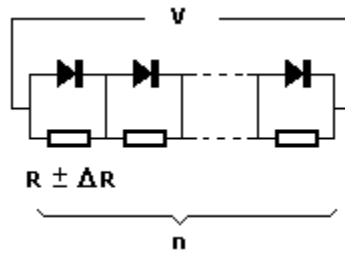
### Connection of diodes in series

The rectifiers of common or controlled commutation (TBJ, GTB, RCS and TRIAC) they support a *voltage of acceptable inverse pick*  $V_{PI}$  to the circulate for them an *acceptable inverse current*  $I_{INVADM}$ . When one needs to tolerate superior voltages to this magnitude

$$V > V_{PI}$$

they prepare in series like it is shown in the figure. This quantity «n» of diode-resistance, jointly

considering their tolerance  $\Delta R$ , it will limit the tensions then.



It can be demonstrated that so that the system works correctly it should be that

$$n > 1 + [(V - V_{PI}) / V_{PI}] (1 + \Delta R / R + R I_{INVADM} / V_{PI}) / (1 - \Delta R / R)$$

or this other way

$$R < \{ [V_{PI} / (1 + \Delta R / R)] - [(V - V_{PI}) / (n - 1) (1 - \Delta R / R)] \} / I_{INVADM}$$

## Design

Be the data ( $V_{PI}$  and  $I_{INVADM}$  can be experienced simply with a high source, for example implemented with a multiplying source and a resistance in series)

$$V = \dots \text{ (or in CA sine wave } V_{pico} = \dots) \quad V_{PI} = \dots \quad I_{INVADM} = \dots$$

We choose a tolerance of the resistors

$$\Delta R / R = \dots$$

and we determine with the equation the quantity of cells to put (to replace in CA sine wave to  $V$  for  $V_{pico}$ )

$$n = \dots > 1 + [(V - V_{PI}) / V_{PI}] (1 + \Delta R / R + R I_{INVADM} / V_{PI}) / (1 - \Delta R / R)$$

and the magnitude of the resistors

$$R = \dots < \{ [V_{PI} / (1 + \Delta R / R)] - [(V - V_{PI}) / (n - 1) (1 - \Delta R / R)] \} / I_{INVADM}$$

verifying the power that it should tolerate

$$P_{(para \text{ CC})} = V^2 / n R = \dots$$

$$P_{(para \text{ CA sinusoidal})} = V_{pico}^2 / 2 n R = \dots$$

## Chap. 08 Power supply stabilized

### Generalities

#### Parallel source with diode Zener

#### **Design**

#### Parallel source with diode programmable Zener

#### Parallel source with diode Zener and TBJ

#### Source series with diode Zener and TBJ

#### Source series with diode Zener, TBJ and preestabilizador

#### Source series for comparison

#### Source series with AOV

#### **Design**

#### Source with integrated circuit 723

#### **Design**

#### Source with integrated circuit 78XX

#### Source commuted series

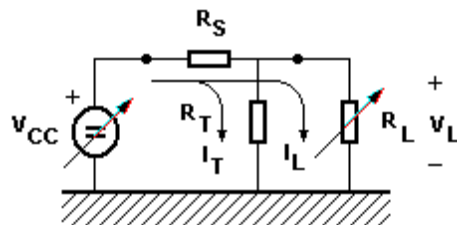
#### **Design**

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### Generalities

In the following figure we observe a alimentation source made with a simple dividing resistive, where their input will be a continuous DC more an undesirable dynamics AC that, to simplify, we opt it is sine wave, as well as to have a variation of the load

$$v_{CC} = V_{CC} \pm \Delta V_{CC} = V_{CC} + V_{pico} \sin \omega t$$
$$i_L = I_L \pm \Delta I_L$$



determining dynamically

$$v_{CC} = v_L + (i_L + i_T) R_S = v_L + v_L R_S / R_T + i_L R_S = v_L (1 + R_S / R_T) + i_L R_S$$

$$v_L = (v_{CC} - i_L R_S) / (1 + R_S / R_T)$$

and consequently *partial factors of stabilization* with respect to the input voltage, to the variations possible of the load and with respect to the ambient temperature

$$\Delta V_L = F_V \Delta V_{CC} + F_I \Delta I_L + F_T \Delta T$$

$$F_V = \partial V_L / \partial V_{CC} = 1 / (1 + R_S / R_T)$$

$$F_I = \partial V_L / \partial I_L = -R_S F_V$$

$$F_T = \partial V_L / \partial T = 0$$

being finally

$$\Delta V_L = (\Delta V_{CC} - R_S \Delta I_L) / (1 + R_S / R_T)$$

### Parallel source with diode Zener

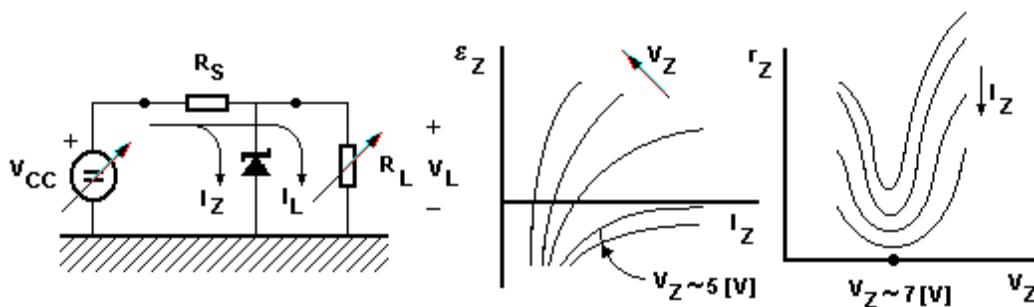
To get small magnitudes of  $F_V$  and  $F_I$  the  $R_T$  it is replaced by a device Zener where their resistance is very small. The advantage resides in that for equal values of  $V_L$  the average  $I_L$  (here  $I_Z$ ) it is not big and therefore uncomfortable and useless dissipations, as well as discharges entrance tensions are avoided.

The behavior equations don't change, since the circuit analyzed dynamically is the same one

$$F_V = \partial V_L / \partial V_{CC} = 1 / (1 + R_S / r_Z)$$

$$F_I = \partial V_L / \partial I_L = -R_S F_V$$

$$F_T = \partial V_L / \partial T = \partial V_Z / \partial T = \varepsilon_Z$$



### Design

Be tha data

$$V_{CCmax} = \dots \quad V_{CCmin} = \dots \quad I_{Lmax} = \dots \quad I_{Lmin} = \dots \quad \geq < 0 \quad V_L = \dots$$

We choose a diode Zener and of the manual we find

$$V_Z = V_L = \dots$$

$$P_{ADM} = \dots \text{ (0,3 [W] for anyone)}$$

$$I_{Zmin} = \dots \text{ (0,001 [A] for anyone of low power it is reasonable)}$$

We choose a  $R_S$  in such a way that sustains the alimentation of the Zener

$$R_S = \dots < (V_{CCmin} - V_L) / (I_{Zmin} + I_{Lmax})$$

and we verify that the power is not exceeded

$$[(V_{CCmax} - V_L) / R_S] - I_{Lmin} = \dots < P_{ADM} / V_Z$$

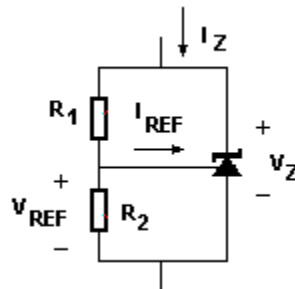
Finally we determine the power that must dissipate the resistance

$$P_{Smax} = (V_{CCmax} - V_L)^2 / R_S = \dots$$

### Parallel source with diode programmable Zener

An integrated electronic circuit is sold that by means of two resistances  $R_1$  and  $R_2$  the  $V_Z$  is obtained (with a maximum given by the maker) with the reference data  $I_{REF}$  and  $V_{REF}$

$$V_Z = V_1 + V_2 = [(V_{REF} / R_2) + I_{REF}] R_1 + V_{REF} = V_{REF} (1 + R_1/R_2) + I_{REF} R_1$$



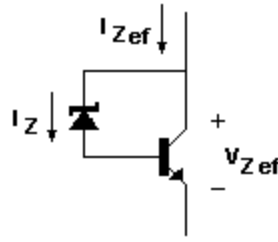
### Parallel source with diode Zener and TBJ

We can increase the power of the effect Zener with the amplifier to TBJ that is shown. The inconvenience is two: that the  $I_{Zmin}$  will increase for this amplification, and another that the resistance dynamic  $r_Z$  it will worsen for the attached of the juncture base-emitter in series with that of the Zener. This species of effective Zener will have the following properties then

$$I_{Zef} \sim \beta I_Z$$

$$V_{Zef} \sim V_Z + 0,6$$

$$r_{Zef} \sim r_Z + h_{11e}$$



of where they are the factors

$$F_V = \partial V_L / \partial V_{CC} = 1 / [1 + R_S / (r_Z + h_{11e})]$$

$$F_I = \partial V_L / \partial I_L = -R_S F_V$$

$$F_T = \partial V_L / \partial T = \partial V_{Zef} / \partial T = \varepsilon_Z + \varepsilon_\gamma \sim \varepsilon_Z - 0,002$$

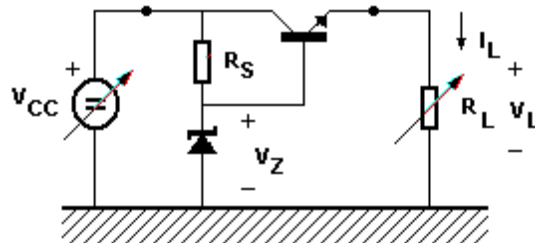
### Source series with diode Zener and TBJ

The following disposition is more used. The low output resistance in common base determines very good stabilization. Here the values are translated to

$$I_L \sim \beta I_B = \beta \{ [(V_{CC} - V_Z) / R_S] - I_Z \}$$

$$V_L \sim V_Z - 0,6$$

$$R_{SAL} \sim (r_Z + h_{11e}) / h_{21e}$$



and in the dynamic behavior

$$V_L \sim V_Z \sim (V_{CC} - I_L R_S h_{21e}^{-1}) / (1 + R_S / r_Z)$$

$$F_V = 1 / (1 + R_S / r_Z)$$

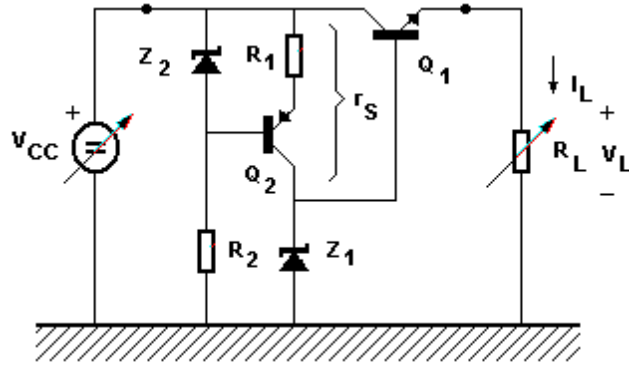
$$F_I = -R_S F_V / h_{21e}$$

$$F_T = \varepsilon_Z - \varepsilon_\gamma \sim \varepsilon_Z + 0,002$$

### Source series with diode Zener, TBJ and preestabilizador

In this circuit it takes advantage the pre-stabilization (for the variations of  $V_{CC}$ ) with a current generator in the place of  $R_S$ . This way, the current in load is practically independent of that of the supply (to remember that  $V_{Z2}$  are produced by  $V_{CC}$ )

$$I_{C1} \sim I_{E1} \sim (V_{Z2} - V_{BE2}) / R_1 \sim (V_{Z2} - 0,6) / R_1 \neq I_{C1}(V_{CC}, R_L)$$



Let us keep in mind that, dynamically for all the practical cases, as much  $R_2$  as the input resistance to the base of  $Q_2$  are very big with respect to that of the Zener

$$R_2 \gg r_{Z2} \ll h_{11e} + (1 + h_{21e}) R_1$$

consequently, the effective resistance  $R_S$  dynamically will be

$$\begin{aligned} r_S &= (v_{CC} - v_{be2} - v_L) / (v_{R1} / R_1) \sim (v_{CC} - v_L) / (v_{R1} / R_1) = \\ &= (v_{CC} - v_L) R_1 / (v_{CC} r_{Z2} / R_2) = (1 - v_L / v_{CC}) R_1 R_2 / r_{Z2} \sim R_1 R_2 / r_{Z2} \end{aligned}$$

where  $v_L$  was simplified in front of  $v_{CC}$  because it is supposed that the circuit stabilizes. Now this equation replaces it in the previous one that thought about for a physical  $R_S$

$$v_L \sim v_Z \sim [v_{CC} - (i_L R_1 R_2 / r_{Z2} h_{21e})] / [1 + (R_1 R_2 / r_{Z1} r_{Z2})]$$

$$F_V = 1 + (R_1 R_2 / r_{Z1} r_{Z2})$$

$$F_I = - (R_1 R_2 / r_{Z2} h_{21e}) F_V$$

$$F_T = \varepsilon_Z - \varepsilon_Y \sim \varepsilon_Z + 0,002$$

### Source series for comparison

An economic and practical system is that of the following figure. If we omit the current for  $R_1$  in front of that of the load, then we can say

$$\begin{aligned} I_L \sim I_{C2} &= \beta_2 (I_0 - I_{C1}) = \beta_2 [I_0 - \beta_1 (V_L - V_{BE1} - V_Z) / R_1] = \\ &= \beta_2 \{ I_0 + [\beta_1 (V_Z + V_{BE1}) / R_1] - (\beta_1 V_L / R_1) \} \end{aligned}$$





$$V_{CCmax} = \dots \quad V_{CCmin} = \dots \quad I_{Lmax} = \dots \quad I_{Lmin} = \dots \geq 0 \quad V_{Lmax} = \dots \quad V_{Lmin} = \dots \geq 0$$

We choose the TBJ or Darlington finding the maximum

$$I_{CADM} = \dots > I_{Lmax}$$

$$V_{CE0} = \dots > V_{CCmax} - V_{CCmin} \text{ (although it would be better only } V_{CCmax} \text{ for if there is a short circuit in the load)}$$

$$P_{CEADM} = \dots > I_{Lmax} V_{CCmax}$$

and then we obtain of the leaf of data

$$T_{JADM} = \dots$$

$$\theta_{JC} = (T_{JADM} - 25) / P_{CEADM} = \dots$$

$$\beta \sim \dots$$

what will allow us to determine for the AOV

$$V_{XX} = \dots \geq V_{Lmax} + V_{BE} = V_{Lmax} + 0,6$$

$$V_{YY} = \dots > 0$$

$$P_{AOVADM} = \dots > I_{Lmax} (V_{XX} - V_{BE}) / \beta = I_{Lmax} (V_{XX} - 0,6) / \beta$$

$$I_{AOVB} = \dots \text{ (let us remember that JFET it is null)}$$

Subsequently the thermal dissipator is calculated as it was seen in the respective chapter

$$\text{surface} = \dots$$

$$\text{position} = \dots$$

$$\text{thickness} = \dots$$

We adopt a diode Zener

$$V_Z = \dots$$

$$P_{ZADM} = \dots$$

$$I_{Zmin} = \dots$$

and we choose  $R_1$  and the potentiometer ( $R_4 + R_5$ ), without dissipating a bigger power that 0,25 [W]

$$R_1 = \dots \ll V_{YY} / 2 I_{AOVB}$$

$$V_{YY}^2 / 0,25 < (R_4 + R_5) = \dots \ll V_{YY} / 2 I_{AOVB}$$

what will allow subsequently to calculate of the gain of the configuration nor-inverter

$$R_2 = R_1 [ (V_{Lmax} / V_Z) - 1 ] = \dots$$

For the project of  $R_3$  we will use the two considerations seen in the stabilization by Zener

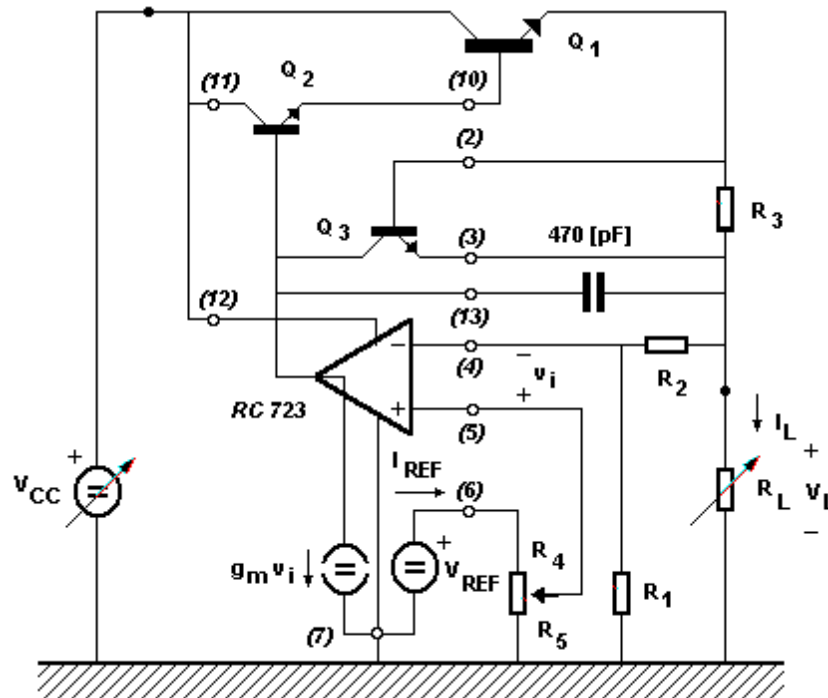
$$R_3 = (V_{CCmin} - V_Z) / [I_{Zmin} + V_Z (R_4 + R_5)^{-1}] = \dots >$$

$$> (V_{CCmax} - V_Z) / [(P_{ZADM} / V_Z) + V_Z (R_4 + R_5)^{-1}]$$

$$P_{R3} = (V_{CCmax} - V_Z)^2 / R_3 = \dots$$

### Source with integrated circuit 723

A variant of the previous case, that is to say with an AOV, it is with the integrated circuit RC723 or similar. It possesses besides the operational one a diode Zener of approximate 7 [V], a TBJ of output of 150 [mA], a second TBJ to protect the short-circuits, and an input capacitive to avoid undesirable oscillations.



The behavior equations will be then

$$V_L = V_{REF} (1 + R_2/R_1) = V_{REF} (1 + R_2G_1) / (1 + R_4G_5)$$

$$V_L = V_{REF} (1 + R_2G_1) / (1 + R_4G_5)$$

and as for the protection

$$I_{LIM} = V_{BE} / R_3 \sim 0,6 / R_3$$

### Design

Be tha data

$$I_{Lmax} = \dots \quad I_{Lmin} = \dots \geq 0 \quad V_{Lmax} = \dots \leq 33 [V] \quad V_{Lmin} = \dots \geq 0$$

and of the manual of data

$$V_{CCADM} \sim 35 [V] \quad I_{REFADM} \sim 0,015 [A] \quad I_{C2ADM} \sim 0,15 [A] \quad V_{REF} \sim 7,1 [V]$$

With the purpose of not dissipating a lot of power in the potentiometers

$$(R_1 + R_2) = \dots (\text{pre-set}) > V_{Lmax}^2 / 0,25$$

$$(R_4 + R_5) = \dots (\text{regulator potentiometer}) > V_{REF}^2 / 0,25$$

and we verify not to exceed the current

$$V_{REF} / (R_4 + R_5) = \dots < I_{REFADM}$$

We choose to source thinking that Q<sub>1</sub> don't saturate; for example 2 [V] because it will be a TBJ of power

$$V_{CC} = \dots = V_{Lmax} + V_{CE1min} \sim V_{Lmax} + 2$$

We adopt the transistor Q1 or Darlington

$$I_{C1max} = I_{Lmax} = \dots$$

$$V_{CE1max} = V_{CC} = \dots < V_{CE01}$$

$$P_{CE1max} = I_{C1max} V_{CE1max} = \dots < P_{CE1ADM}$$

Subsequently the thermal disipator is calculated as it was seen in the respective chapter

$$\text{surface} = \dots$$

$$\text{position} = \dots$$

$$\text{thickness} = \dots$$

We calculate the protective resister

$$R_3 = 0,6 / I_{Lmax} = \dots$$

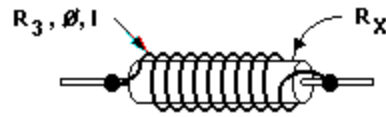
$$P_{R3} = I_{Lmax}^2 R_3 = \dots$$

that to manufacture it, if it cannot buy, it will be willing as coil on another bigger one that serves him as support

$$R_X = \dots \gg R_3$$

$$\varnothing = 0,00035 I_{Lmax}^{1/2} = \dots$$

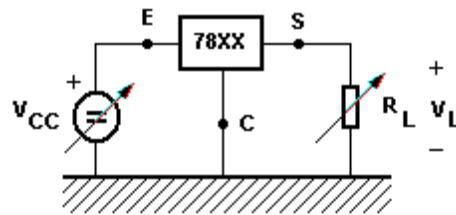
$$l = 45 \cdot 10^6 \varnothing^2 R_3 = \dots$$



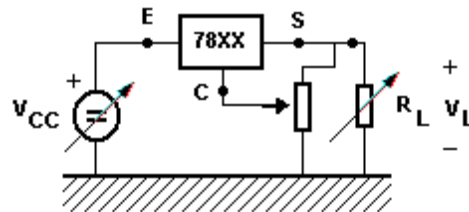
### Source with integrated circuit 78XX

Under the initials 78XX or 79XX, where XX it is the magnitude of output voltage, respectively, positive and negative sources are manufactured.

Demanded with tensions of input and currents of the order of the Ampere with thermal dissipator, they achieve the stabilization of the output voltage efficiently. For more data it is desirable to appeal to their leaves of data.

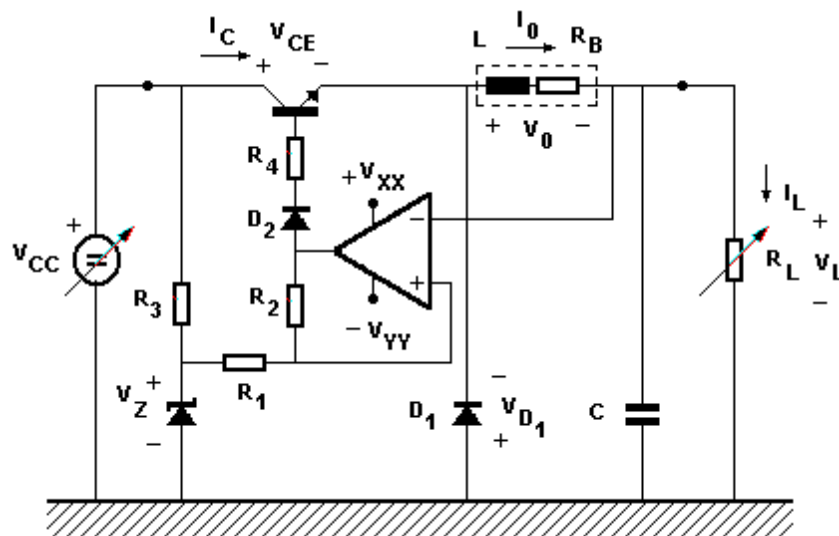


In these chips it is possible to change the regulation voltage if we adjust with one pre-set the feedback, since this integrated circuit possesses an AOV internally

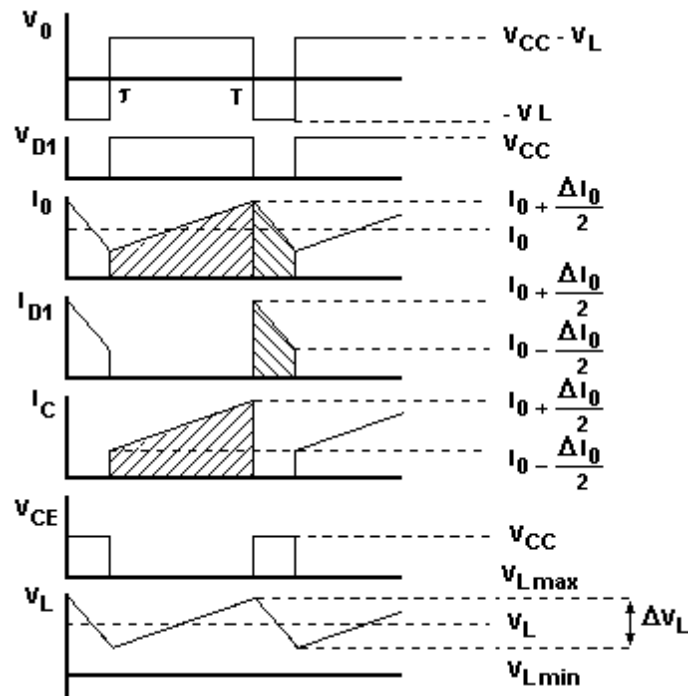


### Source commuted series

With this circuit we can control big powers without demanding to the TBJ since it will work commuted.



In the following graphs we express the operation



It is necessary to highlight that these curves are ideal (that is to say approximate), since it stops practical, didactic ends and of design the voltage has been rejected among saturation collector-emitter  $V_{CES}$  and the 0,6 [V] of the diode rectifier  $D_1$ .

During the interval  $0-\tau$  the magnetic flow of the coil, represented by the current  $I_0$ , circulates him exponentially and, like a constant discharge of time has been chosen it will be a ramp. In the following period  $\tau-T$  the coil discharges its flow exponentially for the diode  $D_1$ .

The AOV simulates the Schmitt-Trigger with  $R_1$  and  $R_2$  and it is then positively realimented to get an effect bistable in the system so that it oscillates.

The  $R_4$  are a current limiter in the base of the TBJ and it allows that the  $V_{XX}$  of the AOV

works with more voltages than the load. In turn, the diode  $D_2$  impedes the inverse voltage to the TBJ when the AOV changes to  $V_{YY}$ .

The voltage of the Zener  $V_Z$  is necessary from the point of view of the beginning of the circuit, since in the first instant  $V_L$  is null. On the other hand, like for  $R_1$  we have current pulses in each commutation, and it is completed that the voltage on her is  $\Delta V_L = V_Z - V_L$  waiting constant  $V_Z$  and  $V_L$ , it will also be then it  $\Delta V_L$ ; for this reason it is convenient to make that  $V_Z$  is the next thing possible to  $V_L$ . If it doesn't have a Zener of the value of appropriate voltage, then it can be appealed to the use of a dividing resistive of the voltage in the load and with it to alimentate the terminal inverter of the AOV.

Let us find some equations that define the behavior of the circuit now.

Let us leave of the fact that we have to work with a period of oscillation where the inductance is sufficiently it reactivates making sure a ramp

$$L / R_B \gg T$$

determining with this

$$I_0 = V_L \tau / L = (T - \tau) (V_{CC} - V_L) / L$$

of where

$$V_L = V_{CC} (1 - \tau/T)$$

También, como

$$\Delta I_0 = C \Delta V_L / \tau + \Delta V_L / R_L \sim C \Delta V_L / \tau$$

and supposing a correct filtrate

$$C R_{Lmin} \gg \tau$$

and on the other hand as

$$V_L = (\pm V_{AOV} - V_Z) R_1 / (R_1 + R_2) + V_Z$$

we have limited the variation

$$\begin{aligned} \Delta V_L &= V_{Lmax} - V_{Lmin} = \\ &= [ (V_{XX} - V_Z) R_1 / (R_1 + R_2) + V_Z ] - [ (-V_{YY} - V_Z) R_1 / (R_1 + R_2) + V_Z ] = \\ &= (V_{XX} + V_{YY}) / (1 + R_2/R_1) \end{aligned}$$

In a same way that when the chapter of sources was studied without stabilizing, we define *critical inductance*  $L_C$  to that limit that would make a change of polarity theoretically  $I_L = \Delta I_0/2$ . Then, combining the previous equations obtains its value

$$L_C = 0,5 T R_{Lmax} [ (V_{CCmin} / V_L) - 1 ]$$

## Design

Be the data

$$I_{Lmax} = \dots \quad I_{Lmin} = \dots \geq 0 \quad V_{CCmax} = \dots \quad V_{CCmin} = \dots \geq 0 \quad V_L = \dots$$

We approach the ranges of the TBJ (to remember that in the beginning  $V_L = 0$ )

$$I_{Cmax} = I_{Lmax} = \dots$$
$$V_{CEmax} = V_{CCmax} = \dots < V_{CE0}$$

and we obtain of their leaves of data

$$V_{CES} = \dots \text{ (approximately 1 [V])}$$

$$I_{CADM} = \dots$$

$$V_{BES} = \dots$$

$$\tau_{apag} = \dots$$

$$\tau_{enc} = \dots$$

$$\beta_{min} = \dots$$

$$T_{JADM} = \dots$$

$$P_{CEADM} = \dots$$

With the conditions of protection of the AOV and commutation of the TBJ we find

$$V_{XX} = \dots > V_{CCmax} + V_{BES} - V_{CES}$$

$$V_{YY} = \dots \leq 36 \text{ [V]} - V_{XX}$$

$$P_{AOVADM} = \dots < V_{XX} I_{Lmax} / \beta_{min}$$

$$I_{B(AOV)} = \dots \text{ (para entrada JFET es nula)}$$

what will allow to calculate at  $R_4$  such that saturates the TBJ; in the worst case

$$R_4 = \beta_{min} ( V_{XX} - 0,6 - V_{BES} + V_{CES} - V_{CCmax} ) / I_{Lmax} = \dots$$

Subsequently we adopt at  $R_1$  of a value anyone, or according to the polarization of their active area in the transition

$$R_1 = \dots \ll V_{YY} / 2 I_{B(AOV)}$$

We choose a small variation of voltage in the load that will be a little bigger than the small among the terminals from the AOV when working actively. A practical magnitude could be 10 [mV]

$$\Delta V_L = \dots \geq 0,01$$



what will allow to clear up of the previous equation

$$R_2 = R_1 [ ( V_{XX} + V_{YY} ) / \Delta V_L - 1 ] = \dots$$

Keeping in mind that to more oscillation frequency the filter will be fewer demanded, and working with sharp flanks for not heating the TBJ, we verify

$$\tau_{enc} + \tau_{apag} = \dots \ll T$$

and consequently we estimate an inductance value and their resistance

$$L = \dots > T V_L (V_{CCmin} / V_L - 1) / 2 I_{Lmin}$$
$$R_B = \dots \ll L / T$$

Subsequently we determine the maximum dynamic current for the inductor

$$\Delta I_{0max} = T V_L ( 1 - V_L / V_{CCmax} ) / L = \dots$$

Now we find the value of the condenser

$$C = \dots \gg \tau / R_{Lmin} = T I_{Lmax} ( V_L^{-1} - V_{CCmax}^{-1} )$$

and we verify the made estimate

$$I_{Lmax} + 0,5 \Delta I_{0max} = \dots < I_{CADM}$$

As in general abacous are not possessed for the determination of the power with pulses on a TBJ (of not being it can be appealed this way to the chapter that it explains and it designs their use), we approach the half value for the worst case ( $\tau \sim T$ )

$$P_{CEmax} \sim V_{CES} ( I_{Lmax} + 0,5 \Delta I_{0max} ) = \dots$$

what will allow to find the thermal disipator

*surface* = ...

*position* = ...

*thickness* = ...

The specifications for the diodes will be

$$I_{RMS1} \sim I_{Lmax} + 0,5 \Delta I_{0max} = \dots$$

$$V_{PEAK REVERSE 1} \sim V_{CCmax} = \dots$$

$$\tau_{RECUP REVERSE 1} = \dots \ll T$$

$$I_{RMS2} \sim I_{Lmax} / \beta_{min} = \dots$$

$$V_{\text{PEAK REVERSE 2}} \sim V_{\text{CCmax}} + V_{\text{YY}} = \dots$$

$$\tau_{\text{RECUP REVERSE 2}} = \dots \ll T$$

With the purpose of that the system begins satisfactorily and let us have good stabilization (we said that in its defect it is necessary to put a Zener of smaller voltage and a dividing resistive in the load that source to the inverter terminal)

$$V_Z \sim > V_L = \dots$$

$$P_{\text{ZADM}} = \dots$$

$$I_{\text{Zmin}} = \dots$$

For not exceeding the current for the AOV the previous adoption it is verified

$$(V_{\text{XX}} - V_Z)(R_1 + R_2)^{-1} + (I_{\text{Lmax}} + 0,5 \Delta I_{0\text{max}}) \beta_{\text{min}}^{-1} = \dots < P_{\text{AOVADM}} / V_{\text{XX}}$$

$$(V_{\text{XX}} + V_Z)(R_1 + R_2)^{-1} = \dots < P_{\text{AOVADM}} / V_{\text{YY}}$$

being also

$$R_3 = (V_{\text{CCmin}} - V_Z) / [I_{\text{Zmin}} + (V_Z + V_{\text{YY}})(R_1 + R_2)^{-1}] = \dots >$$

$$> (V_{\text{CCmax}} - V_Z) / [(P_{\text{ZADM}} / V_Z) - (V_{\text{XX}} - V_Z)(R_1 + R_2)^{-1}]$$

$$P_{\text{R3}} = (V_{\text{CCmax}} - V_Z)^2 / R_3 = \dots$$

## Chap. 09 Amplification of Audiofrequency in low level class A

[Previous theory of the TBJ](#)

[Previous theory of the JFET](#)

[General characteristics of operation](#)

[Bipolar transistor of juncture TBJ](#)

Common emitter

Base common

Common collector

[Transistor of effect of juncture field JFET](#)

Common source

Common gate

Common drain

[Design common emitter](#)

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[Design common collector](#)

[Design common drain](#)

[Adds with AOV](#)

**Design**

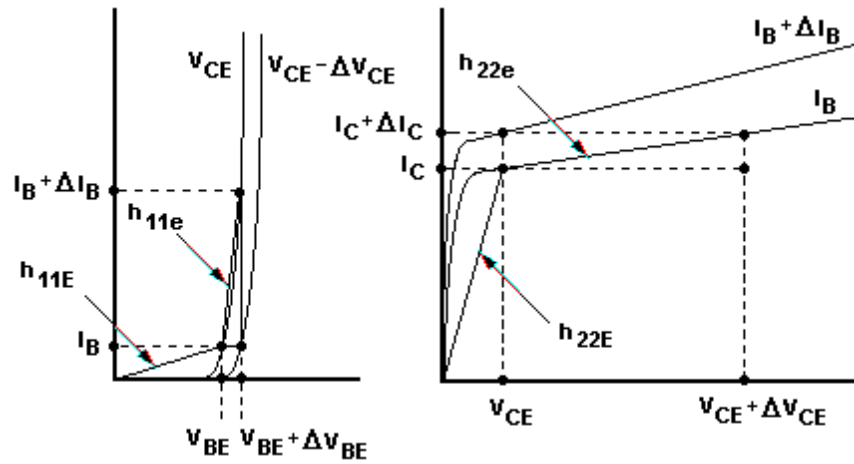
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[Previous theory of the TBJ](#)

Once polarized the transistor, we can understand their behavior with *the hybrid parameters*. In continuous and common emitter is ( $h_{21E} = \beta$ )

$$V_{BE} = h_{11E} I_B + h_{12E} V_{CE}$$

$$I_C = h_{21E} I_B + h_{22E} V_{CE}$$



and dynamically

$$\Delta V_{BE} = h_{11e} \Delta I_B + h_{12e} \Delta V_{CE}$$

$$\Delta I_C = h_{21e} \Delta I_B + h_{22e} \Delta V_{CE}$$

where

$$h_{11e} = \partial V_{BE} / \partial I_B$$

$$h_{12e} = \partial V_{BE} / \partial V_{CE} \sim 0$$

$$h_{21e} = \partial I_C / \partial I_B \sim \beta$$

$$h_{22e} = \partial I_C / \partial V_{CE}$$

or with a simpler terminology

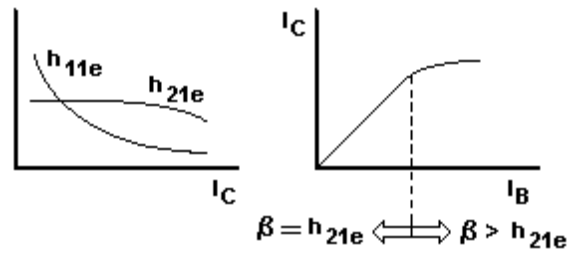
$$v_{be} = h_{11e} i_b + h_{12e} v_{ce} \sim h_{11e} i_b$$

$$i_c = h_{21e} i_b + h_{22e} v_{ce} \sim h_{21e} i_b$$

It will be useful also to keep in mind the transconductance of the dispositive

$$g_m = i_c / v_{be} \sim h_{21e} / h_{11e}$$

We know that these parameters vary with respect to the polarization point, temperature and frequency. Inside a certain area, like sample the figure, we will be able to consider them almost constant.



To measure the parameters of alternating of the simplified transistor, that is to say rejecting  $h_{12e}$  and  $h_{22e}$ , we can appeal to the following circuit, where we will have a short-circuit in the collector if we design

$$h_{22e}^{-1} \gg R_C = \dots \leq 100 [\Omega]$$

and we measure with an oscilloscope for the polarization wanted without deformation (to remember that when exciting with voltage the sign it will be small, because the lineality is only with the current)

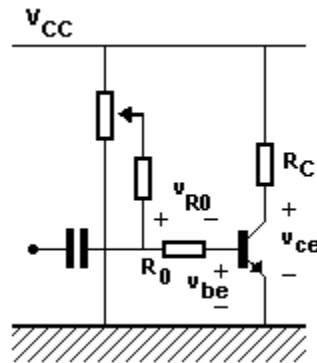
$$V_{CE} = \dots$$

$$I_C = (V_{CC} - V_{CE}) / R_C = \dots$$

$$V_{R0p} = \dots$$

$$V_{cep} = \dots$$

$$V_{bep} = \dots$$



being with it

$$h_{11e} = v_{bep} / i_{bp} = R_0 v_{bep} / V_{R0p} = \dots$$

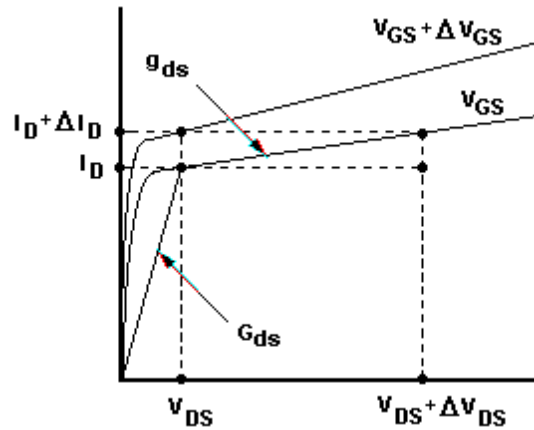
$$h_{21e} = i_{cp} / i_{bp} = R_0 v_{cep} / R_C V_{R0p} = \dots$$

$$g_m = i_{cp} / v_{bep} = \dots$$

### Previous theory of the JFET

Once polarized the JFET one has that

$$I_D = G_m V_{GS} + G_{ds} V_{DS}$$



and taking increments

$$\Delta I_D = g_m \Delta V_{GS} + g_{ds} \Delta V_{DS}$$

$$g_m = \partial I_D / \partial V_{GS}$$

$$g_{ds}^{-1} = r_{ds} = \partial V_{DS} / \partial I_D$$

or with another more comfortable nomenclature

$$i_d = g_m v_{gs} + g_{ds} v_{ds}$$

and if we maintain constant  $I_D$  we find the *amplification factor*  $\mu$

$$0 = g_m v_{gs} + g_{ds} v_{ds}$$

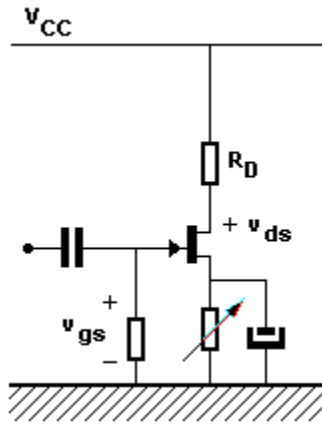
$$\mu = -v_{ds} / v_{gs} = g_m r_{ds}$$

As in general  $r_{ds}$  is big and worthless in front of the resistances in the drain below the 10 [KΩ], it is preferred to use the simplification

$$i_d \sim g_m v_{gs}$$

A practical circuit for the mensuration of the transconductance  $g_m$  is the following one, where it is considered to the connected drain to earth for alternating

$$r_{ds} \gg R_D = \dots \leq 100 [\Omega]$$



and we measure with an oscilloscope

$$V_{DS} = \dots$$

$$I_D = (V_{CC} - V_{DS}) / R_D = \dots$$

$$v_{dsp} = \dots$$

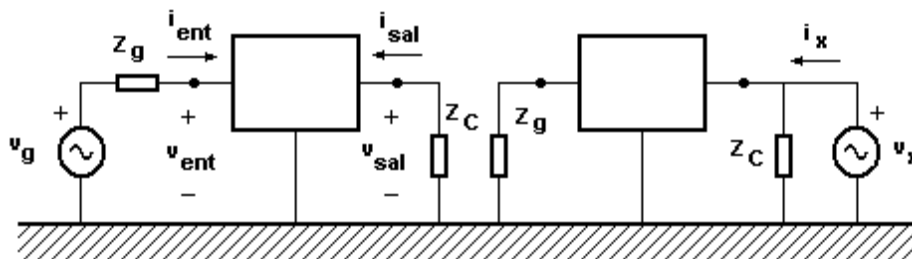
$$v_{gsp} = \dots$$

being with it

$$g_m = i_{dp} / v_{gsp} = R_D v_{dsp} / R_C v_{gsp} = \dots$$

### General characteristics of operation

The methodology that will use responds to the following circuit



### Bipolar transistor of juncture TBJ

#### Common emitter

$$Z_{ent} = v_{ent} / i_{ent} = v_{be} / i_b = h_{11e}$$

$$A_v = v_{sal} / v_{ent} = -i_c Z_C / i_b h_{11e} = -g_m Z_C$$

$$A_i = i_{sal} / i_{ent} = -(v_{sal} / Z_C) / (v_{ent} / Z_{ent}) = -A_v Z_{ent} / Z_C = h_{21e}$$

$$Z_{sal} = v_x / i_x = Z_C$$

### Common base

$$Z_{ent} = v_{ent} / i_{ent} = v_{eb} / i_e = i_b h_{11e} / (i_b + i_b h_{21e}) = h_{11e} / (1 + h_{21e}) \sim g_m^{-1}$$

$$A_v = v_{sal} / v_{ent} = i_c Z_C / i_b h_{11e} = g_m Z_C$$

$$A_i = i_{sal} / i_{ent} = -A_v Z_{ent} / Z_C = h_{21e} / (1 + h_{21e}) \sim 1$$

$$Z_{sal} = v_x / i_x = Z_C$$

### Common collector

$$Z_{ent} = v_{ent} / i_{ent} = [i_b h_{11e} + (i_b + i_b h_{21e}) Z_C] / i_b = h_{11e} + (1 + h_{21e}) Z_C \sim h_{11e} + h_{21e} Z_C$$

$$A_v = v_{sal} / v_{ent} = i_e Z_C / i_b Z_{ent} = [1 + h_{11e} / (1 + h_{21e}) Z_C]^{-1} \approx 1$$

$$A_i = i_{sal} / i_{ent} = -A_v Z_{ent} / Z_C = 1 + h_{21e} \sim h_{21e}$$

$$Z_{sal} = v_x / i_x = Z_C // [i_b (h_{11e} + Z_g) / i_e] \sim Z_C // [(h_{11e} + Z_g) / h_{21e}]$$

### Transistor of effect of juncture field JFET

The considerations are similar that for the TBJ but with  $r_{gs} = h_{11e} = \infty$ .

### Common source

$$Z_{ent} = v_{ent} / i_{ent} = \infty$$

$$A_v = v_{sal} / v_{ent} = -g_m Z_C$$

$$A_i = i_{sal} / i_{ent} = \infty \text{ (no entra corriente)}$$

$$Z_{sal} = v_x / i_x = Z_C$$

### Common gate

$$Z_{ent} = v_{ent} / i_{ent} = g_m^{-1}$$

$$A_v = v_{sal} / v_{ent} = g_m Z_C$$

$$A_i = i_{sal} / i_{ent} = 1$$

$$Z_{sal} = v_x / i_x = Z_C$$

### Common drain

$$Z_{ent} = v_{ent} / i_{ent} = \infty$$

$$A_v = v_{sal} / v_{ent} \approx 1$$

$$A_i = i_{sal} / i_{ent} = \infty \text{ (no entra corriente)}$$

$$Z_{sal} = v_x / i_x = Z_C // g_m^{-1}$$

### Design common emitter



Interested only in dynamic signs, the big capacitances of the circuit will maintain their voltages and they are equivalent to generators of ideal voltage with a value similar to the one that it have in their polarization.

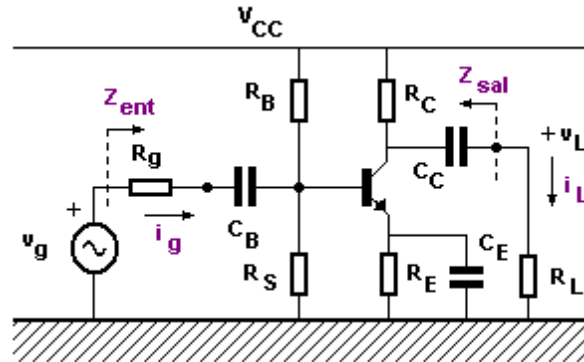
The same as like it has been made previously we find

$$A_v = v_L/v_g = -i_c (R_C//R_L) / i_g [R_g + (h_{11e}/R_B//R_S)] = -g_m (R_C//R_L) / [1 + R_g/(h_{11e}/R_B//R_S)]$$

$$A_i = i_L/i_g = -i_c[(R_C//R_L)/R_L] / i_b[h_{11e}/(h_{11e}/R_B//R_S)] = -h_{21e}/(1 + R_L/R_C)[1 + h_{11e}/(R_B//R_S)]$$

$$Z_{ent} = v_g/i_g = i_g [R_g + (h_{11e}/R_B//R_S)]/i_g = R_g + (h_{11e}/R_B//R_S)$$

$$Z_{sal} = v_{sal}/i_{sal} = R_C/(v_{sal}/i_c) = R_C$$



If the following data are had

$$|A_{vmin}| = \dots \quad R_g = \dots \quad R_L = \dots \quad f_{min} = \dots$$

we choose a TBJ and of the manual or their experimentation we find

$$V_{CE} = \dots \quad I_C = \dots \quad \beta = \dots \quad h_{21e} = \dots \quad h_{11e} = \dots \quad g_m = h_{21e}/h_{11e} = \dots$$

Keeping in mind that seen in the polarization chapter adopts

$$V_{RE} = \dots \geq 1 \text{ [V]}$$

$$1 \leq S_I = \dots \leq 20$$

originating

$$R_C = V_{CE} / I_C = \dots$$

$$R_E \sim V_{RE} / I_C = \dots$$

$$V_{CC} = 2 V_{CE} + V_{RE} = \dots$$

$$R_B = (S_I - 1) R_E V_{CC} / [0,6 + V_{RE} + (S_I - 1) R_E I_C \beta^{-1}] = \dots$$

$$R_S = \{ [(S_I - 1) R_E]^{-1} - R_B^{-1} \}^{-1} = \dots$$

and we verify the gain

$$g_m (R_C//R_L) / [1 + R_g/(h_{11e}/R_B//R_S)] = \dots \geq |A_{vmin}|$$

So that the capacitances of it coupled they don't present comparable voltage in front of the resistance that you go in their terminals, it is

$$R_L \gg 1 / \omega_{\min} C_C \Rightarrow C_C = \dots \gg 1 / \omega_{\min} R_L$$

$$h_{11e} // R_B // R_S \gg 1 / \omega_{\min} C_B \Rightarrow C_B = \dots \gg 1 / \omega_{\min} h_{11e} // R_B // R_S$$

and that of disacoupled

$$h_{11e} \gg (1+h_{21e}) \cdot 1 / \omega_{\min} C_E$$

that it will usually give us a very big  $C_E$ . To avoid it we analyze the transfer of the collector circuit better with the help of the power half to  $\omega_{\min}$ , that is to say  $\sim 3$  [dB]

$$|v_c / v_b| = i_c (R_C // R_L) / [i_b h_{11e} + (i_c + i_b) Z_E] \sim 0,707 g_m (R_C // R_L)$$

of where then

$$C_E = \dots > (h_{21e}^2 + 2h_{21e} h_{11e} / R_E)^{1/2} / h_{11e} \omega_{\min}$$

### Design common base

If we call using Thevenin

$$R_T = R_g // R_E$$

$$v_T = v_g R_E / (R_E + R_g)$$

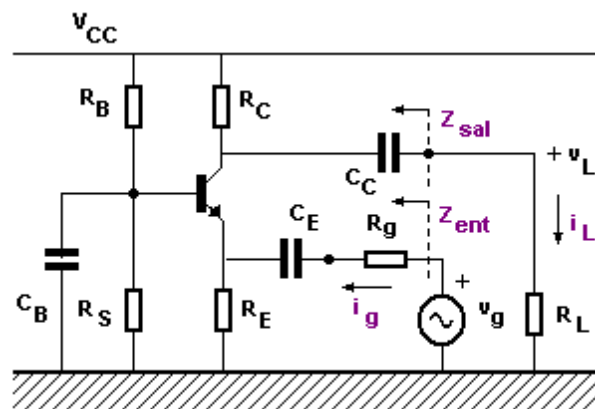
the same as like we have made previously we can find

$$A_v = v_L / v_g = i_c (R_C // R_L) / v_T [1 + (R_g / R_E)] \sim g_m (R_C // R_L) / [1 + R_g / (R_E // g_m)]$$

$$Z_{ent} = v_g / i_g = R_g + [R_E // (v_{eb} / i_e)] \sim R_g + (R_E // g_m)$$

$$A_i = i_L / i_g = A_v Z_{ent} / R_L = (1 + R_L / R_C) (1 + 1 / R_E g_m)$$

$$Z_{sal} = v_{sal} / i_{sal} = R_C // (v_{sal} / i_c) = R_C$$



If the following data are had

$$|A_{vmin}| = \dots \quad R_g = \dots \quad R_L = \dots \quad f_{min} = \dots$$

we choose a TBJ and of the manual or their experimentation we find

$$V_{CE} = \dots \quad I_C = \dots \quad \beta = \dots \quad h_{21e} = \dots \quad h_{11e} = \dots \quad g_m = h_{21e} / h_{11e} = \dots$$

Keeping in mind that explained in the polarization chapter, we adopt

$$V_{RE} = \dots \geq 1 \text{ [V]} \\ 1 \leq S_I = \dots \leq 20$$

originating

$$R_C = V_{CE} / I_C = \dots \\ R_E \sim V_{RE} / I_C = \dots \\ V_{CC} = 2 V_{CE} + V_{RE} = \dots \\ R_B = (S_I - 1) R_E V_{CC} / [0,6 + V_{RE} + (S_I - 1) R_E I_C \beta^{-1}] = \dots \\ R_S = \{ [(S_I - 1) R_E]^{-1} - R_B^{-1} \}^{-1} = \dots$$

and we verify the gain

$$g_m (R_C // R_L) / [1 + R_g / (R_E // g_m)] = \dots \geq |A_{vmin}|$$

So that the capacitances of it coupled they don't present comparable voltage in front of the resistance among their terminals it is

$$R_L \gg 1 / \omega_{min} C_C \Rightarrow C_C = \dots \gg 1 / \omega_{min} R_L$$

and that of desacoupled

$$h_{11e} \gg 1 / \omega_{min} C_B \Rightarrow C_B = \dots \gg 1 / \omega_{min} h_{11e}$$

and in a similar way we reason with the condenser of the emitter

$$R_E // g_m \gg 1 / \omega_{min} C_E$$

but that it will usually give us a very big  $C_E$ . To avoid it we analyze the transfer better from the collector circuit to base with the power half to  $\omega_{min}$ , that is to say  $\sim 3$  [dB]

$$|v_c / v_{ent}| = i_c (R_C // R_L) / [i_b h_{11e} + (i_c + i_b) Z_E] \sim 0,707 g_m (R_C // R_L)$$

of where then

$$C_E = \dots > 1 / \omega_{\min} g_m$$

### Design common collector

If we call using Thevenin

$$R_T = R_g // R_B // R_S$$

$$v_T = v_g (R_B // R_S) / (R_g + R_B // R_S)$$

the same as like it has been made previously we find

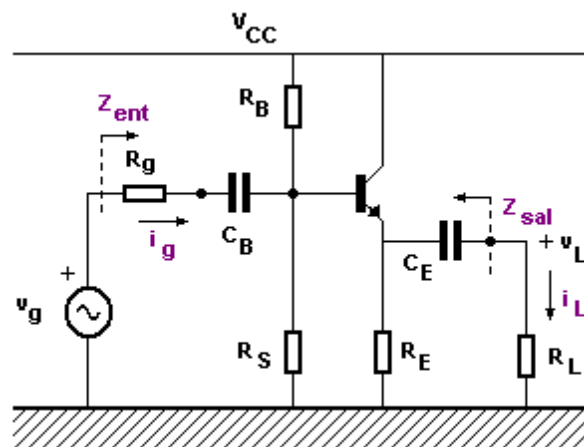
$$A_v = v_L / v_g = i_e (R_E // R_L) / v_T [(R_g + R_B // R_S) / (R_B // R_S)] \sim$$

$$\sim [h_{21e} R_E // R_L / (h_{11e} + h_{21e} R_E // R_L)] \cdot \{ (1 + R_g / R_B // R_S) [1 + R_T / (h_{11e} + h_{21e} R_E // R_L)] \}^{-1} \sim 1$$

$$Z_{ent} = v_g / i_g = R_g + R_B // R_S // [(i_b h_{11e} + i_e R_E // R_L) / i_b] \sim R_g + R_B // R_S // (h_{11e} + h_{21e} R_E // R_L)$$

$$A_i = i_L / i_g = A_v Z_{ent} / R_L \approx h_{21e}$$

$$Z_{sal} = v_{sal} / i_{sal} = R_E // [(i_b (h_{11e} + R_T) / i_e)] \sim R_E // [(h_{11e} + R_g // R_B // R_S) / h_{21e}]$$



If the following data are had

$$R_{ent} = \dots \quad R_g = \dots \quad R_L = \dots \quad f_{\min} = \dots$$

we choose a TBJ and of the manual or their experimentation we find

$$V_{CE} = \dots \quad I_C = \dots \quad \beta = \dots \quad h_{21e} = \dots \quad h_{11e} = \dots \quad g_m = h_{21e} / h_{11e} = \dots$$

Keeping in mind that seen in the polarization chapter adopts

$$V_{RE} = V_{CE} = \dots \geq 1 \text{ [V]}$$

$$1 \leq S_I = \dots \leq 20$$

originating

$$R_E \sim V_{RE} / I_C = \dots$$

$$V_{CC} = 2 V_{CE} = \dots$$

$$R_B = (S_I - 1) R_E V_{CC} / [0,6 + V_{RE} + (S_I - 1) R_E I_C \beta^{-1}] = \dots$$

$$R_S = \{ [(S_I - 1) R_E]^{-1} - R_B^{-1} \}^{-1} = \dots$$

and we verify the input resistance

$$R_g + R_B // R_S // (h_{11e} + h_{21e} R_E // R_L) = \dots \geq R_{ent}$$

So that the capacitances of it couples they don't present comparable voltage in front of the resistance in their terminals, it is

$$R_L \gg 1 / \omega_{min} C_E \Rightarrow C_E = \dots \gg 1 / \omega_{min} R_L$$

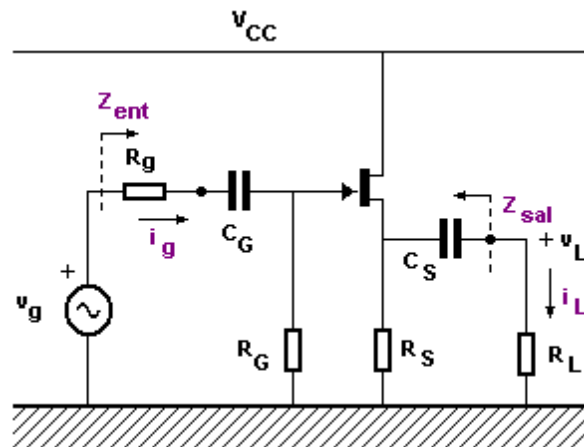
$$R_{ent} \gg 1 / \omega_{min} C_B \Rightarrow C_B = \dots \gg 1 / \omega_{min} [R_g + R_B // R_S // (h_{11e} + h_{21e} R_E // R_L)]$$

### Design common drain

Generally used to adapt impedances, that is to say with the purpose of not loading to the excitatory generator, the following circuit will be the one proposed. Let us find their main ones then characteristic

$$Z_{ent} = v_g / i_g = R_g + R_G$$

$$Z_{sal} = v_{sal} / i_{sal} = R_E // (v_{sg} / i_d) = R_E // g_m$$



If then we have the following data

$$R_{ent} = \dots \quad R_{sal} = \dots \quad R_g = \dots \quad R_L = \dots \quad f_{min} = \dots$$

we choose a JFET and of the manual or their experimentation we find

$$V_P = \dots \quad I_{DSS} = \dots \quad I_{G0} = \dots$$

If we keep in mind that  $R_{sal} = g_m^{-1}$  in this configuration, and that

$$g_m = \partial I_D / \partial V_{GS} \sim 2 I_{DSS} (1 + V_{GS} / V_P) / V_P$$

we make

$$0 > V_{GS} = \dots \geq [V_P (V_P / 2 I_{DSS} R_{sal})] - 1 = \dots$$

$$I_D = I_{DSS} (1 + V_{GS} / V_P)^2 = \dots$$

$$V_{DS} = \dots \geq V_P$$

$$V_{CC} = V_{DS} - V_{GS} = \dots$$

$$R_S = -V_{GS} / I_D = \dots$$

$$R_{ent} \leq R_G = \dots \ll -V_{GS} / I_{G0}$$

We design the capacitors of it coupled so that to the minimum frequency they have worthless reactance

$$R_L \gg 1 / \omega_{min} C_S \Rightarrow C_S = \dots \gg 1 / \omega_{min} R_L$$

$$R_G \gg 1 / \omega_{min} C_G \Rightarrow C_G = \dots \gg 1 / \omega_{min} R_G$$

### Adds with AOV

Commonly to this circuit it denominates it to him *mixer*. In the following figure we see a possible implementation, where it is observed that it is not more than an amplifier inverter of «n» entrances, and that it possesses a filter of high frequencies in their feedback circuit.

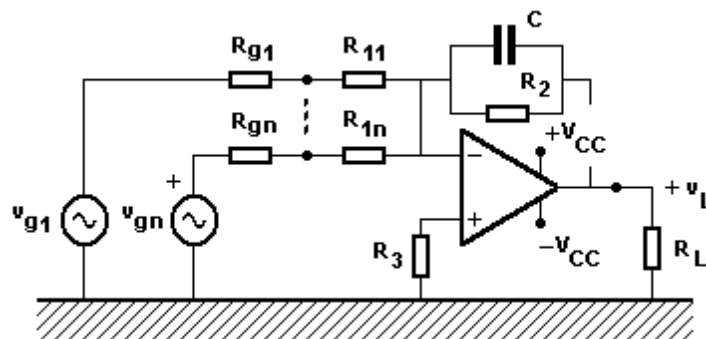
Their behavior equations are

$$R_1 = R_{g1} + R_{1n}$$

$$R_{ent n} = R_{1n}$$

$$A_{vn} = Z_2 / R_1$$

$$v_L = (v_{g1} + v_{g2} + \dots v_{gn}) A_{vn} = - (v_{g1} + v_{g2} + \dots v_{gn}) (R_2 // sC^{-1}) / R_1$$



## Design

Be the data

$$R_{g1} = \dots \quad R_{g2} = \dots \quad R_{gn} = \dots$$
$$f_{\max} = \dots \quad A_{v0} = \dots \text{ (minimum gain in the band pass)}$$

We choose an AOV and of the manual or their experimentation we obtain

$$V_{CC} = \dots \quad I_B = \dots \text{ (con JFET } I_B = 0)$$

therefore

$$R_2 = \dots \ll V_{CC} / 2 I_B$$
$$R_{1n} = (R_2 / A_{v0}) - R_{gn} = \dots$$
$$R_3 \sim R_2 // (R_1 / n) = R_2 // [(R_2 / A_{v0}) / n] = \dots \text{ (con JFET } R_3 = 0)$$

If we design the capacitor so that it produces the power half to the minimum specified frequency

$$A_{v(\omega \max)} = 0,707 A_{v0} = R_2 / R_1 [1 + (\omega_{\max} C R_2)^2]^{1/2} \Rightarrow C = 1 / \omega_{\max} R_2 = \dots$$

As for any AOV the maximum advisable power is of the order of 0,25 [W], we prevent

$$0,25 > V_{CC}^2 / R_L \Rightarrow R_L = \dots \geq 1 \text{ [K}\Omega\text{]}$$

## Chap. 10 Amplification of Audiofrequency on high level classes A and B

### Generalities

Efficiency of a stage

Linearity of the amplification

Maximum dissipated power

Amplifier without coupled (class A)

Amplifier with inductive coupled (class A)

Design

Amplifier with coupled capacitive (class B)

Design

Differential variant

Amplifier with the integrated circuit 2002

Speakers and acoustic boxes

Design

Acoustic filters

Design

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### Generalities

#### Efficiency of a stage

Considering to an amplifier like energy distributor, we observe the following thing

$P_{ENT}$  power surrendered by the power supply

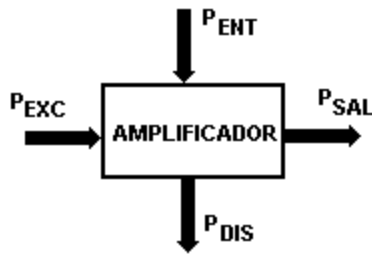
$P_{EXC}$  excitatory power (worthless magnitude)

$P_{SAL}$  output power on the useful load

$P_{DIS}$  power dissipated by the amplifier (their exit component/s)

$$P_{ENT} \sim P_{SAL} + P_{DIS}$$





and we denominate their *efficiency* to the relationship

$$\eta = P_{SAL} / P_{ENT}$$

of where it is also deduced

$$P_{DIS} = P_{ENT} - P_{SAL} = P_{SAL} (\eta^{-1} - 1)$$

### Lineality of the amplification

To study the behavior here of the exit transistors with parameters of low sign doesn't make sense. It will be made with those of continuous.

It is also important to the transistors to excite them with current and not with voltage, since their lineality is solely correct with the first one.

To improve all lineality of the amplifications there is that feed-back negatively. The percentage of harmonic distortion D decreases practically in the factor  $1+GH$ .

### Maximum dissipated power

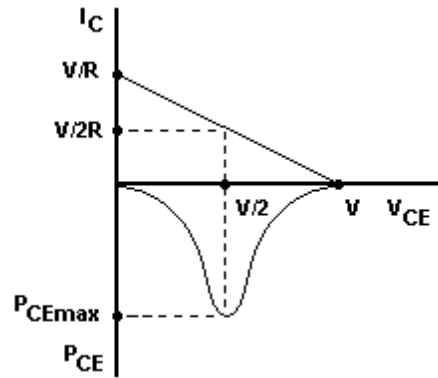
When a TBJ possesses an operation straight line like sample the figure, the power among collector-emitter goes changing measure that the work point moves, and there will be a maximum that we want to find. Their behavior equations are the following ones

$$I_C = (V - V_{CE}) / R$$

$$P_{CE} = I_C V_{CE} = V V_{CE} / R - V_{CE}^2 / R$$

$$\partial P_{CE} / \partial V_{CE} = V / R - 2 V_{CE} / R$$

$$[\partial P_{CE} / \partial V_{CE}]_{P_{CEmax}} = 0 \Rightarrow P_{CEmax} = V^2 / 4 R$$



### Amplifier without coupled (class A)

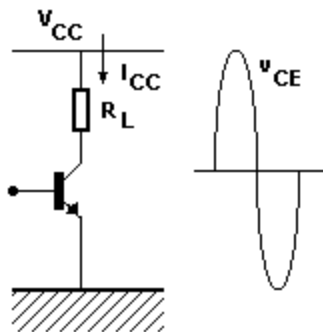
Although this is not a practical circuit due to their bad *efficiency*, yes it will be didactic for our studies. Subsequently we express their behavior equations

$$P_{SALmax} = P_{Lmax} = (0,707 v_{Lp})^2 / R_L = (0,707 V_{CC}/2)^2 / R_L = V_{CC}^2 / 8 R_L$$

$$P_{ENTmax} = V_{CC} I_{Cmed} = V_{CC} (V_{CC} / 2 R_L) = V_{CC}^2 / 2 R_L$$

$$\eta = P_{SALmax} / P_{ENTmax} = 0,25$$

$$P_{DISmax} = P_{CEmax} = P_{SALmax} (\eta^{-1} - 1) = 3 P_{SALmax} = 0,375 V_{CC}^2 / R_L$$



### Amplifier with inductive coupled (class A)

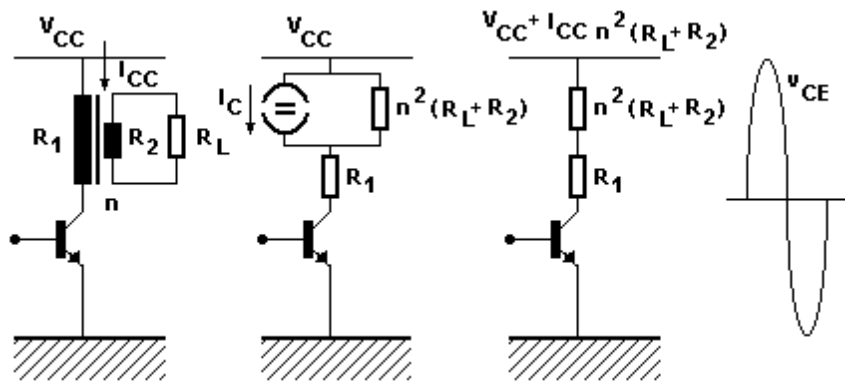
The circuit is the following, where the effect of over-voltage of the inductance magnetic that will improve the *efficiency* of the stage. This way they are the equations

$$P_{SALmax} = P_{Lmax} = (0,707 V_{CC})^2 / n^2 R_L = V_{CC}^2 / 2 n^2 R_L$$

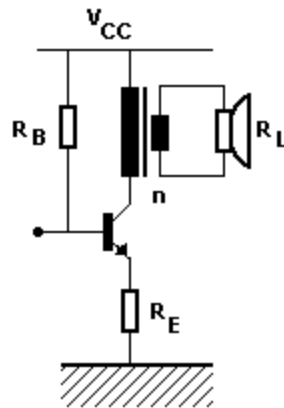
$$P_{ENTmax} = V_{CC} I_{Cmed} = V_{CC} [(2 V_{CC} / n^2 R_L) / 2] = V_{CC}^2 / n^2 R_L$$

$$\eta = P_{SALmax} / P_{ENTmax} = 0,5$$

$$P_{DISmax} = P_{CEmax} = P_{SALmax} (\eta^{-1} - 1) = P_{SALmax} = V_{CC}^2 / 2 n^2 R_L$$



In the practice a small resistance is usually put in the emitter  $R_E$  with two ends: first, so that the voltage in the base excites for current ( $R_{EN} \sim \beta R_E$ ) and not for voltage to avoid deformations the sign (the  $\beta$  is only lineal in the TBJ), and second to stabilize the work point since the transistor will be hot.



## Design

Be the data

$$R_L = \dots \quad P_{Lmax} = \dots \text{ (power for a single tone)} \quad f_{max} = \dots \quad f_{min} = \dots$$

We adopt a convenient source

$$V_{CC} = \dots$$

what implies

$$n = N_1 / N_2 = (V_{CC}^2 / 2 R_L P_{Lmax})^{1/2} = \dots$$

and then we determine the winding of the transformer according to that seen in their respective chapter

$$R_1 = \dots \ll n^2 R_L$$

$$R_2 = \dots \ll R_L$$

$$\begin{aligned}\varnothing_1 &= \dots > 0,00065 [ (I_{CC}^2 + I_{Cef}^2)^{1/2} ]^{1/2} \sim 0,0001 (P_{Lmax} / n^2 R_L)^{1/2} \\ \varnothing_2 &= \dots > 0,00065 (n I_{Cef})^{1/2} \sim 0,00077 (P_{Lmax} / n R_L)^{1/2}\end{aligned}$$

we choose an inductance that verifies the generating effect of current and guarantee  $2V_{CC}$  (we call L to the magnetic inductance of the primary  $L_1$ )

$$\omega_{min} L \gg n^2 R_L \Rightarrow L = \dots \gg n^2 R_L / \omega_{min}$$

and to verify their magnitude the equations and abacous they could be used that were presented in the respective chapter

$$\begin{aligned}H_Q &= N_1 I_C / l_{Fe} \\ \Delta B &= V_{CC} / S N_1 \omega \\ L &= N_1^2 S \Delta \mu_{ef} / l_{Fe} = N_1^2 S / [ (l_{Fe} / \Delta \mu_{ef}) + (l_A / \mu_A) ]\end{aligned}$$

or appealing to the empiric mensurations.

We find the data next to choose the TBJ

$$\begin{aligned}I_C &= V_{CC} / n^2 R_L = \dots \\ V_{CE} &\sim V_{CC} / 2 = \dots \\ I_{Cmax} &= 2 I_C = \dots \\ V_{CEmax} &= 2 V_{CC} = \dots \\ P_{CEmax} &= V_{CC}^2 / 2n^2 R_L = \dots\end{aligned}$$

and we obtain of the same one

$$\begin{aligned}T_{JADM} &= \dots \\ P_{CEADM} &= \dots \\ \theta_{JC} &= (T_{JADM} - 25) / P_{CEADM} = \dots \\ \beta &\sim \dots\end{aligned}$$

and for the dissipator

$$\begin{aligned}surface &= \dots \\ position &= \dots \\ thickness &= \dots\end{aligned}$$

Subsequently we choose a small feedback in the emitter that doesn't affect the calculations

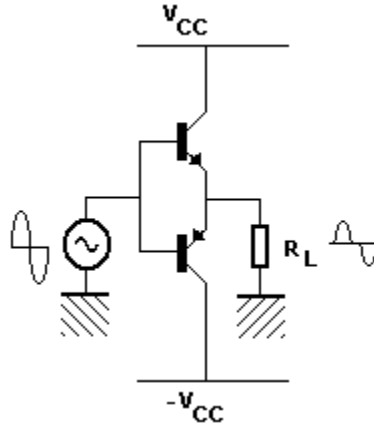
$$\begin{aligned}R_E &= \dots \ll n^2 R_L \\ P_{REmax} &= (I_{CC}^2 + I_{Cef}^2) R_E \sim 1,5 I_C R_E = \dots\end{aligned}$$

and we finish polarizing

$$R_B = (V_{CC} - 0,6 - I_C R_E) \beta / I_C = \dots$$

### Amplifier with coupled capacitive (class B)

The circuit following typical class B *complementary symmetry* is denominated. The coupled capacitive to the load is carried out through the condenser of the negative source of power supply (not indicated in the drawing).



For the ideal system we have the following equations for an unique tone of sine wave

$$P_{SALmax} = P_{Lmax} = (0,707 V_{CC})^2 / R_L = V_{CC}^2 / 2R_L$$

$$P_{ENTmax} = 2 V_{CC} I_{Cmed} = 2 V_{CC} (i_{Lp} / \pi) = 2 V_{CC} (V_{CC} / \pi R_L) = 2 V_{CC}^2 / \pi R_L$$

$$\eta = P_{SALmax} / P_{ENTmax} = \pi / 4 \sim 0,78$$

$$P_{DISmax} = 2 P_{CEmax} = P_{SALmax} (\eta^{-1} - 1) \sim 0,28 P_{SALmax} = 0,14 V_{CC}^2 / R_L$$

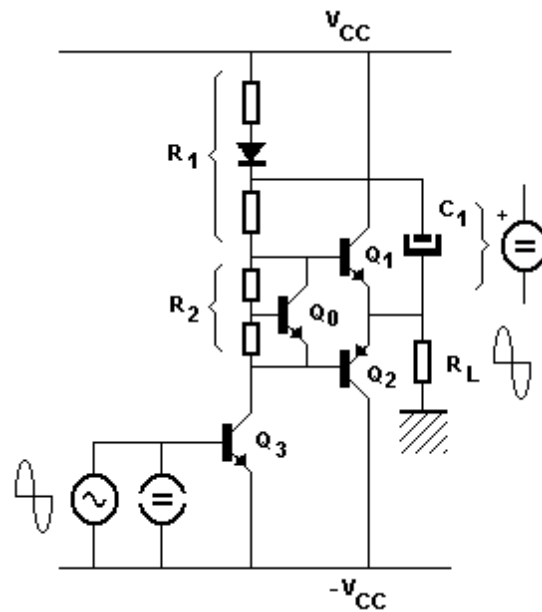
modifying for a square sign as it was seen previously in class A

$$P_{CEmax} = V_{CC}^2 / 4 R_L$$

$$P_{DISmax} = 2 P_{CEmax} = 0,5 V_{CC}^2 / R_L$$

The following circuit perfects to the previous one to be more elaborated. This circuit if it didn't have at  $R_2$  it has a distortion for not polarizing the bases, and that it is denominated *crusade distortion* —previously view in the previous implementation. Added this, the asymmetry of the complementary couple's excitement possesses a deficiency in the positive signs on the load; for it is designed it a source of extra power that this behavior increases, and whose responsible it is the capacitor  $C_1$  denominated bootstrap (because it produces feedback: "to throw of the cord of the boots to put on shoes"). The diode impedes the discharge of  $C_1$  if its positive plate rises in voltage  $V_{CC}$  it has more than enough. If this reforzador is not, it would be necessary to design a  $R_1$  very small and it would polarize in class A at  $Q_3$  being inefficient the system

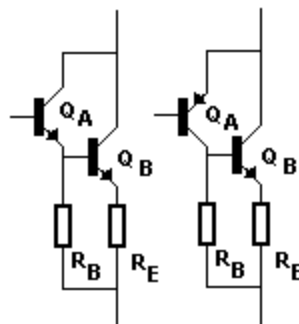
$$R_1 \ll V_{CC} / I_{B1max} \sim \beta_1 R_L$$



A fourth transistor  $Q_0$  has been connected. This will improve the stabilization of the landslides of the polarization in class B of the exit couple when powers are managed in the load superiors at 5 [W]. This is because they are, before sign, hot. This way their currents of losses collector-base will affect to the base of  $Q_0$  increasing their current  $I_{C0}$  and it will avoid that the first one enters to the complementary couple's bases. For this reason it will be advised, although not necessary, that  $Q_0$  are coupled thermally to the dissipator of the complementary ones, this way their own one  $I_{CB00}$  will be added to the effect and it will regulate the work point in class B.

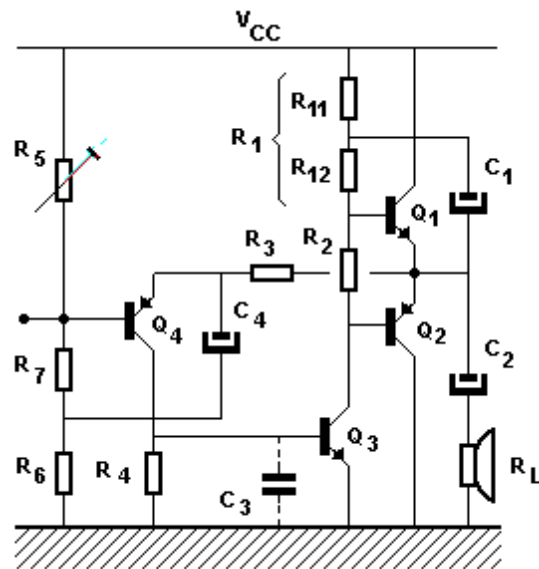
It is also usual to connect small resistors in the exit couple's emitters. For further powers it is connected in Darlington the complementary couple, but with NPN in their exits for the economic cost, reason why it is denominated to the system of *quasi-complementary symmetry*. There are many variants with the connections of resistors and diodes. The following one belongs only one to them, where the proyect approaches for  $R_E$  and  $R_B$  can be consulted in the chapter of *polarization of dispositives*.

$$\beta \sim \beta_A \beta_B$$



The final circuit that we will design is presented next. We have eliminated the double source for their substitution the condenser  $C_2$ . The transistor  $Q_4$  provides the necessary negative feedback and it serves from preamplifier to  $Q_3$ , and its adjustment of polarization  $R_5$  polarizes to the complete

circuit. The condenser  $C_3$  is optional since it will eliminate oscillations and undesirable interferences.



The behavior equations are for the polarization (to remember that to practical ends the voltages base-emitter varies between 0,6 and 0,75 [V] and they approach all in 0,6 [V])

$$R_1 = R_{11} + R_{12}$$

$$I_{R12} = I_{C3} = I_{B1} + (2 \cdot 0,6 / R_2) = \text{it is constant for the bootstrap}$$

$$V_{C1} = I_{C3} R_{12} + 0,6$$

for the excitement to full positive load

$$I_{C3min} = 0$$

$$I_{B1max} = I_{R12}$$

$$I_{C1max} = I_{B1max} / \beta_1 = I_{R12} / \beta_1$$

to full load negative

$$I_{C3max} = I_{R12} + I_{B2max}$$

$$I_{B2max} = I_{C2max} / \beta_2 = v_{Lmax} / \beta_2 R_L = [(V_{CC} / 2) - 0,6] / \beta_2 R_L$$

$$I_{C2max} = I_{B2max} / \beta_2 = [(V_{CC} / 2) - 0,6] / R_L$$

and for the sign

$$A_v = v_L / v_{b4} \sim 1 + R_3 / R_6$$

$$R_{ent} \sim R_5$$

## Design

Be the data

$$R_L = \dots \quad P_{Lmax} = \dots \text{ (power for a single tone)} \quad f_{max} = \dots \quad f_{min} = \dots \quad A_v = \dots$$

We calculate a convenient alimentation having present the double source and the possible affection of the voltage base-emitter

$$P_{SALmax} = P_{Lmax} = [ (V_{CC} - 0,6) / 2 ]^2 / 2R_L \Rightarrow V_{CC} = ( 8 P_{Lmax} R_L )^{1/2} + 0,6 = \dots$$

Subsequently we obtain the exit couple's data in the worst case

$$P_{CE1max} = (V_{CC} / 2)^2 / 4 R_L = V_{CC}^2 / 16 R_L = \dots$$

$$I_{C1max} = V_{CC} / 2 R_L = \dots$$

$$V_{CE1max} = V_{CC} = \dots$$

and of the manual polarizing them practically to the cut with  $V_{CE1} = V_{CC} / 2$

$$I_{C1} = \dots$$

$$\beta_1 \sim \dots$$

$$T_{JADM1} = \dots$$

$$P_{CEADM1} = \dots$$

$$\theta_{JC1} = (T_{JADM1} - 25) / P_{CEADM1} = \dots$$

and for the dissipator of each one

$$surface = \dots$$

$$position = \dots$$

$$thickness = \dots$$

We calculate

$$I_{B1max} = [ (V_{CC} / 2) - 0,6 ] / \beta_1 R_L = \dots$$

We adopt the important current of the system

$$I_{C3} = \dots > I_{B2max} = I_{B1max}$$

originating

$$R_2 = 2 \cdot 0,6 / (I_{C3} - I_{B1}) = 1,2 / [I_{C3} - (I_{C1} / \beta_1)] = \dots$$

$$R_{12} = (V_{BE1max} - V_{BE1}) / (I_{C3} - I_{B1max}) \approx 0,15 [V] / (I_{C3} - I_{B1max}) = \dots$$

$$P_{R12} \sim I_{C3}^2 R_{12} = \dots$$

$$R_{11} = R_1 - R_{12} = \{ [ (V_{CC} / 2) - 0,6 ] / I_{C3} \} - R_{12} = \dots$$

$$P_{R11} \approx V_{CC}^2 / 8 R_{11} = \dots \quad (R_{11} \text{ it is practically in parallel with } R_L)$$



For the calculation of the limits of  $Q_3$  it suits to remember that their operation line is not in fact that of a straight line. Consequently we determine their demand

$$\begin{aligned} I_{C3\max} &= I_{R12} + I_{B2\max} = [ ( I_{C1} / \beta_1 ) + ( 2 \cdot 0,6 / R_2 ) ] + I_{B1\max} = \dots \\ V_{CE3\max} &= V_{CC} = \dots \\ P_{CE3\max} &\sim ( V_{CE3\max} / 2 ) ( I_{C3\max} / 2 ) = \dots \end{aligned}$$

and like it is

$$V_{CE3} = ( V_{CC} / 2 ) - 0,6 = \dots$$

then the transistor is chosen and it is

$$\beta_3 = \dots$$

being able to need a small dissipator in some cases.

For not altering the made calculations it is chosen

$$I_{C4} = \dots \ll I_{C1}$$

and for the approach seen in the *polarization* chapter we adopt

$$\begin{aligned} V_{R3} &= \dots \geq 1 \text{ [V]} \\ I_{R5} &= \dots \sim I_{C4} \end{aligned}$$

what will determine finally

$$\begin{aligned} R_3 &= V_{R3} / I_{C4} = \dots \\ R_4 &= V_{R4} / I_{R4} \sim 0,6 / I_{C4} = \dots \\ R_5 &= V_{R5} / I_{R5} = [ 0,6 + V_{R3} + ( V_{CC}/2 ) ] / I_{R5} = \dots \text{ (to choose it bigger to be an adjustment)} \\ R_6 &= R_3 / ( A_v - 1 ) = \dots \\ R_7 &\sim ( V_{CC} - I_{R5} R_5 ) / I_{R5} = \dots \end{aligned}$$

The condenser bootstrap will possess the following voltage

$$V_{C1} = I_{C3} R_{12} + 0,6 = \dots$$

and their discharge that we will avoid it will be totally on a circuit alineality that it will go changing in each hemicicle. We can approach it considering that to the minimum frequency (of the worst case) it will have a great constant of time

$$\begin{aligned} C_1 [ R_{11} // ( R_{12} + h_{11E1} ) ] &>> 1 / f_{\min} && \rightarrow \text{positive hemicilce in the load} \\ C_1 [ R_{11} // ( R_{12} + R_2 ) ] &>> 1 / f_{\min} && \rightarrow \text{negative hemicilce in the load} \end{aligned}$$

reason why it is better to experience their magnitude and to avoid big equations that are not very necessary.

As  $C_2$  it is the biggest and expensive in the condensers, we calculate it so that it produces the *power half* (the resistance of output of the amplifier is worthless to be of exit in common collectors and feedback to be negatively)

$$C_2 = \dots \geq 1 / \omega_{\min} R_L$$

$$V_{C2} = V_{CC} / 2 = \dots$$

The capacitor  $C_3$  will be optional and experimental, being able to choose of 0,1 [mF]. As for  $C_4$ , this will always be a short circuit in front of the series  $R_3$ - $R_6$  of feedback

$$C_4 = \dots \gg 1 / \omega_{\min} (R_3 + R_6)$$

### Differential variant

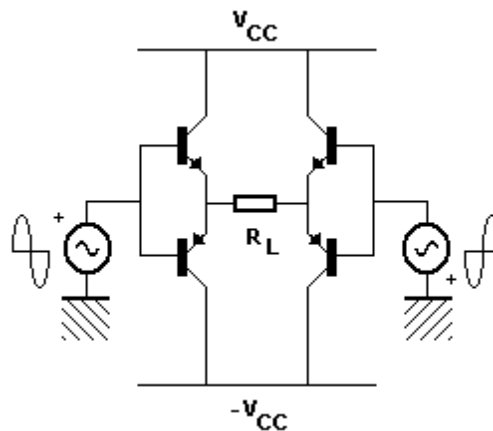
The implementation shows a coupled without condenser and with transistors working in class B operating in anti-parallel way. The behavior equations are the following ones for a single tone

$$P_{SALmax} = P_{Lmax} = (0,707 \cdot 2V_{CC})^2 / R_L = 2 V_{CC}^2 / R_L$$

$$P_{ENTmax} = 2 V_{CC} \cdot 2 I_{Cmed} = 2V_{CC} (2 i_{Lp} / \pi) = 2 V_{CC} (4V_{CC} / \pi R_L) = 8V_{CC}^2 / \pi R_L$$

$$\eta = P_{SALmax} / P_{ENTmax} = \pi / 4 \sim 0,78$$

$$P_{DISmax} = 4 P_{CEmax} = P_{SALmax} (\eta^{-1} - 1) \sim 0,07 P_{SALmax} = 0,14 V_{CC}^2 / R_L$$



and where it can be compared with respect to the ordinary complementary symmetry that for the same source the power in the load multiplies for four

$$P_{SALmax} (4 \text{ TBJ}) / P_{SALmax} (2 \text{ TBJ}) = 4$$

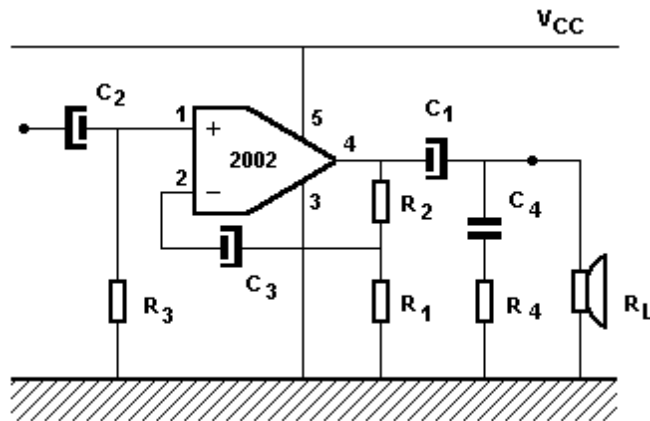
and that for same load power the transistors are demanded in half

$$P_{CEmax} (4 \text{ TBJ}) / P_{CEmax} (2 \text{ TBJ}) = 0,5$$

## Amplifier with the integrated circuit 2002

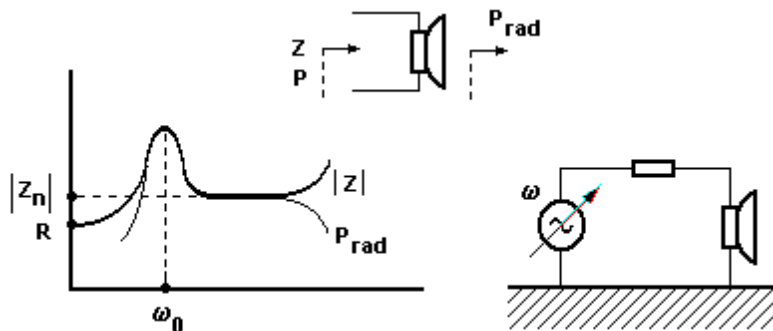
This integrated circuit allows until approximate 10 [W] on the load. It summarizes the explanations that we have made. The negative feedback is made to the terminal 2 and, in this case, it is already predetermined by the maker's design. The differential entrance is JFET reason why the same losses of  $C_2$  they polarize it, although in this circuit it has become physical with  $R_3$ . The advised magnitudes are

$R_1 = 2,2 [\Omega]$	$C_1 = 1 [\text{mF}]$	$V_{CC} = 12 [\text{V}]$
$R_2 = 220 [\Omega]$	$C_2 = 10 [\mu\text{F}]$	Usar disipador térmico
$R_3 = 1 [\text{M}\Omega]$	$C_3 = 470 [\mu\text{F}]$	$4 [\Omega] \leq R_L \leq 8 [\Omega]$
$R_4 = 1 [\Omega]$	$C_4 = 100 [\text{nF}]$	



## Speakers and acoustic boxes

A magnetic speaker (non piezoelectric) it presents, approximately, the characteristics that are shown when being experienced to the air —without box. The frequency of auto-resonance  $\omega_0$  can be measured with the enclosed circuit detecting maximum amplitude with a simple tester on the speaker.



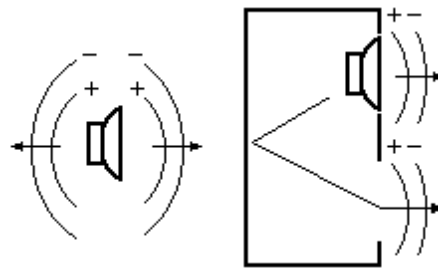
It is common to listen to say that a speaker has an impedance of certain magnitude. This

means that it has measured it to him inside the band in passing. In the practice, this value is more or less constant and it has denominated it to him here for  $Z_n$  that, for a quick calculation, it can approach it with regard to the value of continuous (that is to say measured with the ohmeter of a simple tester)

$$Z_n \sim 1,5 R$$

Something similar we have with regard to the power that the transducer tolerates. The specification of her measures it to him with a tone of sine wave (when not, lately in these decades and making bad use of the honesty, it measures it to him in an instantaneous one transitory) inside the spectrum of plane and typical power of 1 [KHz]. It is also necessary to highlight that although the power here truly is *apparent*, but it approaches it to *active*.

With the purpose of taking advantage of the back wave fronts in the emission, to match the pick of auto-resonance and to protect of it interprets it to the speaker, the acoustic cabinets or *baffles* are used.

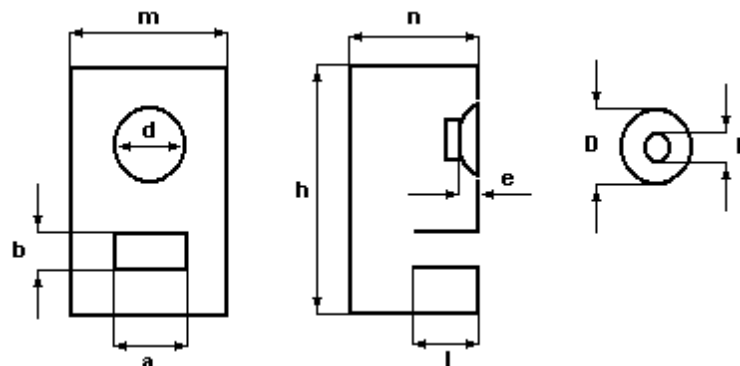


Subsequently we attach some design equations for fans (wooden box)

$$V = n m h \sim [ 4360 A / f_0^2 ( A^{1/2} + 2,25 l ) ] + 0,4 e d^2 \quad \text{volume of the box: } V$$

$$0,5 d^2 \leq A = a b \leq 0,86 d^2 \quad \text{area of the window: } A$$

$$d \sim ( D^2 - R^2 )^{1/2} \leq a \leq 1,1 d \quad \text{effective diameter of the cone: } d$$



where the diameter «d» it represents the section to make in the box and that it will be similar to the useful section of the wave, that is to say that this diameter will be smaller than that of the speaker's front

With regard to the aesthetics and external practice, it is generally accustomed to be adopted

$$m = 3 h / 4$$

$$n = h / 2$$

## Design

Be the data

$$f_0 = \dots \quad D = \dots \quad R = \dots \quad e = \dots$$

Considering the equations for a wooden box

$$d \sim (D^2 - R^2)^{1/2} = \dots$$

$$d \leq a = \dots \leq 1,1 d$$

$$0,25 d^2 / a \leq b = \dots \leq 0,86 d^2 / a$$

$$m = \dots > a$$

$$h = 4 m / 3 = \dots$$

$$n = h / 2 = \dots$$

$$l = 0,44 \{ [ 4360 a b / f_0^2 ( m n h - 0,4 d^2 e ) ] - ( a b )^{1/2} \} = \dots$$

## Acoustic filters

The acoustic spectrum can be divided in three bands (very approximately)

- low frequencies (until 400 [Hz])
- medium frequencies (from 400 until 4000 [Hz])
- high frequencies (from 4000 [Hz] in more)

and in general the technology of the reproductive electro-acoustic determines an accessible cost with a limited spectrum range, and they are designed completing these bands. Their respective names are

- *woofer* (low frequencies)
- *squawker* (medium frequencies)
- *tweeter* (high frequencies)

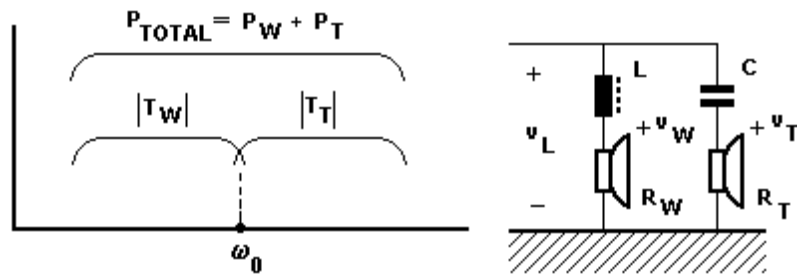
We will always consider in our studies to these speakers with an impedance that is pure resistiva, being quite valid this approach in the practice.

Firstly we present a design without control of medium frequencies (*squawker*)

$$T_w = v_w / v_L = R_w / (R_w + X) = \omega_0 / (s + \omega_0); \quad \omega_0 = R_w / L; \quad T_{w(\omega_0)} \sim 0,707$$

$$T_T = v_T / v_L = R_T / (R_T + X) = s / (s + \omega_0); \quad \omega_0 = 1 / R_T C; \quad T_{T(\omega_0)} \sim 0,707$$

$$P_{TOTAL} = |T_{T(\omega_0)}|^2 + |T_{W(\omega_0)}|^2 = 1$$

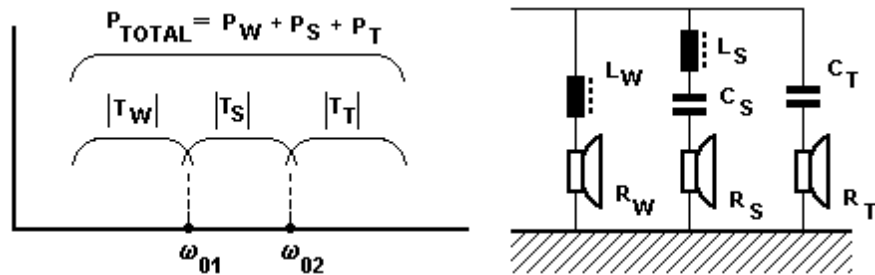


and now with a reproducer of medium frequencies

$$T_W = \omega_0 / (s + \omega_{01}); \quad \omega_{01} = R_W / L_W; \quad T_{W(\omega_{01})} \sim 0,707$$

$$T_T = s / (s + \omega_{02}); \quad \omega_{02} = 1 / R_T C_T; \quad T_{T(\omega_{02})} \sim 0,707$$

$$T_S = \alpha s / (s^2 + s \xi \omega_n + \omega_n^2) = \alpha s / (s + \omega_{01}) (s + \omega_{02})$$



where

$$\alpha = R_S / L_S$$

$$\omega_n = (L_S C_S)^{-1/2}$$

$$\xi = \alpha / \omega_n$$

$$\omega_{01} = (\alpha / 2) \cdot \{ [1 - (4 L_S / R_S^2 C_S)]^{1/2} \}$$

$$\omega_{02} = (\alpha / 2) \cdot \{ [1 + (4 L_S / R_S^2 C_S)]^{1/2} \}$$

that for the design conjugated poles will be avoided and with it undesirable syntonies

$$R_S^2 C_S > 4 L_S$$

## Design

Be the data for a design of two filters

$$f_0 = \dots \quad P_{Lmax} = \dots \quad R_W = \dots \quad R_T = \dots$$

We calculate for the equations seen

$$C = 1 / R_T \omega_0 = \dots$$

$$L = R_W / \omega_0 = \dots$$

We find the effective maximum current for the *woofer*

$$I_{\text{efwmax}} = (P_{L\text{max}} / R_w)^{1/2} = \dots$$

what will determine a minimum diameter of the inductor. If we adopt a current density for him of 3 [A/mm<sup>2</sup>]

$$\varnothing = \dots \geq 0,00065 I_{\text{efwmax}}^{1/2} = \dots$$

being able to manufacture the reel according to that explained in the *inductores* chapter.

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## Chap. 11 Amplification of Radiofrequency in low level class A

### Generalities

#### Effect Miller

#### Model of the TBJ in RF

#### Factors of over-value and reactivity

#### Passages of meshes series to parallel

#### Filter impedance

#### Response of width and phase of a transfer

### Amplifier of simple syntony

#### Design

### Amplifier multi-stages of same simple syntony

#### Design

### Amplifier multi-stages of simple syntonies, for maximum plain

#### Design

### Amplifier multi-stages of simple syntonies, for same undulation

### Amplifier of double syntony, for maximum plain

#### Design

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### Generalities

#### Effect Miller

This effect is applied networks amplifiers and voltage inverters to voltage. We can see the following thing here

$$A_v = v_{sal} / v_{ent} < 0$$

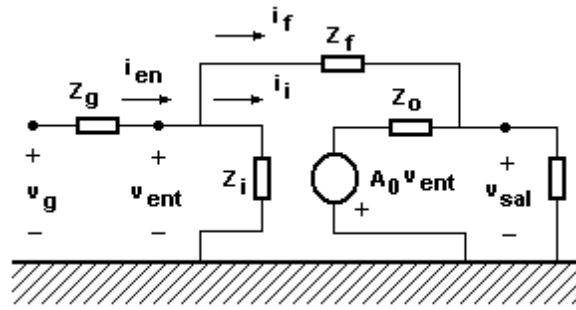
$$i_i = Y_i v_{ent}$$

$$i_f = (v_{ent} - v_{sal}) Y_f$$

$$i_f / i_i = (v_{ent} - v_{sal}) Y_f / Y_i v_{ent} = (1 - A_v) Y_f / Y_i$$

$$Y_{ent} = i_{ent} / v_{ent} = (i_f + i_i) / i_i Z_i = Y_i + Y_f (1 - A_v)$$





and like it is in general

$$|A_v| \gg 1$$

it is

$$Y_{ent} \sim Y_i - Y_f A_v$$

and in a similar way we can demonstrate

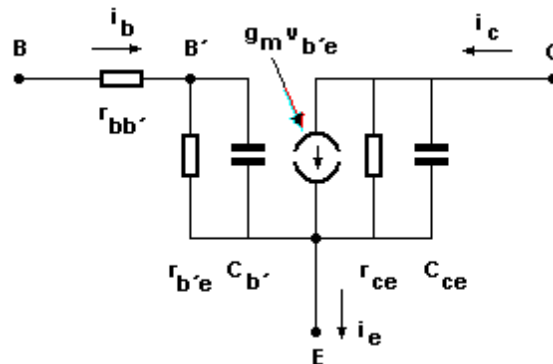
$$Y_{sal} \sim Y_o$$

### Model of the TBJ in RF

It is common two types of models of the transistor in radiofrequency, that is: the  $\pi$  (or also denominated *Giacoletto*) and that of *admittance parameters*. The first one expresses it next, where the  $C_{b'}$  is the sum of the capacitances among B'E and B'C amplified by the effect Miller. This model possesses parameters that will change with the frequency and the polarization, and the makers of devices have not made it frequent use in their data, surely for the difficult of the same one; for what we will try to replace it for the second model in this chapter. Their basic equations are

$$C_{b'} = C_{b'e} + C_{b'c} (1 + A_v) \sim C_{b'e} + C_{b'c} \cdot v_{ec} / v_{b'e}$$

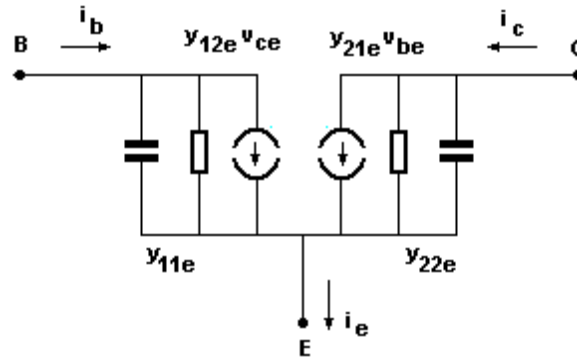
$$g_m = \partial I_C / \partial V_{BE} \approx \beta \partial [I_{BE0} (1 - e^{V_{BE}/V_T})] / \partial V_{BE} = \beta I_{BE0} e^{V_{BE}/V_T} / V_T = I_C / V_T \sim 20 I_C$$



The admittance pattern is more general, and it adapts meetly for the amplifications of low sign.

Their system of equations is the following one

$$\begin{aligned} i_b &= y_{11e} v_{be} + y_{12e} v_{ce} \\ i_c &= y_{21e} v_{be} + y_{22e} v_{ce} \end{aligned}$$



and their parameters usually specify according to the frequency and the polarization.

For the frequencies and magnitudes that we will work we will be able to simplify this model in the following way (the same as with the hybrid pattern)

$$y_{12e} \sim 0$$

$$y_{22e} \approx 0$$

being with it the TBJ a dispositive of unidirectional transmittance.

### Factors of over-value and reactivity

Given a polynomial of second degree in the way

$$P(s) = s^2 + s a + b = (s^2 + s \xi \omega_n + \omega_n^2) = (s + \alpha)(s + \alpha^*) = |P| e^{\varphi}$$

we define in him

$$\delta = b^{1/2} / a$$

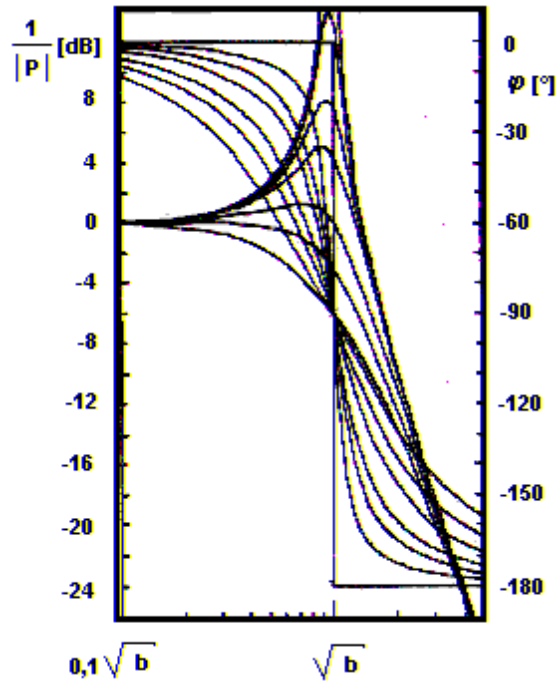
$$\xi = 1 / 2 \delta$$

$$f_n = 2 \pi / \omega_n$$

*over-value factor (of current or voltage)*

*coefficient of damping*

*natural frequency of the polynomial system*



Subsequently take the simple example of passive components; for example an inductance in series with a resistance and let us find their total apparent power

$$\begin{aligned} Z &= R + sL \leftrightarrow R + j\omega L && \text{impedance} \\ S &= P + jR && \text{apparent power} = \text{active} + \text{reactive} \end{aligned}$$

and let us define a factor of merit reactive that we will denominate *factor of quality*

$$Q = R / P \quad \text{factor of merit reactive}$$

that for us it will be  $\omega L/R$ .

We will see the relationship that exists among these factors  $\delta$  and  $Q$  in a syntony circuit subsequently; or, said in a more appropriate way, in a transfer of second order of conjugated poles.

### Passages of meshes series to parallel

An impedance series  $Z_s = R_s \pm jX_s$  can behave, in certain range of frequencies where the  $Q$  stays constant, similarly to another parallel  $Z_p = R_p // jX_p$  and opposedly. If the dipole is inductive its equivalences are the following ones

$$\begin{aligned} Y_p &= G_p + (sL_p)^{-1} = (R_s + sL_s)^{-1} \\ R_p &= R_s (1 + Q^2) \\ L_p &= L_s (1 + Q^{-2}) \\ Q &= Q_s = Q_p = \omega L_s / R_s = R_p / \omega L_p \end{aligned}$$

and in a same way for the capacitive

$$Y_p = G_p + sC_p = [R_s + (sC_s)^{-1}]^{-1}$$

$$R_p = R_s (1 + Q^2)$$

$$C_p = C_s [(1 + Q^2)^{-1}]$$

$$Q = Q_s = Q_p = \omega C_p / R_p = 1 / \omega C_s R_s$$

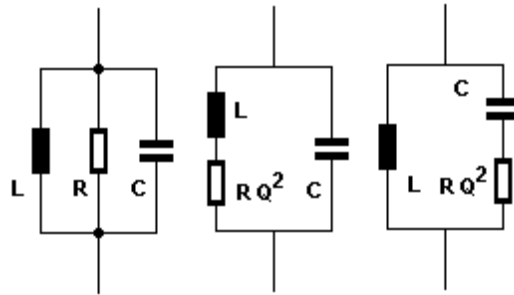
and generalizing has finally to remember with easiness

$$Q \geq 4$$

$$R_p = R_s Q^2$$

$$L_p = L_s$$

$$C_p = C_s$$



### Filter impedance

The typical resonant circuit that has just shown in the previous figure presents a  $Q > 10$  with easiness in frequencies above the 100 [KHz]. In its band pass the following equations are completed

$$Z = (R_s + sL_s) // (sC)^{-1} \sim R // sL // (sC)^{-1} = C^{-1} \cdot s / (s^2 + s\omega_0/\delta + \omega_0^2) =$$

$$= C^{-1} \cdot s / (s + \alpha)(s + \alpha^*) \equiv R / [1 + jQ_0(\omega/\omega_0 - \omega_0/\omega)] \sim$$

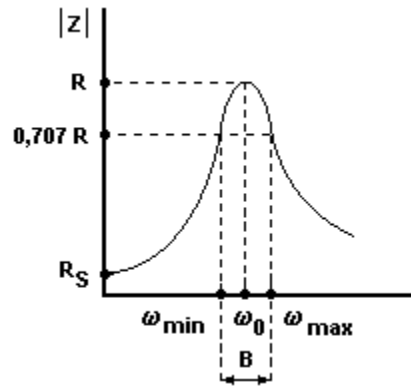
$$\sim R / [1 + j2(\omega - \omega_0)/B]$$

$$\omega_0 = (LC)^{-1/2}$$

$$Q_0 = \delta = \omega_0 L / R \text{ (in resonance the factors coincides)}$$

$$a = \sigma_a + j\omega_a = (\omega_0 / 2Q_0) [1 + j(4Q_0^2 - 1)^{1/2}] \sim (B/2)(1 + j2Q_0)$$

$$B = \omega_0 / Q_0 \text{ (wide of band to power half)}$$



## Response of width and phase of a transfer

The spectral characteristic of the module and the phase of a transfer that it doesn't distort should be in the band pass B plain for the first one, and a crescent or in decline straight line for second. To see this we take an example like the following one

$$T_{(\omega)} = |T| e^{j\phi}$$

$$|T| = K \quad \text{constant}$$

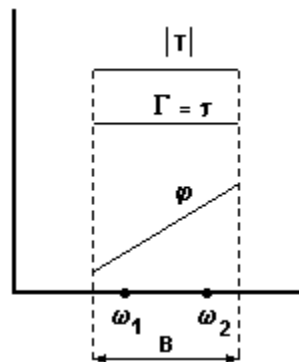
$$\phi = \omega \tau + \phi \quad \text{straight line with constant angle } \phi$$

to which we apply him two tones to their entrance

$$v_1 = v_{1p} e^{j\omega_1 t}$$

$$v_2 = v_{2p} e^{j\omega_2 t}$$

$$v_{\text{ent}} = v_1 + v_2$$



being then to their exit

$$v_{\text{sal}} = T v_{\text{ent}} = K v_{1p} e^{j[\omega_1(t+\tau) + \phi]} + K v_{2p} e^{j[\omega_2(t+\tau) + \phi]}$$

where observe that the amplitudes of the signs have changed proportionally the same as their angles. This last it is equal to say that their *temporary retard*  $\Gamma$  is constant

$$\Gamma = \partial \phi / \partial \omega = \tau$$

## Amplifier of simple syntony

Their behavior equations are the same ones that we have done with the filter impedance inside the area of the band pass B

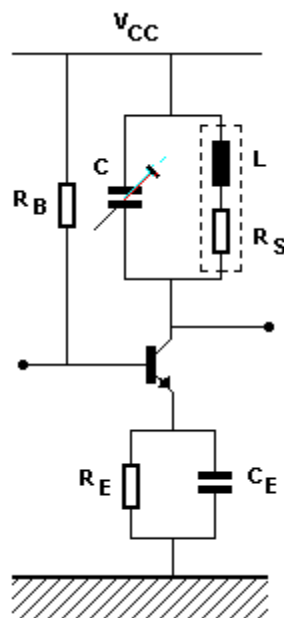
$$A_v = v_{sal} / v_{ent} = y_{21e} Z \sim y_{21e} R / [1 + j 2 (\omega - \omega_0) / B] = |A_v| e^{j\varphi}$$

$$|A_v| \approx |A_{v(\omega_0)}| = y_{21e} R \quad \text{approximately constant}$$

$$\varphi = -\arctan [2 (\omega - \omega_0) / B] \sim -2 (\omega - \omega_0) / B \quad \text{straight line}$$

$$Q_0 = \omega_0 L / R_s = \omega_0 C R = R / \omega_0 L = \omega_0 / B \geq 4$$

$$a \sim (B / 2) (1 + j 2 Q_0)$$



Llamamos *producto ganancia por ancho de banda* PGB al área definida por la ganancia a potencia mitad y el ancho de banda pasante

$$PGB = |A_v| B \approx |A_{v(\omega_0)}| B = |y_{21e}| / C$$

## Design

Be the data (underisables capacitances without considering)

$$|A_v| = \dots \quad f_{\max} = \dots \quad f_{\min} = \dots \quad f_0 = \dots$$

We begin adopting a transistor and with an elected polarization we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$\begin{aligned}\beta &= \dots \\ y_{11e} &= \dots \\ y_{21e} &= \dots \\ y_{12e} &= \dots \\ y_{22e} &= \dots\end{aligned}$$

and we polarize it

$$\begin{aligned}V_{RE} &= \dots \geq 1 \text{ [V]} \\ R_E &= V_{RE} / I_C = \dots \\ V_{CC} &= V_{CE} + V_{RE} = \dots \\ R_B &= \beta (V_{CC} - V_{RE} - 0,6) / I_C = \dots\end{aligned}$$

Of the precedent data we obtain

$$\begin{aligned}Q_0 &= f_0 / (f_{\max} - f_{\min}) = \dots \\ C_{22e} &= b_{22e} / \omega_0 = \dots\end{aligned}$$

and if we adopt

$$L = \dots$$

we will be able to find

$$\begin{aligned}R_s &= (\omega_0 L)^2 [ (Q_0 \omega_0 L)^{-1} - g_{22e} ] = \dots \\ C &= (\omega_0^2 L)^{-1} - C_{22e} = \dots\end{aligned}$$

and to verify

$$|y_{21e}| Q_0 \omega_0 L = \dots > |A_v|$$

Of the chapter of *oscillators* we verify the possible undesirable oscillation

$$|y_{21e}| [g_{22e} + (R_s Q_0^2)^{-1}]^{-1} \cdot |y_{12e}| (g_{11e} + R_B^{-1})^{-1} = \dots < 1$$

So that the emitter is to earth potential

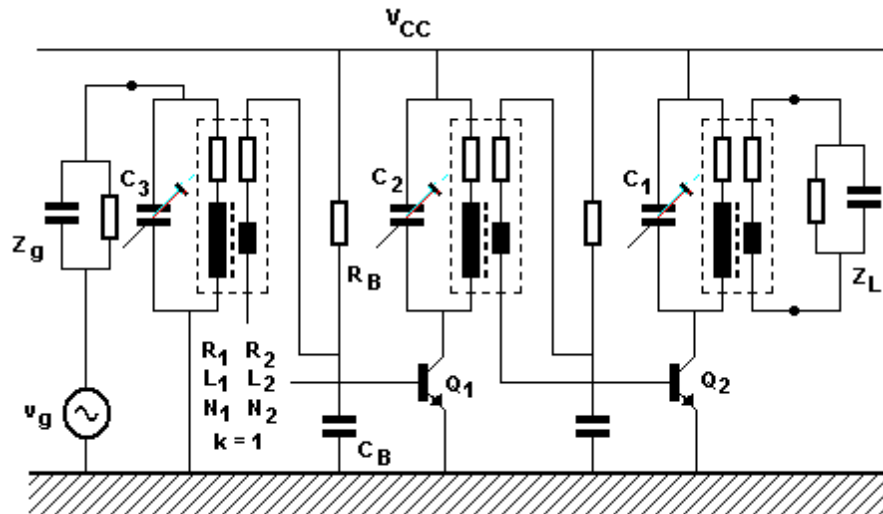
$$|y_{11e}|^{-1} \gg 1 / \omega_0 C_E \Rightarrow C_E = \dots \gg |y_{11e}| / \omega_0$$

or any experimental of 0,1 [μF] it will be enough.

### Amplifier multi-stages of same simple syntony

Placing in having cascade «n» stages of simple syntony syntonized to the same frequency

- bigger gain
- decrease of the band width
- increase of the selectivity (flanks  $\partial |A_v| / \partial \omega$  more abrupt)
- increase of the product gain for wide of band


$$\begin{aligned} A_{\text{vef}} &= |A_{\text{vef}}| e^{j\varphi_{\text{ef}}} \\ |A_{\text{vef}}| &= |A_V|^n \approx (g_m R)^n \\ \varphi_{\text{ef}} &= n\varphi = -n \cdot 2(\omega - \omega_0)/B \end{aligned}$$
$$\begin{aligned} |A_{\text{vef}}|_{(\omega_{\text{max}}; \omega_{\text{min}})} &= 0,707 (g_m R)^n = (g_m R) / \{1 + [2(\omega_{\text{max}} - \omega_0)/B]^2\}^n \\ \omega_{\text{max}}; \omega_{\text{min}} &= \omega_0 \{1 \pm [(2^{1/n} - 1)^{1/2} / 2Q_0]\} \\ B_{\text{ef}} = \omega_{\text{max}} - \omega_{\text{min}} &= B (2^{1/n} - 1)^{1/2} \end{aligned}$$
$$Q_{0ef} = \omega_0 / B_{ef} = Q_0 / (2^{1/n} - 1)^{1/2}$$
$$PGB_{\text{ef}} = |A_{\text{vef}}|_{(\omega_0)} B_{\text{ef}} = (g_m R)^{n-1} (2^{1/n} - 1)^{1/2} PGB$$

Be the data (underisables capacitances without considering)



$$|A_v| = V_L / V_g = \dots \quad f_{\max} = \dots \quad f_{\min} = \dots \quad f_0 = \dots \quad C_L = \dots \quad Z_g = \dots \quad n = \dots$$

We begin adopting a transistor and with an elected polarization we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$\beta = \dots$$

$$y_{11e} = \dots$$

$$y_{21e} = \dots$$

$$y_{12e} = \dots$$

$$y_{22e} = \dots$$

and we polarize it

$$V_{CC} = V_{CE} = \dots$$

$$R_B = \beta (V_{CC} - 0,6) / I_C = \dots$$

Of the precedent data we obtain

$$B = (\omega_{\max} - \omega_{\min}) / (2^{1/n} - 1)^{1/2} = \dots$$

$$Q_0 = \omega_0 / B = \dots$$

and if we adopt

$$L_1 = \dots$$

$$R_1 = \dots$$

to simplify the calculations

$$R_L = g_{11e}^{-1} = \dots$$

where of having been  $R_L$  a fact, then the polarization could have been changed or to place a resistance in series or derivation to this.

Subsequently we can find the such relationship of spires that satisfies the  $Q_0$  that we need if we make

$$Q_0 = [R_1(\omega_0 L_1 / R_1)^2 // g_{22e}^{-1} // g_{11e}^{-1}(N_1/N_2)^2] / \omega_0 L_1$$

of where

$$N_1 / N_2 = \{ g_{11e} / [ (Q_0 \omega_0 L_1)^{-1} - R_1(\omega_0 L_1)^{-2} - g_{22e} ]^{-1} \}^{1/2} = \dots$$

and that it allows us to verify the previously made adoption (same diameter of wires is supposed between primary and secondary)

$$g_{11e}^{-1} = \dots \gg R_2 = R_1 (N_2/N_1)^2$$

We express the individual gains now

$$R_{ent} = R_1(\omega_0 L_1 / R_1)^2 // g_{11e}^{-1} (N_1/N_2)^2 = [R_1(\omega_0 L_1)^2 + g_{11e} (N_2/N_1)^2]^{-1} = \dots$$

$$A_1 = R_{ent} / (R_g + R_{ent}) = \dots$$

$$A_2 = A_4 = A_6 = N_2/N_1 = \dots$$

$$A_3 = A_5 = |y_{21e}| Q_0 \omega_0 L_1 = \dots$$

should complete the fact

$$A_1 A_2 A_3 A_4 A_5 A_6 = \dots > |A_v|$$

According to what is explained in the chapter of oscillators, it is considered the stability of each amplifier

$$A_3 |y_{12e}| / \{ [R_g^{-1} + R_1(\omega_0 L_1)^2] (N_1/N_2)^2 + g_{11e} \} = \dots < 1 \quad \text{estabilidad de } Q_1$$

$$A_5 |y_{12e}| Q_0 \omega_0 L_1 (N_2/N_1)^2 = \dots <$$

1

*estabilidad de  $Q_2$*

To estimate the values means of the syntonies (has those distributed in the inductor is presented, in the cables, etc.) we calculate

$$C_1 = (\omega_0^2 L_1)^{-1} - C_{22e} - C_L (N_2/N_1)^2 = \dots$$

$$C_2 = (\omega_0^2 L_1)^{-1} - C_{22e} - (C_{11e} + C_{22e} A_4) (N_2/N_1)^2 = \dots$$

$$C_3 = (\omega_0^2 L_1)^{-1} - C_g - (C_{11e} + C_{22e} A_2) (N_2/N_1)^2 = \dots$$

and that of disacoupled

$$C_B = \dots \gg |y_{11e}| / \omega_0$$

or any experimental of 0,1 [μF] it will be enough.

### Amplifier multi-stages of simple syntonies, for maximum plain

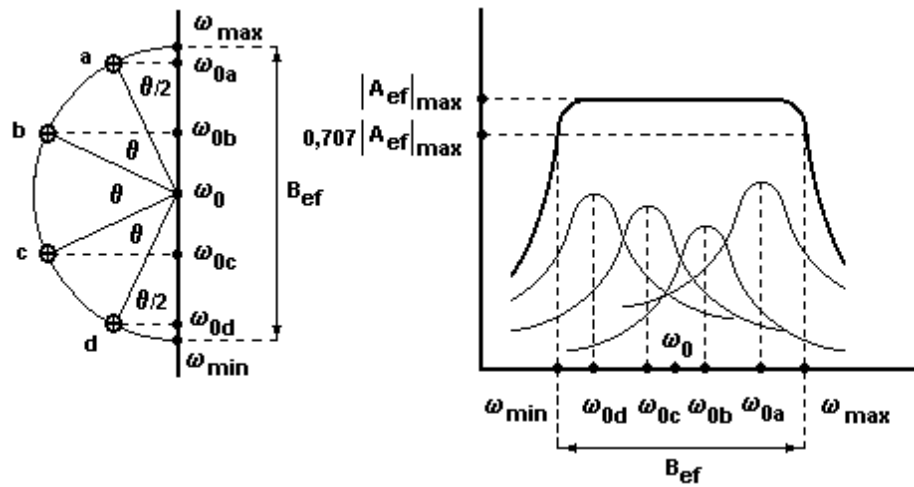
The advantages of this implementation in front of the previous one (the circuit is the same one) they are the following

- perfect spectral plain of the gain
- bigger selectivity

and their disadvantage

- not so much gain

In synthesis, this method consists on to implement the same circuit but to syntonize those «n» stages in different frequencies  $\omega_{0i}$  ( $i = a, b, c, \dots$ ) to factors of appropriate merits  $Q_{0i}$ .

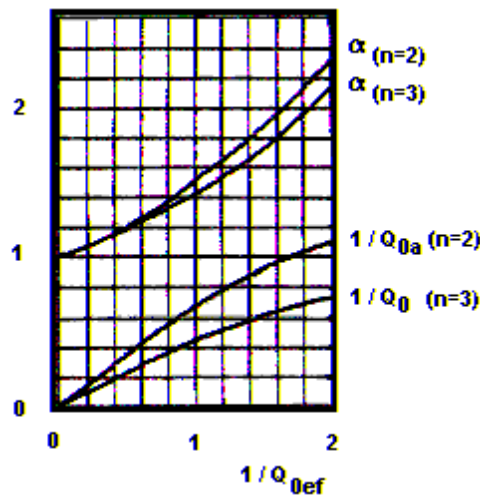


When one works in a *short band* s this plain characteristic it is obtained if the poles of the total transfer are located symmetrically in the perimeter of a circumference

$$\theta = \pi / n$$

$$Q_{0ef} = \omega_0 / B_{ef} \geq 10 \quad (\text{short band})$$

$$\omega_0 = (\omega_{\max} \omega_{\min})^{1/2} \sim (\omega_{\max} - \omega_{\min}) / 2$$



When we speak of *wide band* the method it will be another

$$Q_{0ef} = \omega_0 / B_{ef} < 10 \quad (\text{wide band})$$

$$\omega_0 = (\omega_{\max} \omega_{\min})^{1/2} \neq (\omega_{\max} - \omega_{\min}) / 2$$

If we want to calculate some of these cases, they are offered for it abacus for any  $Q_{0ef}$ , where

	<b>n = 2</b>	<b>n = 3</b>
	$Q_{0a} \sim Q_{0b}$	$Q_{0a} \sim Q_{0b}$ $Q_{0c} = Q_{0ef}$
$\omega_{0a}$	$\omega_0 \alpha$	$\omega_0 \alpha$
$\omega_{0b}$	$\omega_0 / \alpha$	$\omega_0 / \alpha$
$\omega_{0c}$	—	$\omega_0$

## Design

Be the data (underisables capacitances without considering)

$$f_{\max} = \dots \quad f_{\min} = \dots \quad f_0 = \dots \quad C_L = \dots \quad Z_g = \dots \quad n = 2$$

We begin adopting a transistor and with an elected polarization we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$\beta = \dots$$

$$y_{11e} = \dots$$

$$y_{21e} = \dots$$

$$y_{12e} = \dots$$

$$y_{22e} = \dots$$

and we polarize it

$$V_{CC} = V_{CE} = \dots$$

$$R_B = \beta (V_{CC} - 0,6) / I_C = \dots$$

Of the precedent data we obtain

$$B_{ef} = (\omega_{\max} - \omega_{\min}) = \dots$$

$$\omega_0 (\omega_{\max} + \omega_{\min}) / 2 = \dots$$

$$Q_{0ef} = \omega_0 / B_{ef} = \dots$$

and of the abacus

$$\alpha = \dots$$

$$Q_{0a} = \dots$$

what determines

$$\begin{aligned} Q_{0b} &= Q_{0a} = \dots \\ \omega_{0a} &= \omega_0 \alpha = \dots \\ \omega_{0b} &= \omega_0 / \alpha = \dots \end{aligned}$$

If we adopt

$$\begin{aligned} L_1 &= \dots \\ R_1 &= \dots \end{aligned}$$

as

$$\begin{aligned} Q_{0a} &= [ R_g // R_1(\omega_0 L_1 / R_1)^2 // g_{11e}^{-1}(N_1/N_2)^2 ] / \omega_{0a} L_1 \\ Q_{0b} &= [ R_1(\omega_0 L_1 / R_1)^2 // g_{22e}^{-1} // R_L(N_1/N_2)^2 ] / \omega_0 L_1 \end{aligned}$$

they are

$$\begin{aligned} N_1 / N_2 &= \{ g_{11e} / [ (Q_{0a} \omega_{0a} L_1)^{-1} - R_1(\omega_{0a} L_1)^{-2} - R_g^{-1} ]^{-1} \}^{1/2} = \dots \\ R_L &= (N_1/N_2)^2 / [ (Q_{0b} \omega_{0b} L_1)^{-1} - R_1(\omega_{0b} L_1)^{-2} - g_{22e} ] = \dots \end{aligned}$$

According to what is explained in the chapter of *oscillators*, it is considered the stability of each amplifier

$$\begin{aligned} A_G &= |y_{21e}| / [ R_1(\omega_0 L_1)^2 + g_{22e} + R_L^{-1}(N_2/N_1)^2 ] = \dots \\ A_H &= |y_{12e}| / \{ [ R_g^{-1} + R_1(\omega_0 L_1)^2 ] (N_2/N_1)^2 + g_{11e} \} = \dots \\ A_G A_H &= \dots < 1 \end{aligned}$$

To estimate the values means of the syntonies (has those distributed in the inductor is presented, in the cables, etc.) we calculate

$$\begin{aligned} C_1 &= (\omega_{0b}^2 L_1)^{-1} - C_{22e} - C_L(N_2/N_1)^2 = \dots \\ C_2 &= (\omega_{0a}^2 L_1)^{-1} - C_g = \dots \end{aligned}$$

and being for effect Miller that the capacity in base will vary along the spectrum, it will be advisable to become independent of her with the condition

$$(N_2/N_1)^2 [ C_{11e} + C_{22e} \cdot |y_{21e}| Q_{0b} \omega_{0b} L_1 ] = \dots \ll C_2$$

El de desacople

$$C_B = \dots \gg |y_{11e}| / \omega_0$$

or any experimental of  $0,1 \text{ } [\mu\text{F}]$  it will be enough.

### Amplifier multi-stages of simple syntonies, for same undulation

The advantage of this answer type in front of that of maximum plain is

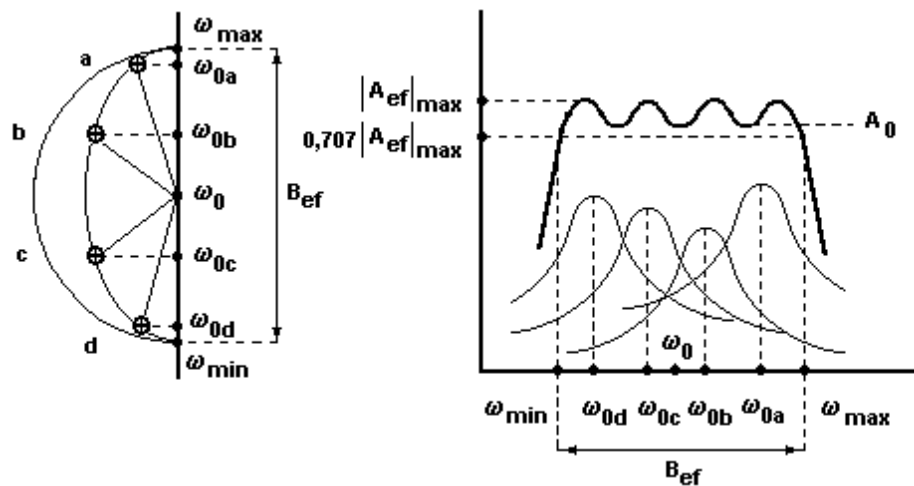
— bigger selectivity

and the disadvantages

— undulation in the gain

— without straight line in the phase

For any  $Q_{0\text{ef}}$  this characteristic is achieved minimizing, in oneself quantity, all the widths of individual band  $B_i$ . Subsequently it is expressed the diagram of poles and answer for the case of *short band*.



We define the *undulation factor* here of the gain

$$\text{FO [dB]} = 20 \log |A_{\text{vef}}|_{\text{max}} / A_0$$

that it will allow by means of the square to find this factor  $\gamma$

$$B_i = B_{i \text{ (max horiz)}} \cdot \gamma$$

or

$$\sigma_i = \sigma_{i \text{ (max horiz)}} \cdot \gamma$$

FO	$\gamma (n = 2)$	$\gamma (n = 3)$	$\gamma (n = 4)$
0	1	1	1
0,01	0,953	0,846	0,731
0,03	0,92	0,786	0,662
0,05	0,898	0,75	0,623
0,07	0,88	0,725	0,597
0,1	0,859	0,696	0,567
0,2	0,806	0,631	0,505
0,3	0,767	0,588	0,467
0,4	0,736	0,556	0,439
0,5	0,709	0,524	0,416

### Amplifier of double syntony, for maximum plain

We will find the equations of behavior of a doubly syntonized transformer

$$A_v = v_{sal} / v_{ent} = H \cdot s / (s^4 + s^3 A + s^2 B + s C + D) =$$

$$= H \cdot s / [(s + a)(s + a^*)(s + b)(s + b^*)]$$

$$Q_p = \omega_p L_p / R_p = 1 / \omega_p C_p R_p$$

$$Q_s = \omega_s L_s / R_s = 1 / \omega_s C_s R_s$$

$$\omega_p = (L_p C_p)^{-1/2}$$

$$\omega_s = (L_s C_s)^{-1/2}$$

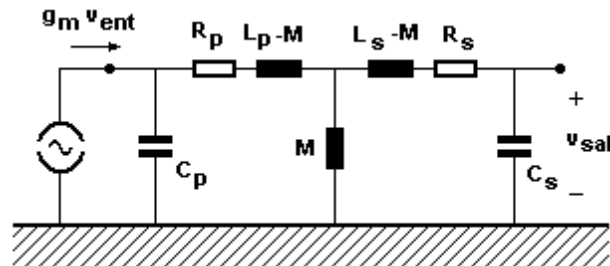
$$H = \omega_p \omega_s g_m / (k^{-1} - k)(C_p C_s)^{1/2}$$

$$A = (1 - k^2)^{-1} [(\omega_p / Q_p) + (\omega_s / Q_s)]$$

$$B = (1 - k^2)^{-1} [\omega_p^2 + \omega_s^2 + (\omega_p \omega_s / Q_p Q_s)]$$

$$C = (1 - k^2)^{-1} \omega_p \omega_s [(\omega_p / Q_p) + (\omega_s / Q_s)]$$

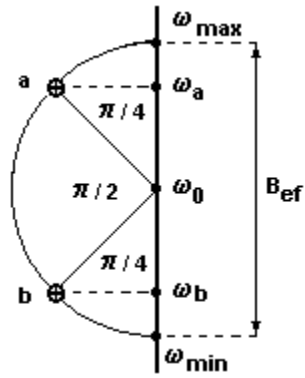
$$D = (1 - k^2)^{-1} \omega_p^2 \omega_s^2$$



and if we call

$$a = \sigma_a + j\omega_a$$

$$b = \sigma_a + j\omega_b$$



we arrive to

$$A = 4 \sigma_a$$

$$B = 6 \sigma_a^2 + \omega_a^2 + \omega_b^2$$

$$C = 4 \sigma_a^3 + 2 \sigma_a (\omega_a^2 + \omega_b^2)$$

$$D = \sigma_a^4 + \sigma_a^2 (\omega_a^2 + \omega_b^2) + \omega_a^2 \omega_b^2$$

If we simplify all this based on the conditions

$$\omega_0 = \omega_p = \omega_s$$

$$Q_0 = Q_p = Q_s \quad (\text{this implies short band } Q_{0ef} \geq 0)$$

it is

$$\omega_a^2 + \omega_b^2 = B - 6 \sigma_a^2 \sim 2\omega_0^2(1-k^2)^{-1}$$

$$\omega_a^2 \omega_b^2 = D - \sigma_a^4 - \sigma_a^2 (\omega_a^2 + \omega_b^2) \sim \omega_0^2(1-k^2)^{-1}$$

$$\omega_a; \omega_b = \pm \omega_0 [(1 \pm k)(1-k^2)^{-1}]^{1/2} = \pm \omega_0 (1 \pm k)^{-1/2}$$

and that for the simplification

$$k < 0,1$$

it is

$$\omega_a; \omega_b \sim \pm \omega_0 (1 \pm 0,5k)$$

and deducing geometrically is

$$k = 1 / Q_0 \quad (\text{coefficient of critical coupling for maximum plain})$$

$$B_{ef} \sim 1,41 k \omega_0$$

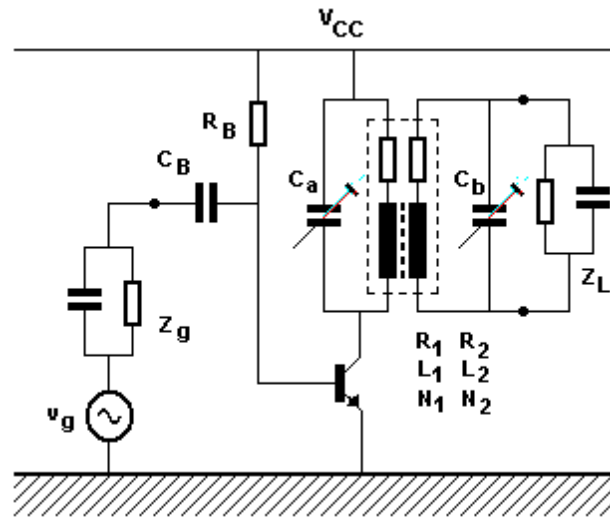
**Design**



Be the data (underisables capacitances without considering)

$$f_{\max} = \dots \quad f_{\min} = \dots \quad Z_L = \dots \quad Z_g = \dots$$

$$(f_{\max} f_{\min})^{1/2} / (f_{\max} + f_{\min}) = \dots > 10 \quad (\text{condition of short band})$$



We begin adopting a transistor and with an elected polarization we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$\beta = \dots$$

$$Y_{11e} = \dots$$

$$Y_{21e} = \dots$$

$$Y_{12e} = \dots$$

$$Y_{22e} = \dots$$

and we polarize it

$$V_{CC} = V_{CE} = \dots$$

$$R_B = \beta (V_{CC} - 0,6) / I_C = \dots$$

Of the precedent data we obtain

$$\omega_0 (\omega_{\max} + \omega_{\min}) / 2 = \dots$$

$$B_{ef} = (\omega_{\max} - \omega_{\min}) = \dots$$

$$Q_0 = 1,41 \omega_0 / B_{ef} = \dots$$

$$k = 1 / Q_0 = \dots$$

If we adopt

$$L_1 = \dots$$

$$L_2 = \dots$$

being

$$Q_0 = \{ [ (\omega_0 L_1)^2 / R_1 ] // g_{22e}^{-1} \} / \omega_0 L_1$$

$$Q_0 = \{ [ (\omega_0 L_2)^2 / R_2 ] // R_L \} / \omega_0 L_2$$

they are

$$R_1 = (\omega_0 L_1)^2 [ (Q_0 \omega_0 L_1)^{-1} - g_{22e} ] = \dots$$

$$R_1 = (\omega_0 L_2)^2 [ (Q_0 \omega_0 L_2)^{-1} - R_L^{-1} ] = \dots$$

To estimate the values means of the syntonies (has those distributed in the inductor is presented, in the cables, etc.) we calculate

$$C_a = (\omega_0^2 L_1)^{-1} - C_{22e} = \dots$$

$$C_b = (\omega_0^2 L_2)^{-1} - C_L = \dots$$

The one of it couples

$$1/\omega_0 C_B \ll |y_{11e} + R_B^{-1}| \Rightarrow C_B = \dots$$

or any experimental of 0,1 [μF] it will be enough.

According to what is explained in the chapter of *oscillators*, it is considered the stability of the amplifier

$$A_G = |y_{21e}| / [ R_1 (\omega_0 L_1)^2 + g_{22e} ] = \dots$$

$$A_H = |y_{12e}| / \{ [ R_g^{-1} + R_B^{-1} + g_{11e} ] \} = \dots$$

$$A_G A_H = \dots < 1$$

## Chap. 12 Amplification of Radiofrequency in low level class C

### Generalities

#### Design

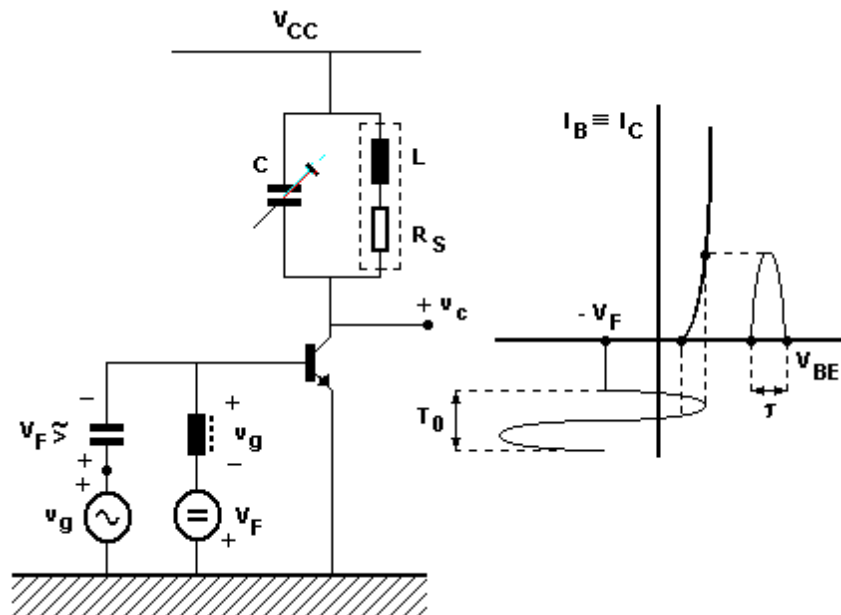
#### Design variant

### Generalities

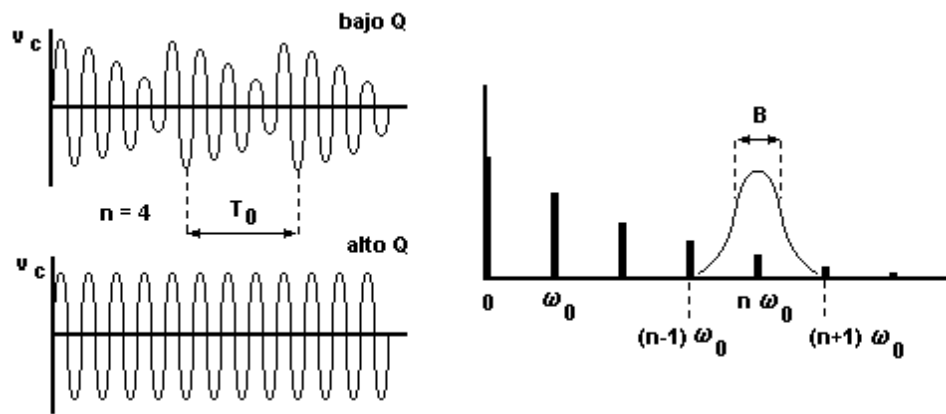
The circuit of the figure shows a typical implementation in class C. The sine wave of the generator it only transmits its small peak picks in the TBJ due to the negative polarization in its base

$$v_g = v_{gp} \sin \omega_0 t$$

$$v_b \sim v_g - V_F = v_{gp} \sin \omega_0 t - V_F$$



For a sign clipped as the one that is illustrated we will have many harmonics. Calling  $w_0$  to the fundamental one with period  $T_0$ , their order harmonics  $n$  ( $n = 1$  are the fundamental one) they will be captured an or other according to the high  $Q$  of the syntonized circuit, determining for it a sine wave another time in the collector

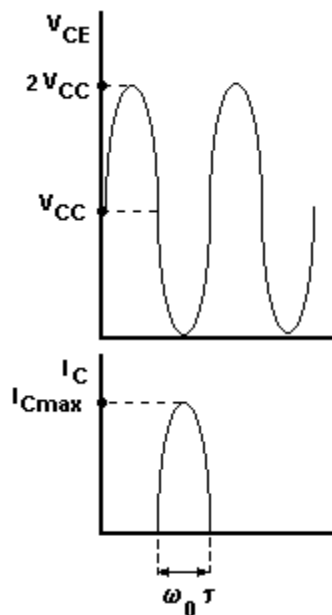


This way we can say that for the harmonic that we want to be captured it will complete

$$Q \gg n \omega_0 / B_{\max} = n \omega_0 / [(n+1)\omega_0 - (n-1)\omega_0] = n / 2$$

Subsequently we will find the characteristics of use of the energy in the system class C. For this end we appeal to the syntony of the fundamental and we design it for a maximum possible trip in the collector. It will be obtained it increasing the gain to resonance  $g_m R_s Q^2$  and it will show us the following approach that will give us the efficiency of the harmonic (or fundamental) to syntonize

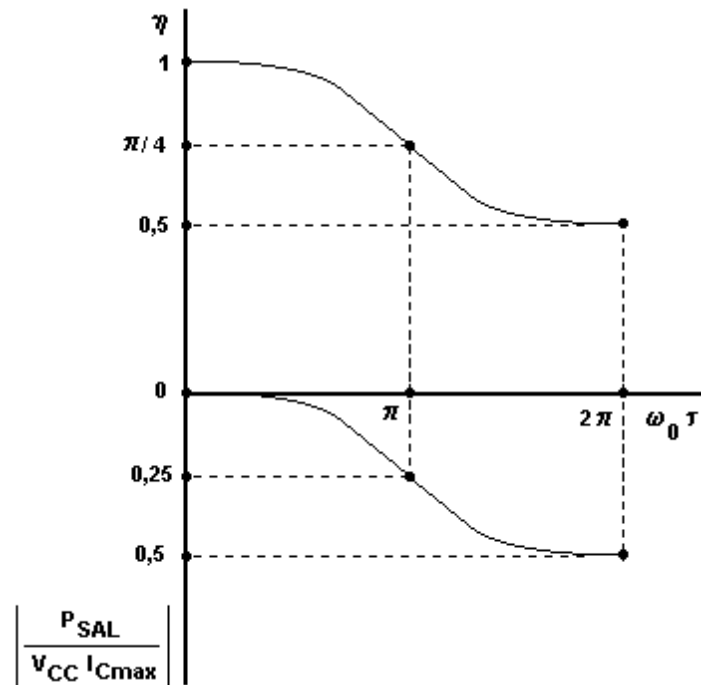
$$\tau \sim T_0 / 2n$$



Then, the equations will be

$$\begin{aligned} P_{\text{ENTmax}} &= V_{CC} I_{C\text{med}} = \\ &= V_{CC} \cdot 2 \left[ (1/2\pi) \int_0^{\omega_0 \tau/2} I_{C\text{max}} (\cos \omega_0 t - ) d\omega_0 t \right] = \\ &= (V_{CC} I_{C\text{max}} / \pi) ( \sin \omega_0 \tau/2 - \omega_0 \tau/2 \cdot \cos \omega_0 \tau/2 ) \end{aligned}$$

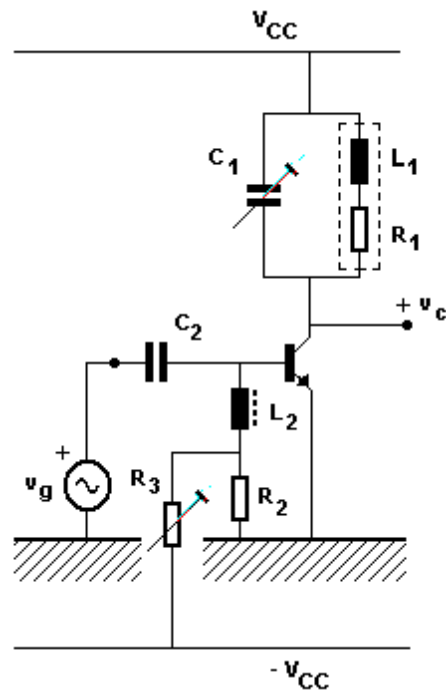
$$\begin{aligned}
 P_{DISmax} &= P_{CEmax} = 2 \left[ \int_0^{\omega_0 \tau / 2} (V_{CC} - V_{CC} \cos \omega_0 t) I_{Cmax} (\cos \omega_0 t - \cos \omega_0 \tau / 2) \partial \omega_0 t \right] = \\
 &= (V_{CC} I_{Cmax} / \pi) [\sin \omega_0 \tau / 2 - \omega_0 \tau / 2 (0,5 + \cos \omega_0 \tau / 2) + 0,25 \sin \omega_0 \tau / 2] \\
 P_{SALmax} &= P_{Lmax} = P_{ENTmax} - P_{DISmax} = (V_{CC} I_{Cmax} / \pi) (\omega_0 \tau - \sin \omega_0 \tau) \\
 \eta &= P_{SALmax} / P_{ENTmax} = (\omega_0 \tau - \sin \omega_0 \tau) / (4 \sin \omega_0 \tau / 2 - 2 \omega_0 \tau - \cos \omega_0 \tau / 2)
 \end{aligned}$$



## Design

Be the data (underisables capacitances without considering)

$$f_0 = \dots \quad n = \dots \quad v_g = \dots$$



We begin adopting a transistor and with a polarization practically of court, we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$y_{22e} = \dots$$

Adoptamos

$$L_1 = \dots$$

and we calculate their losses having present a high Q

$$\begin{aligned} n/2 &<< \{ [(n\omega_0 L_1)^2 / R_1] // g_{22e}^{-1} \} / n\omega_0 L_1 \\ \Rightarrow R_1 &= \dots >> (n\omega_0 L_1)^2 [2(n^2\omega_0 L_1)^{-1} - g_{22e}] \end{aligned}$$

also being able to estimate the syntony capacitor (to have present those distributed in the inductor, of the conductors, etc.)

$$C_1 = [(n\omega_0)^2 L_1]^{-1} - C_{22e} = \dots$$

We obtain the estimated period of conduction

$$\tau \sim T_0 / 2n = \pi / n\omega_0 = \dots$$

for what we find

$$V_F = v_{gp} \sin \{ \omega_0 [(T/2) - \tau] / 2 \} = v_{gp} \sin [ \omega_0 \tau (n - 1) / 2 ] = \dots$$

We choose a  $R_2$  that it is worthless in front of the inverse resistance of the diode base-emitter

$$R_2 = \dots \leq 100 \text{ [K}\Omega\text{]}$$

of where we obtain

$$V_F = V_{CC} R_2 / R_3 + R_2 \Rightarrow R_3 = [ (V_{CC} / V_F) - 1 ] R_2 = \dots$$

Now, so that  $L_2$  behave as a choke of RF it will be

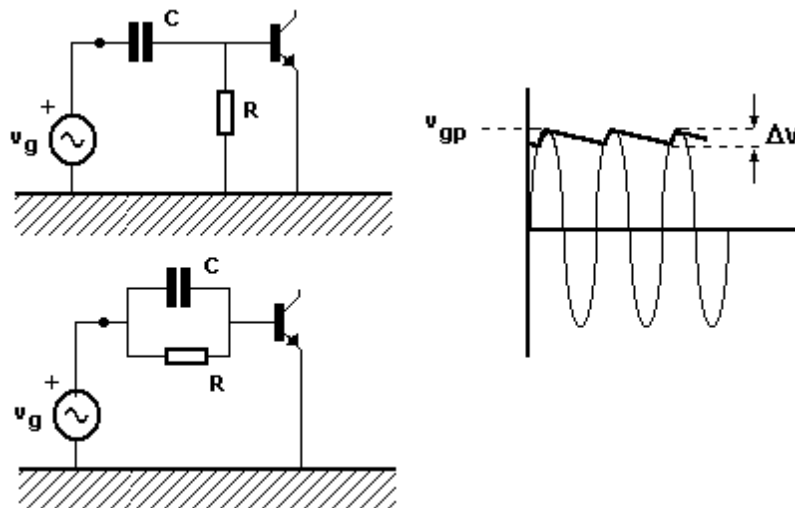
$$\omega_0 L_2 \gg R_2 // R_3 \Rightarrow L_2 = \dots \gg (R_2 // R_3) / \omega_0$$

determining with it a capacitor of it coupling

$$\omega_0 L_2 \gg 1 / \omega_0 C_2 \Rightarrow C_2 = \dots \gg 1 / \omega_0^2 L_2$$

### Design variant

Other forms of implementing the designed circuit omitting the negative source, are those that are drawn next. The load of the capacitor with the positive hemicycle of the generator is quick for the conduction of the diode, and slow its discharge because it makes it through the high resistance  $R$ . It understands each other an implementation of other applying the theorem of Thevenin. The critic that is made to this circuit in front of the one polarized that we saw, it is that their economy is a question today in completely overcome day.



The biggest difficulty in the design of these implementations consists on esteem of the conduction angle that will exist due to the curve of the diode. There are abacous of Shade that express the harmonic content for these situations, but that here they are omitted because in amplifications of low sign they are not justified these calculations. In synthesis, it seeks advice to

experience the values of R and C.

A quick way to project an esteem of this would be, for example, if previously we adopt a condenser

$$C = \dots$$

and we observe their discharge

$$v_{gp} - \Delta V = v_{gp} e^{-T_0/RC}$$

with

$$V_F = v_{gp} - \Delta V/2$$

we find the maximum value of the resistance then

$$R = \dots < T_0 / [C \ln (2V_F/v_{gp} - 1)^{-1}]$$

since the minimum will be given by a correct filtrate

$$RC = \dots \gg T_0$$



## Chap. 13 Amplifiers of continuous

### Generalities

#### Amplifier with AOV in differential configuration

#### Design

#### Amplifier with sampling

#### Nano-ammeter

#### Design

### Generalities

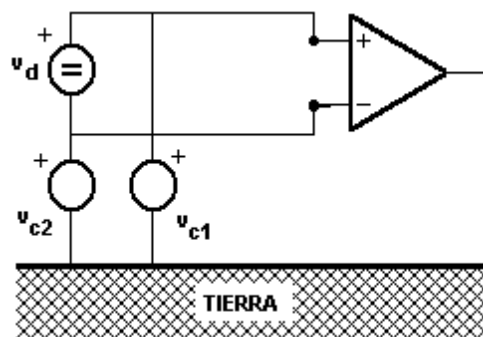
The problem of the current amplifiers or continuous voltage has been, and it will be, the offset for temperature. Added this, when the sign is of very low magnitude the problems of line interference they become present worsening the situation, because although they make it dynamically, they alter the polarizations of the stages.

For these reasons, they have been defined two ways of input of the sign: the entrance in common way and the differential entrance. Next they are defined an and other, also offering the denominated *Relationship of Rejection to the Common Mode* (RRMC) as factor of merit of all amplifier

$$V_{entMC} = (v_{c1} + v_{c2}) / 2$$

$$V_{entMD} = v_d$$

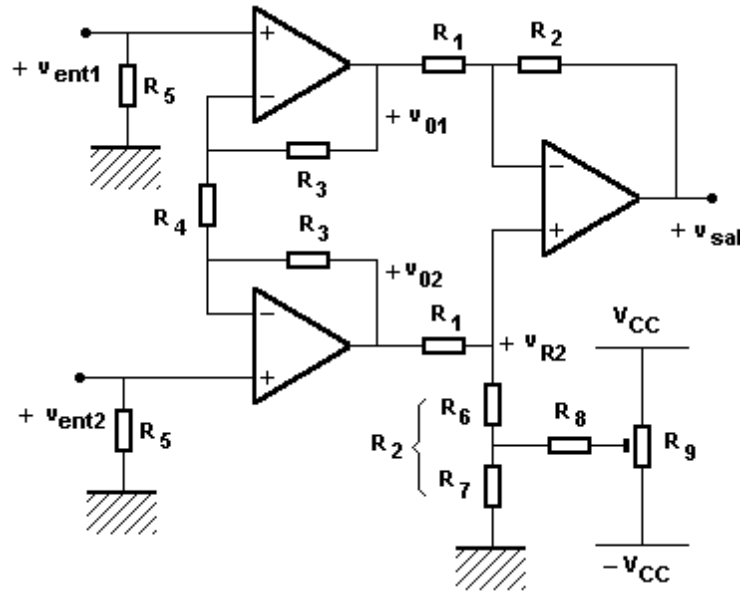
$$RRMC [dB] = 20 \log (v_{entMC} / v_{entMD})$$



#### Amplifier with AOV in differential configuration

The following configuration, today in day implemented with JFET to avoid bigger offsets, a high RRMC presents because the first stage is inside paired integrated circuit, and then its offsets is proportional and the difference to its exit diminishes. A third AOV follows it in subtraction configuration that will amplify the sign.

It is usual the use of this implementation for electromedical applications —electroencephalography, electrocardiography, etc.



A second important property of this configuration is its great gain. A single integrated circuit containing the three AOV offers with some few resistances high confiability and efficiency.

Having present that the entrance of each differential possesses a practically null voltage, the equations  $v_{01}$  and  $v_{02}$  correspond to the exits inverter and non-inverter. Their basic equations are the following

$$v_{01} = v_{en1} (-R_3/R_4) + v_{en2} (1+R_3/R_4)$$

$$v_{02} = v_{en2} (-R_3/R_4) + v_{en1} (1+R_3/R_4)$$

then

$$v_{R2} = v_{01} / (1+R_1/R_2)$$

$$R_2 \sim R_6 + R_7 \quad (\text{donde se optó excitar con corriente: } R_8 \gg R_7)$$

to the exit of the substractor

$$v_{sal} = v_{01} (-R_2/R_1) + v_{R2} (1+R_2/R_1) = (v_{en1} - v_{en2}) R_2/R_1$$

and finally the gain in differential mode

$$A_{VMD} = v_{sal} / v_{entMD} = v_{sal} / (v_{en2} - v_{en1}) = (1+2R_3/R_4) R_2/R_1$$

A third advantage consists in that can design to will the entrance resistance in common and differential mode (connecting to earth the mass of the circuit)

$$R_{entMC} = R_5 / 2$$

$$R_{entMD} = 2 R_5$$

The other property that we will comment is the high value of the RRM, in general bigger than 50 [dB]. We can do this with the following equations

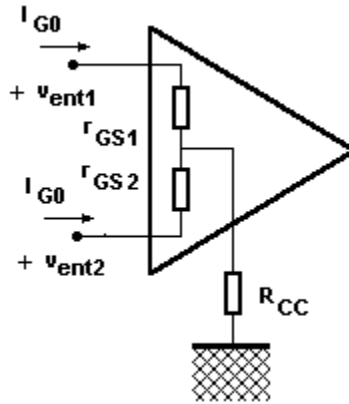
$$v_{entMD} = v_{en2} - v_{en1}$$

$$v_{entMC} = (v_{en2} + v_{en1}) / 2$$

$$A_{VMD} = v_{sal} / v_{entMD}$$

$$A_{VMC} = v_{sal} / v_{entMC}$$

$$RRMC = A_{VMD} / A_{VMC} = v_{entMC} / v_{entMD} \sim \\ \sim \{ [ I_{G0}(r_{GS1}+R_{CC})+I_{G0}(r_{GS2}+R_{CC}) ] / 2 \} / (I_{G0}r_{GS1}-I_{G0}r_{GS2})$$



it is

$$RRMC = [ (r_{GS1}+r_{GS2})/2 + R_{CC} ] / (r_{GS1}-r_{GS2}) \rightarrow \infty$$

The response in frequency of this amplifier is limited (usually below the 100 [Hz]) due to the stability of the AOV, reason why they usually take small condensers —small capacitors of polyester in the terminals of the AOV and electrolytic in the half point of the cursor of the pre-set.

An important fact for the design consists on not giving a high gain to the first stage, that is to say to  $(1+2R_3/R_4)$ , because the sign this way amplified it can be cut by the source. Similar effect can be given if the planner adjusts the offset in the first stage and you exceeds the range of continuous acceptable.

## Design

Be the data

$$R_{entMD} = \dots \quad A_{VMD} = \dots$$

We adopt a paired AOV (f.ex.: TL08X) and of the manual or their experimentation

$$V_{CC} = \dots \quad V_{OS} = \dots$$

We choose a gain of the differential stage (for it should be kept it in mind the esteem of the maxim differential entrance as we said)

$$A_1 = 1 + 2 R_3 / R_4 = \dots$$

and if we choose for example

$$R_2 = \dots$$

$$R_3 = \dots$$

they are

$$R_4 = 2 R_3 / (A_1 - 1) = \dots$$

$$R_1 = R_2 A_1 / A_{VMD} = \dots$$

$$R_5 = 2 R_{entMD} = \dots$$

For not affecting the calculations we make

$$R_6 = R_2 = \dots$$

$$R_7 = \dots \gg R_6$$

We calculate the resistance that will correct the offset in the case of maximum trip of the pre-set

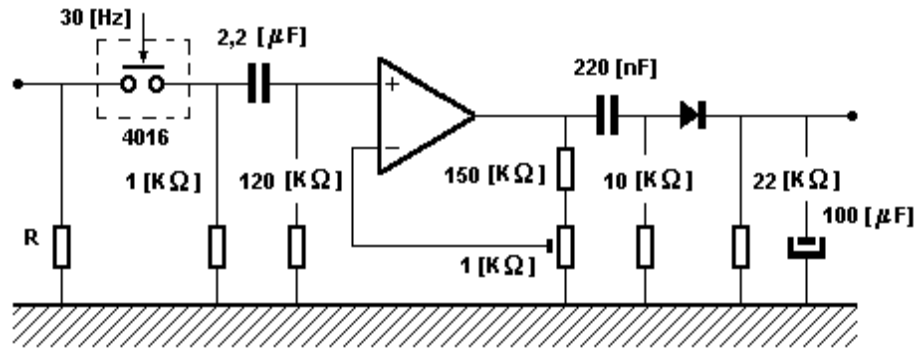
$$R_8 = \dots \leq (V_{CC} - V_{R7}) / I_{R8} \sim (V_{CC} - V_{OS}) / (V_{OS} / R_7) \sim V_{CC} R_7 / V_{OS}$$

and for not dissipating a high power in the pre-set

$$R_9 = \dots \leq 0,25 / (2 V_{CC})^2$$

### Amplifier with sampling

A way to overcome the offsets of continuous is sampling the sign making it pass to alternating, and then to amplify it dynamically with coupling capacitive, it stops then to demodulate it with a rectificator and filter. The following circuit, designed for of the author of the present book, it is used satisfactorily in the amplifications of termocuples voltages. Their entrance resistance is given by the value of the resister R, in this case of 47 [ $\Omega$ ], and could be increased for other applications. With the pre-set the gain, and the adjusted of zero it is not necessary. Another advantage of this system consists in that it doesn't care polarity of the continuous tension of entrance.

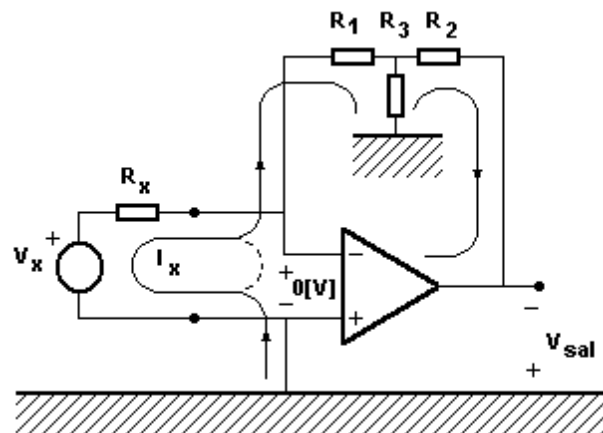


### Nano-ammeter

To translate and to measure so low currents, so much of alternating as of continuous fixed or dynamics presents the following circuit. The designer will compensate his offset when it is of continuous for some of the techniques explained in the chapter of *polarization* of the AOV. Their equations are the following

$$R_1 \gg R_3 \ll R_2$$

$$0 \sim I_x R_1 - (V_{sal} / R_2) R_3$$



then

$$V_{sal} \sim I_x R_1 R_2 / R_3$$

### Design

Be tha data

$$I_{x\max} = \dots \text{ (continuous, dynamics or alternates)}$$

We choose an AOV with entrance to JFET to have smaller offset

$$\pm V_{CC} = \dots$$

and if we adopt for example

$$R_1 = \dots \text{ (v.g.: } 1 \text{ [M}\Omega\text{])}$$

$$R_3 = \dots \text{ (v.g.: } 1 \text{ [M}\Omega\text{])}$$

and we determine

$$V_{\text{salmax}} = \dots$$

we will be able to calculate

$$R_3 = I_{\text{xmax}} R_1 R_2 / V_{\text{salmax}} = \dots$$

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## Chap. 14 Harmonic oscillators

### Generalities

#### Type phase displacement

#### Design

#### Type bridge of Wien

#### Design

#### Type Colpitts

#### Design

#### Tipo Hartley

#### Variant with piezoelectric crystal

#### Type syntonized input-output

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### Generalities

Basically, these oscillators work with lineal dipositives.

Let us suppose a transfer logically anyone that has, poles due to their inertia. For example the following one

$$T = v_{sal} / v_{ent} = K / (s + \alpha_0)$$

$$\alpha_0 = \sigma_0 + j \omega_0$$

where it will be known that this it is a theoretical example, since in the practice the complex poles are always given conjugated. This has been chosen to simplify the equations.

To the same one it is applied a temporary step of amplitude V (transitory of polarization for example). consequently, it is their exit

$$v_{ent} = V \rightarrow V / s$$

$$v_{sal} = KV (1 - e^{-\alpha_0 t}) / \alpha_0 =$$

$$= KV \{ \sigma_0 [1 + (e^{-\sigma_0 t} / \sin \phi_1) \sin(\omega_0 t + \phi_1) + j \omega_0 [(e^{-\sigma_0 t} / \sin \phi_2) \sin(\omega_0 t + \phi_2) - 1] \} / (\sigma_0^2 + \omega_0^2)$$

$$\phi_1 = \arctan (-\sigma_0 / \omega_0)$$

$$\phi_2 = \arctan (\sigma_0 / \omega_0)$$

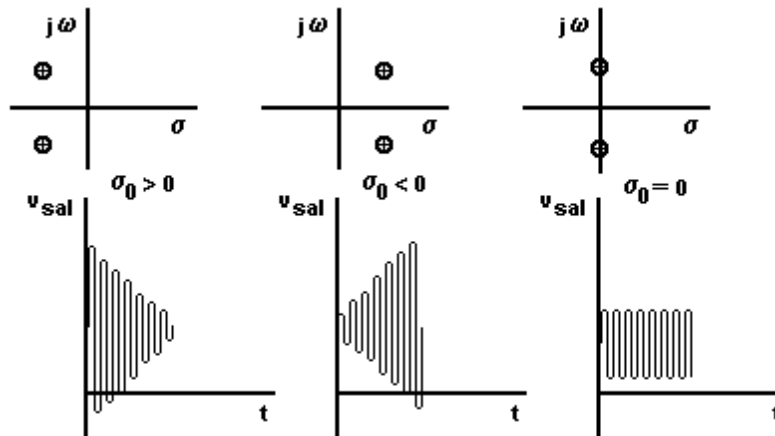
being able to happen three cases

$$I) \quad \sigma_0 > 0$$

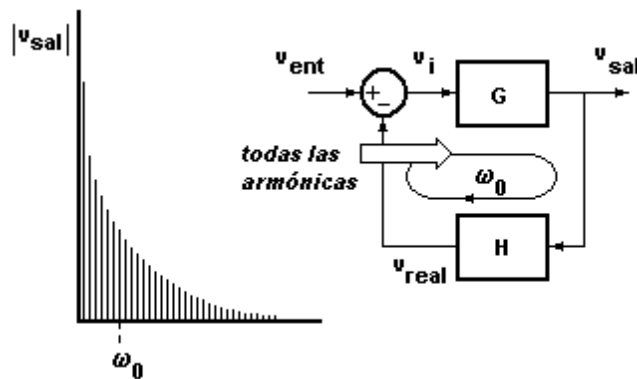
*stable exit*

- II )  $\sigma_0 < 0$  *unstable exit*  
 III )  $\sigma_0 = 0$   
 $v_{sal} = (KV/\omega_0) [\sin(\omega_0 t + \pi/2) - 1]$  *oscillatory or unstable exit to  $\omega_0$*

where conceptually we express these results in the following graphs for a transfer of conjugated poles.



Of this analysis we can define to an ideal harmonic oscillator saying that it is that system that doesn't possess transitory attenuation in none of their poles (inertias). The name of this oscillator type resides in that in the  $v_{sal}$  spectrum in permanent régime, "only" the harmonic  $\omega_0$  has been captured and it becomes present to the exit. The purity of the same one resides in the selectivity of the syntonies that /they prevent to capture other harmonic. The following drawing represents what we are saying.



Another way to understand the operation of a *harmonic* oscillator, perhaps more didactic, it is considering a feedback transfer where G wins what H loses and it injects in phase its own change, all this to the frequency  $\omega_0$  and not to another —to remember that as much G as H change module and phase with the frequency. This way, the behavior equations will be

$$T = v_{sal} / v_{ent} = v_{sal} / (v_i + v_{real}) = (v_{sal} / v_i) / [1 + (v_{real} / v_{sal})] = G / (1 + GH)$$

that for the conditions of harmonic oscillation



$$G_{(\omega_0)} H_{(\omega_0)} = -1 + j 0$$

$$T_{(\omega_0)} \rightarrow \infty$$

and then the high transfer of closed loop will go increased the amplitude of  $\omega_0$  until being limited by the own nonlinearity of the electronic components when they end up being limited by its feeding supply. This determines an important practical consideration, and that it consists in that the suitable critical point sees that it is theoretical, and that in the practice it should make sure

$$|G_{(\omega_0)} H_{(\omega_0)}| \sim 1$$

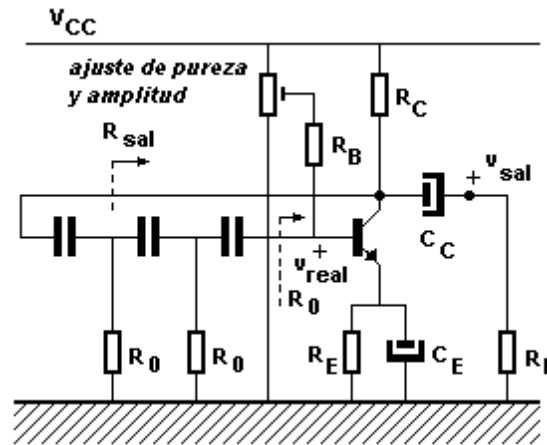
for a correct operation, and more pure of the sine wave; this will be the more close it is of the critical point.

### Type phase displacement

It is shown a typical implementation subsequently. Their behavior equations are

$$H = v_{\text{real}} / v_{\text{sal}} = 1 / [1 - 5(\omega R_0 C_0)^{-2} + j [(\omega R_0 C_0)^{-3} - 6(\omega R_0 C_0)^{-1}]]$$

$$G = v_{\text{sal}} / v_{\text{real}} \sim -g_m R_C // R_L$$



that it will determine an oscillation for pure real G in

$$0 = (\omega_0 R_0 C_0)^{-3} - 6(\omega_0 R_0 C_0)^{-1} \Rightarrow \omega_0 \sim 0,408 / R_0 C_0$$

being finally

$$H_{(\omega_0)} \sim 0,035$$

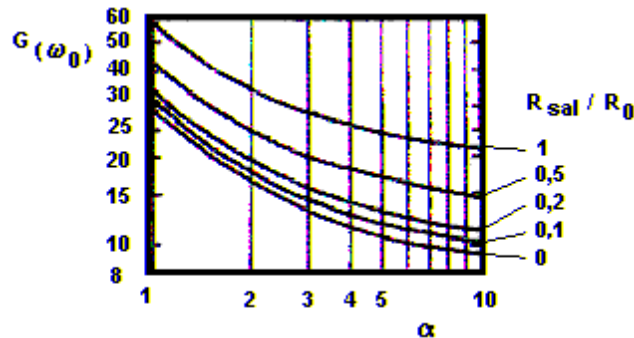
$$G_{(\omega_0)} = -1 / H_{(\omega_0)} = -29$$

In a more general way we will be able to have with the abacus

$$R_{sal} = R_C // R_L$$

$$\omega_0 = 1 / R_0 C_0 [ 3 + 2/\alpha + 1/\alpha^2 + (R_{sal}/R_0)(2 + 2/\alpha) ]$$

$$G_{(\omega_0)} = 8 + 12/\alpha + 7/\alpha^2 + 2/\alpha^3 + (R_{sal}/R_0)(9 + 11/\alpha + 4/\alpha^2) + (R_{sal}/R_0)^2(2 + 2/\alpha)^2$$



## Design

The following data are had

$$R_L = \dots \quad f_0 = \dots$$

we choose a TBJ and of the manual or their experimentation we find

$$V_{CE} = \dots \quad I_C = \dots \quad \beta = \dots \quad h_{21e} = \dots \quad h_{11e} = \dots \quad g_m = h_{21e} / h_{11e} = \dots$$

Keeping in mind that explained in the *polarization* chapter adopts

$$V_{RE} = \dots \geq 1 \text{ [V]}$$

$$I_{RN} = \dots \ll I_C / \beta$$

originating

$$R_C = V_{CE} / I_C = \dots$$

$$R_E \sim V_{RE} / I_C = \dots$$

$$V_{CC} = 2 V_{CE} + V_{RE} = \dots$$

$$R_B = [ (V_{CC}/2) - 0.6 ] \beta / I_C = \dots$$

$$R_0 = 1 / (h_{11e}^{-1} + R_B^{-1}) = \dots$$

and we will be able to verify (we work with gain modules to simplify the nomenclatures)

$$G_{(\omega_0)} \sim g_m R_L // R_C = \dots > 29 + 24/[R_0/(R_L^{-1} + R_C^{-1})] + \{2/[R_0/(R_L^{-1} + R_C^{-1})]\}^2$$

We calculate the oscillation capacitor and we verify that it doesn't alter the made calculations

$$C_0 = 1 / \omega_0 R_0 \{6 + 4/[R_0/(R_L^{-1} + R_C^{-1})]\} = \dots \ll 1 / \omega_0 R_C // R_L$$

and those of coupling and disacoupling

$$R_C // R_L \gg 1 / \omega_0 C_C \Rightarrow C_C = \dots \gg 1 / \omega_0 R_C // R_L$$

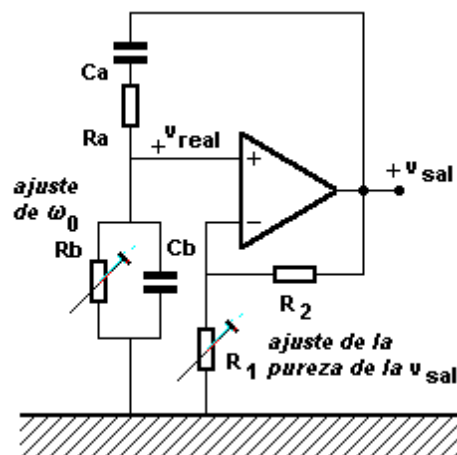
$$h_{11e} \gg (1 + h_{21e}) \cdot 1 / \omega_0 C_E \Rightarrow C_E = \dots \gg g_m / \omega_0$$

last expression that if it gives us a very big  $C_E$ , it will be necessary to avoid it conforming to with another minor and that it diminishes the gain, or to change circuit. The result truly should not be alarming, because any condenser at most will diminish the amplitude of the exit voltage but it won't affect in great way to the oscillation. Some designers usually place even a small resistance in series with the emitter without disacoupling capacitor dedicating this end to improve the quality of the wave although it worsens the amplitude of the oscillation.

### Type bridge of Wien

With the implementation of any circuit amplifier that completes the conditions of an AOV, that is: high gain, low output resistance and high of differential input, we will be able to, on one hand feedback negatively to control their total gain, and for another positively so that it is unstable. The high differential gain will make that with the transitory of supply of polarization outburst it takes to the amplifier to the cut or saturation, depending logically on its polarity, and it will be there to be this an unstable circuit.

The process changes favorably toward where we want, that is to say as harmonic oscillator, if we get that this instability is to a single frequency  $\omega_0$  and we place in the one on the way to the positive feedback a filter pass-tone. In the practice the implementation is used that is shown, where this filter is a pass-band and for it many times the oscillation is squared and of smaller amplitude that the supply —is to say that it is not neither to the cut neither the saturation.



The name of this circuit like "bridge" comes for the fact that the differential entrance to the AOV, for which doesn't circulate current and neither it possesses voltage, it shows to the implementation like a circuit bridge in balance.

The behavior equations are

$$H = v_{\text{real}} / v_{\text{sal}} = 1 / [ 1 + R_a/R_b + C_b/C_a + j (\omega R_a C_a - 1/\omega R_b C_b) ]$$

$$G = v_{\text{sal}} / v_{\text{real}} = - ( 1 + R_2/R_1 )$$

that it will determine an oscillation for pure real G in

$$0 = \omega_0 R_a C_a - 1/\omega_0 R_b C_b \Rightarrow \omega_0 = ( R_a R_b C_a C_b )^{-1/2}$$

being finally

$$H_{(\omega_0)} = 1 / ( 1 + R_a/R_b + C_b/C_a )$$

$$G_{(\omega_0)} = -1 / H_{(\omega_0)} = - ( 1 + ( 1 + R_a/R_b + C_b/C_a ) )$$

or

$$R_2/R_1 = R_a/R_b + C_b/C_a$$

This circuit made its fame years ago because it was used to vary its frequency sine wave by means of a potentiometer in R2. This, implemented by the drain-source of a JFET that presents a lineal resistance in low sign amplitudes, it was feedback in the dispositive through a sample of continuous of the exit amplitude. This has already been in the history, because with the modern digital synthesizers they are overcome this annoying and not very reliable application thoroughly.

## Design

The following data are had

$$R_L = \dots \quad f_0 = \dots$$

we choose an AOV and we adopt

$$\pm 9 \text{ [V]} \leq \pm V_{CC} = \dots \leq \pm 18 \text{ [V]}$$

$$C_a = C_b = \dots$$

$$R_2 = \dots$$

of where we obtain

$$R_a = R_b = 1 / \omega_0 C_a = \dots$$

$$R_1 = R_2 / 2 = \dots$$

## Type Colpitts

We could say that  $\pi$  responds to a filter with capacitives inpt-output.  
The behavior equations are (a short circuit of  $C_3$  could be made that is of it couples and for it

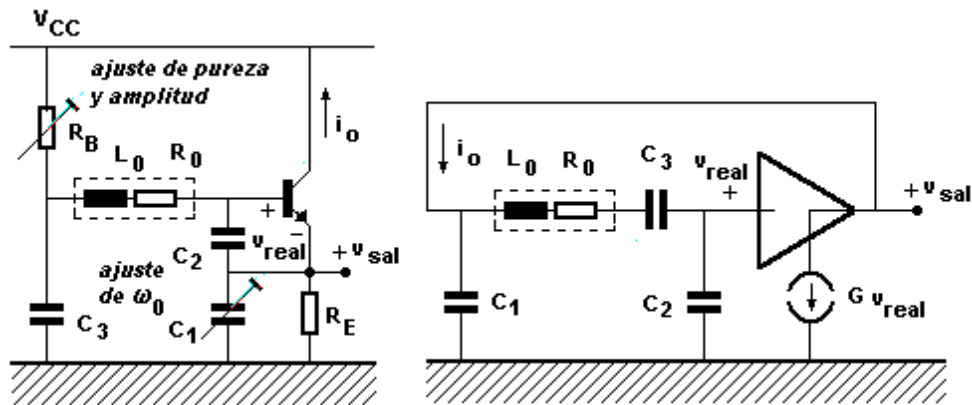
the equations are the same ones with  $C_3 \rightarrow \infty$ )

$$H = v_{\text{real}} / i_o = 1 / [ -\omega^2 R_0 C_1 C_2 + j \omega (C_1 + C_2 + C_1 C_2 / C_3 - \omega^2 C_1 C_2 L_0) ]$$

$$0 = \omega_0 (C_1 + C_2 + C_1 C_2 / C_3 - \omega_0^2 C_1 C_2 L_0) \Rightarrow \omega_0 = (L_0 C_1 // C_2 // C_3)^{-1/2}$$

$$H_{(\omega_0)} = -L_0 / R_0 (C_1 + C_2)$$

$$G_{(\omega_0)} = -1 / H_{(\omega_0)} = R_0 (C_1 + C_2 + C_1 C_2 / C_3) / L_0 \sim g_m R_0 Q_0^2$$



In this implementation the  $R_E$  can be changed by a choke of RF; the result will be better because it will provide to the oscillator bigger exit voltage and a better syntony selectivity because it increases the  $Q_0$ . Another common variant is to replace the inductor for a crystal at  $\omega_0$  (saving the polarization logically) so that the frequency is very much more stable.

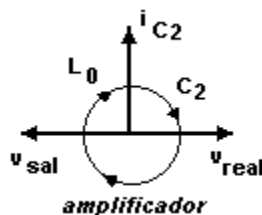
A simpler way and didactics regarding the principle of operation of the Colpitts, we will have it if we consider all ideal and without the one it couples  $C_3$ . The  $C_1$  syntonize at  $L_0$  and it produces an inductive current for the coil in backwardness ninety degrees, that again will be ahead whit the voltage on  $C_2$ . The inconvenience of this focus is that the magnitude of the attenuations is not appreciated

$$\omega_0 \sim (L_0 C_1)^{-1/2}$$

$$\omega_0 L_0 \gg 1 / \omega_0 C_2 \Rightarrow v_{\text{salp}} \gg v_{\text{realp}} \sim 0$$

$$C_1 \ll C_2$$

$$g_{11e}^{-1} \gg 1 / \omega_0 C_2 \Rightarrow i_{C2p} \gg i_{bp} \sim 0$$



## Design

They are had the following fact (parasitics capacitances not considered)

$$f_0 = \dots$$

We begin adopting a transistor and with an elected polarization we obtain

$$V_{CE} = \dots$$

$$I_C = \dots$$

$$\beta = \dots$$

$$y_{11e} = \dots (\sim h_{11e}^{-1})$$

$$y_{21e} = \dots (\sim g_m = h_{11e}/h_{21e})$$

$$y_{22e} = \dots (\sim h_{22e})$$

and we polarize it

$$R_E = V_{CE} / I_C = \dots \text{ (the biggest thing possible because it is in parallel with the inductor)}$$

$$V_{CC} = 2 V_{CE} = \dots$$

$$R_B = \beta (V_{CC} - 0,6 - V_{CE}) / I_C = \dots$$

We choose an inductor with the  $Q_0$  (we speak logically of the  $Q_{0ef}$ ) bigger possible (that can replace it for a crystal and a crash of RF in derivation)

$$L_0 = \dots$$

$$R_0 = \dots$$

what will allow us to have idea of the half value of the syntony capacitor

$$R_E // g_{22e} \ll 1 / \omega_0(C_1 + C_{22e})$$

$$C_1 = 2 C_{1med} \sim 2 / \omega_0^2 L_0 = \dots \ll (g_{22e} + R_E^{-1}) / \omega_0 - C_{22e}$$

We calculate and we verify that the characteristics of the circuit don't alter the theoretical equations (this in the practice can be omitted, since the same circuit will surely oscillate)

$$\omega_0 L_0 \gg 1 / \omega_0 C_3 \Rightarrow C_3 = \dots \gg 1 / \omega_0^2 L_0$$

The condenser that it lacks  $C_2$  can be keeping in mind experimentally that it is related practically with  $C_{1med}$  in the same times that the gain of tension of the TBJ; that is to say, some ten times, or with the requirement (we reject the effect Miller)

$$g_{11e}^{-1} \gg 1 / \omega_0(C_2 + C_{11e}) \Rightarrow C_2 = \dots \gg g_{11e} / \omega_0 - C_{11e}$$

and that, like it was said, if one chooses a very big value it will diminish the gain and therefore also the amplitude of the output signal —increased their purity.

Although the circuit will oscillate without inconveniences, we verify the oscillation condition

$$R_0 (C_1/2 + C_{22e} + C_2 + C_{11e}) / L_0 = \dots < |y_{21e}|$$

### Type Hartley

We could say that  $\pi$  responds to a filter with inductive input-output. It will also be it although these are coupled.

The behavior equations are

$$H = v_{\text{real}} / i_0 =$$

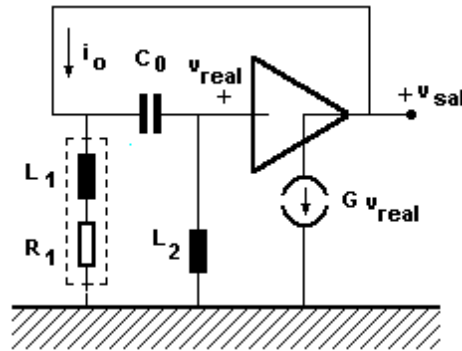
$$= L_2 C_0 \omega^2 \{ R_1 (\omega^2 L_2 C_0 - 1) + j \omega [R_1^2 C_0 + L_1 [\omega^2 (L_1 + L_2) C_0 - 1]] \} / \{ [\omega^2 (L_1 + L_2) C_0 - 1]^2 + (\omega R_1 C_0)^2 \}$$

$$0 = \omega [R_1^2 C_0 + L_1 [\omega^2 (L_1 + L_2) C_0 - 1]] \Rightarrow \omega_0 = [(L_1 + L_2)^{-1} (C^{-1} - R_1^2 / L_1)]^{-1/2}$$

$$H(\omega_0) = -L_2 R_1 (L_1 - R_1^2 C_0) (L_1^2 + L_2 R_1^2 C_0) / [(R_1^2 C_0 / L_1)^2 + R_1^2 C_0 (1 - R_1^2 C_0 / L_1) / (L_1 + L_2)]$$

$$G(\omega_0) = -1 / H(\omega_0) =$$

$$= [(R_1^2 C_0 / L_1)^2 + R_1^2 C_0 (1 - R_1^2 C_0 / L_1) / (L_1 + L_2)] / L_2 R_1 (L_1 - R_1^2 C_0) (L_1^2 + L_2 R_1^2 C_0)$$

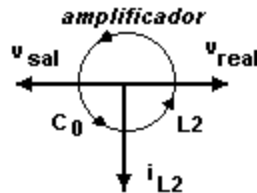


A simpler way and didactics regarding the principle of operation of the Hartley, we will have it if we consider all ideal. The  $L_1$  syntonize with  $C_0$  and it produces an courant capacitive for the condenser in advance ninety degrees that again will be late with the on  $L_2$ . The inconvenience of this focus is that the magnitude of the attenuations is not appreciated

$$\omega_0 \sim (L_0 C_1)^{-1/2}$$

$$\omega_0 L_2 \ll 1 / \omega_0 C_0 \Rightarrow v_{\text{salp}} \gg v_{\text{realp}} \sim 0$$

$$L_1 \gg L_2$$



### Variant with piezoelectric crystal

With the name of *piezoelectric crystal* it is known in Electronic to a dipole that presents the

characteristics of the figure, being able to or not to possess overtones, and that it is characterized to have high stability of their equivalent electric components and to be highly reactive. This way their equations are

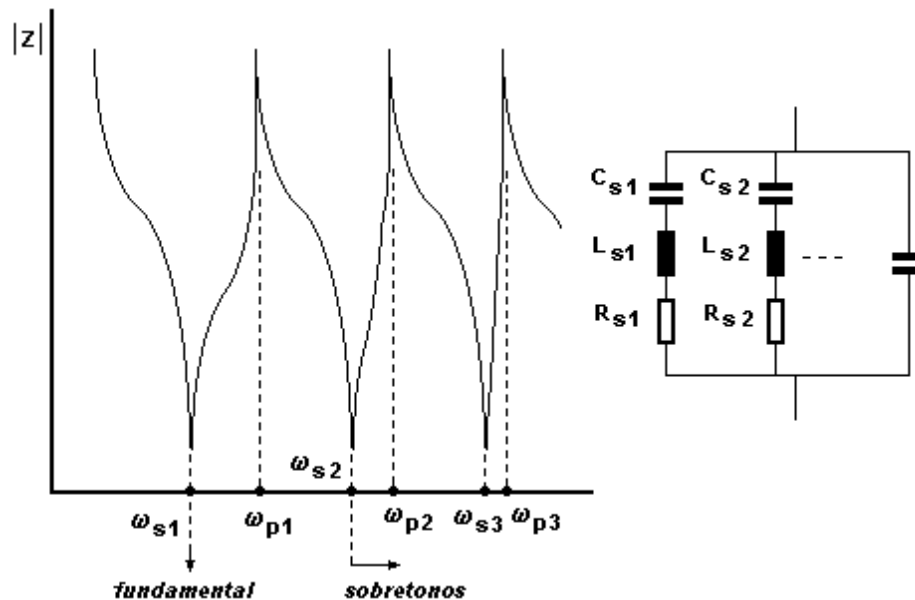
$$C_p \gg C_s$$

$$Q = \omega L_s / R_s \text{ (varios miles, tanto la sintonía serie } \omega_s \text{ como paralelo } \omega_p)$$

$$Z = \omega L_p // (R_s + \omega L_s + \omega C_s^{-1}) = \omega C_p^{-1} [(\omega_s/\omega)^2 - 1 + j Q^{-1}] / \{Q^{-1} - j [(\omega_p/\omega)^2 - 1]\}$$

$$\omega_s = (L_s C_s)^{-1/2}$$

$$\omega_p = [L (C_p^{-1} + C_s^{-1})^{-1}]^{-1/2}$$



This way, with the crystal used in the one on the way to oscillation and replacing for example to the inductor in the Colpitts or the capacitor in the Hartley, an oscillator of high performance can be gotten, where the final Q of the whole oscillator will be given by the stable and high of the crystal.

### Type syntonized input-output

It responds to a transfer simply G in feedback with H. for example implemented with the bidirectional property of a TBJ, it usually uses in the converters of frequency. Their general equations of operation are the following

$$\begin{aligned} GH &= (i_o/v_i) (i_{real}/v_i) = (i_o/v_o) (i_{real}/v_i) = Y_2 (-Y_1) = \\ &= -R_1^{-1} [1 + j2Q_1(\omega - \omega_1)/\omega_1] \cdot R_2^{-1} [1 + j2Q_2(\omega - \omega_2)/\omega_2] = \\ &= - (1/R_1 R_2) \{ 1 - 4Q_1 Q_2 (\omega/\omega_1 - 1)(\omega/\omega_2 - 1) + j 2 [Q_1(\omega/\omega_1 - 1) + Q_2(\omega/\omega_2 - 1)] \} \end{aligned}$$

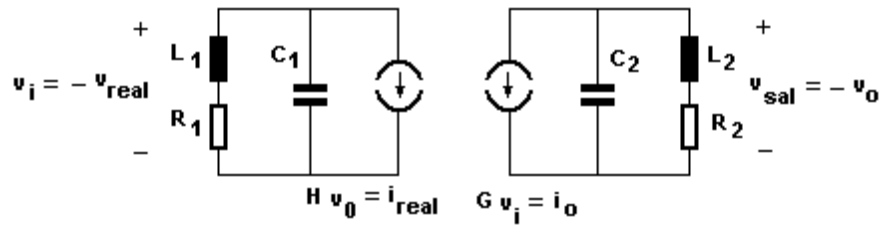
$$Q_1 = \omega_1 L_1 / R_1 = 1/\omega_1 C_1 R_1$$

$$Q_2 = \omega_2 L_2 / R_2 = 1/\omega_2 C_2 R_2$$

$$\omega_1 = (L_1 C_1)^{-1/2}$$

$$\omega_2 = (L_2 C_2)^{-1/2}$$





and designing to simplify

$$\omega_0 = \omega_1 = \omega_2$$

$$R_0 = R_1 = R_2$$

it is

$$GH = - (1/R_0^2) \{ 1 - 4Q_1Q_2(\omega/\omega_0-1)^2 + j 2 [(Q_1+ Q_2)(\omega/\omega_0-1)] \}$$

$$G_{(\omega_0)}H_{(\omega_0)} = - (1/R_0^2)$$

$$H_{(\omega_0)} = -Y_1 = - 1/R_0$$

$$G_{(\omega_0)} = Y_2 = 1/H_{(\omega_0)} = 1/R_0$$

## Chap. 15 Relaxation oscillators

### Generalities

#### Type TUJ

#### Design

#### Type multivibrator

#### Type harmonic-relaxation

#### Converters and Inverters

### Generalities

#### Inverter of a TBJ and a transformer

#### Inverter of two TBJ and a transformer

#### Design

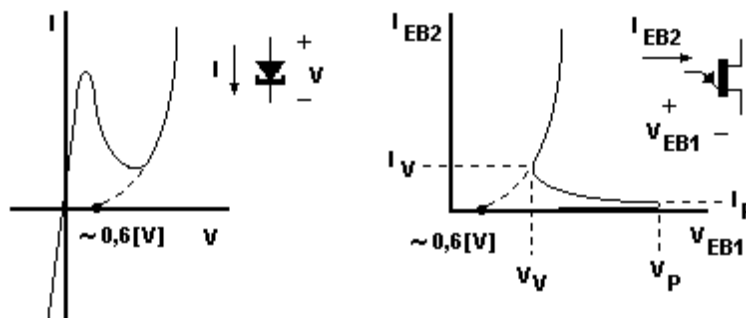
#### Inverter of two TBJ and two transformers

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### Generalities

Basically, these oscillators work with aperiodic diodes. We can classify them in two types, that is: those that work without feedback (with negative resistance) or those that yes they are (astable multivibrator).

The first ones that possess an area of negative resistance, are par excellence two: the diode tunnel and the transistor uni-junction (conventional TUJ or programmable TUP); the first one is stable to the voltage and the second to the current in their critical areas; that is to say, they should be excited, respectively, with voltage generators and of current.



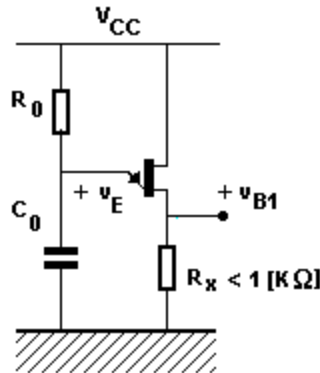
It is necessary to explain in these diodes that it is not that they have a true "negative" resistance, but rather to the being polarized in this area they take energy of the source and they only offer there this characteristic, that is to say dynamically.

## Type TUJ

Truly, already forgotten this dispositive with the years and highly overcome by the digital benefits, it doesn't stop to be historically instructive. For such a reason we won't deepen in their study, but we will only comment some general characteristics to design it if the occasion determines it. It will still be of easy and efficient use in applications of simple phase regulators and in timers of high period.

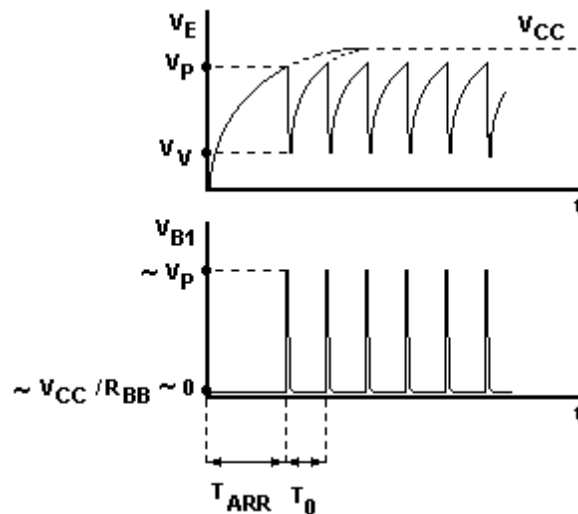
The following implementation is classic. A typical resistance of 390 [ $\Omega$ ] has been omitted in series with the second base to compensate the offset of temperature, but that to practical ends it doesn't affect for anything its use and it hinders our studies. For any TUJ their characteristics are approximately the same ones and they are

$\eta \sim 0,6$	<i>attenuation factor among bases <math>R_{B1}/R_{BB}</math></i>
$V_V \sim 1,5$ [V]	<i>barrier voltage</i>
$I_V \approx 1$ [mA]	<i>barrier current</i>
$I_P \approx 1$ [ $\mu$ A]	<i>current of the shot pick</i>
$V_{BB} \approx 10$ [V]	<i>voltage among bases</i>
$R_{BB} \approx 10$ [K $\Omega$ ]	<i>resistance among bases <math>R_{B1}+R_{B2}</math></i>
$V_P = 0,6 + \eta V_{CC}$	<i>voltage of the shot pick</i>



and the equations of time of operation like timer ( $T_{ARR}$  starts up) or oscillator ( $T_0$ ) we find them outlining the load and discharge from  $C_0$  to the tensions  $V_P$  and  $V_V$  (the discharge  $\sim C_0 R_X$  is omitted to be worthless)

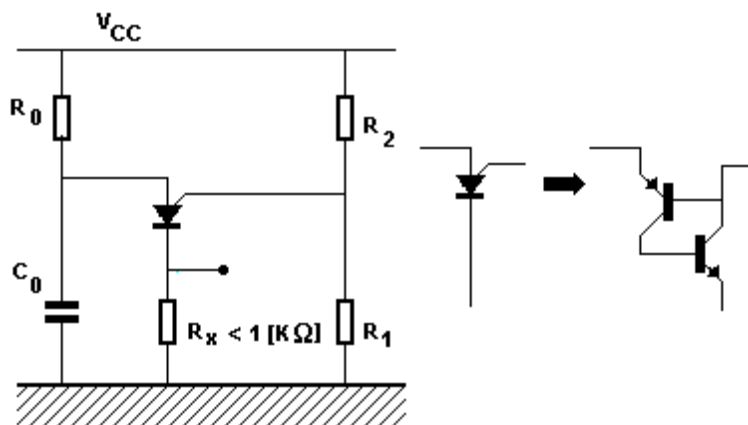
$$T_{ARR} = 1 / R_0 C_0 \ln (1 - V_P / V_{CC})^{-1} \sim 1 / R_0 C_0 \ln (V_{CC} - 1,5) / (0,4 V_{CC} - 0,6)$$
$$T_0 = 1 / R_0 C_0 \ln (V_{CC} - V_V) / [V_{CC} (1 - \eta) - 0,6] \sim 1 / R_0 C_0 \ln (V_{CC} - 1,5) / (0,4 V_{CC} - 0,6)$$



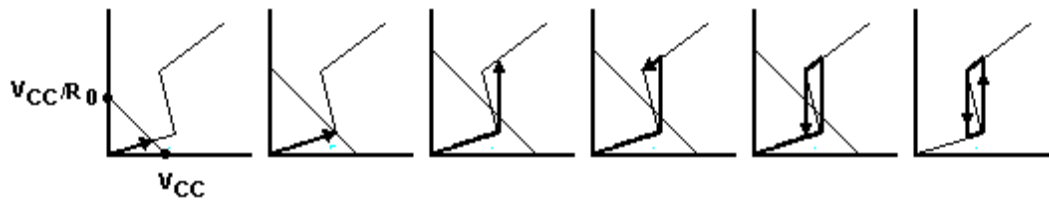
A programmable variant of the  $\eta$  of this circuit is with the device *denominated transistor programmable unijunction* or TUP. This is not more than kind of a controlled rectifier of silicon RCS (or *unidirectional thyristor*) but of anodic gate, and almost perfectly replaceable with TBJ PNP-NPN'S couple like it is shown. The following circuit offers the same properties that the previous one but with the possibility of programming him the  $\eta$

$$\eta = R_1 / (R_1 + R_2) \text{ factor of programmable attenuation}$$

$$V_p = 0,6 + \eta V_{CC} \text{ voltage of the shot pick}$$



For the design of these dispositive, and without going into explanatory details, it will polarize the straight line of operation in such a way that cuts the area of negative slope; in their defect: or it won't shoot for not arriving to  $V_p$ , or it will have a behavior monostable  $T_{ARR}$ . The following oscillation graphs explain how the work point travels for the characteristics of the device, without never ending up resting in the polarization point.



## Design

Be the data (circuit with TUJ)

$$f_0 = \dots \quad V_{CC} = \dots$$

On one hand we respect for not modifying the theoretical data

$$R_x = \dots \leq 1 \text{ [K}\Omega\text{]} \quad (R_x \text{ can be an inductor, a transformer of pulses, or to be in series with the base-emitter juncture of a TBJ})$$

We determine the voltage shot pick

$$V_p = 0,6 + \eta V_{CC} \sim 0,6 + 0,6 V_{CC} = \dots$$

and then we calculate the resistance in such a way that shoots the TUJ but that it is not very big and make to the circuit monostable

$$(V_{CC} - V_V)/I_V \sim 10^3(V_{CC} - 1,5) < R_0 = \dots < (V_{CC} - V_P)/I_P \sim 10^6(V_{CC} - V_P)$$

for then to determine the condenser (it is convenient that it is of the biggest possible voltage and of mark of grateful production to avoid faulty losses)

$$C_0 = T_0 / R_0 \ln (V_{CC} - V_V)/[V_{CC} (1-\eta)-0,6] \sim T_0 / R_0 \ln (V_{CC} - 1,5)/[0,4V_{CC}-0,6] = \dots$$

where  $V_{CC} \geq 6 \text{ [V]}$  practically it is  $C_0 \sim T_0 / R_0$

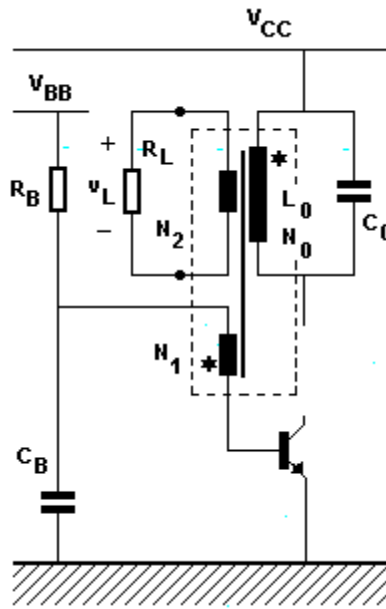
## Type multivibrator

It studies it to him and it designs in the *multivibrators* chapter.

## Type harmonic-relaxation

Basically, these oscillators work with lineal and alineal dipositives. Being a mixed of both, infinite ways exist of implementing them.

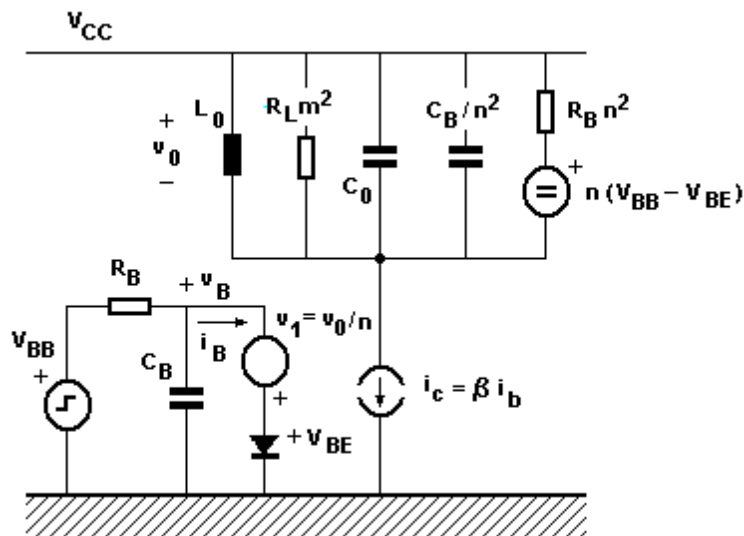
We will see the denominated type for *atoblocking*. The following one is a typical configuration and only one of the possible ones.



When being given alimentation  $V_{BB}$ , the  $C_B$  leaves loading until polarizing the TBJ in direct and that it will drive without being saturated (because  $V_{BB}$  and  $R_B$  are not designed for this). This transitory one in the collector will make that the syntonized circuit captures its harmonica  $\omega_0$ , and the circuit is in the following analogy

$$n = N_0 / N_1$$

$$m = N_0 / N_2$$



If to simplify the equations we make

$$C_0 \gg C_B / n^2$$

$$R_0 = R_L m^2 \ll R_B n^2$$

$$1/n \ll \beta$$

they will be approximately

$$Z = v_{ec} / i_c = (1/C_0) s / [(s+\tau_0)^2 + \omega_0^2]$$

$$\tau_0 = 2 R_0 C_0$$

$$\omega_0 = 1 / [(R_0 C_0 C_0)^{-2} - (\tau_0)^{-2}]^{1/2}$$

and like it is applied an excitement step

$$i_c = \beta (V_{BB} - V_{BE}) / R_B \rightarrow [\beta (V_{BB} - V_{BE}) / R_B] / s$$

$$v_{ec} = Z i_c = [\beta (V_{BB} - V_{BE}) / R_B C_0] / [(s+\tau_0)^2 + \omega_0^2] \rightarrow$$

$$\rightarrow [\beta (V_{BB} - V_{BE}) / R_B C_0 \omega_0] e^{-t/\tau_0} \cdot \sin \omega_0 t = v_{op} e^{-t/\tau_0} \cdot \sin \omega_0 t$$

expression that shows that for not having undesirable oscillations (that is to say a under-damping exit) it should be

$$R_0 C_0 / L_0 > 0,25$$

If we still summarize more the expressions

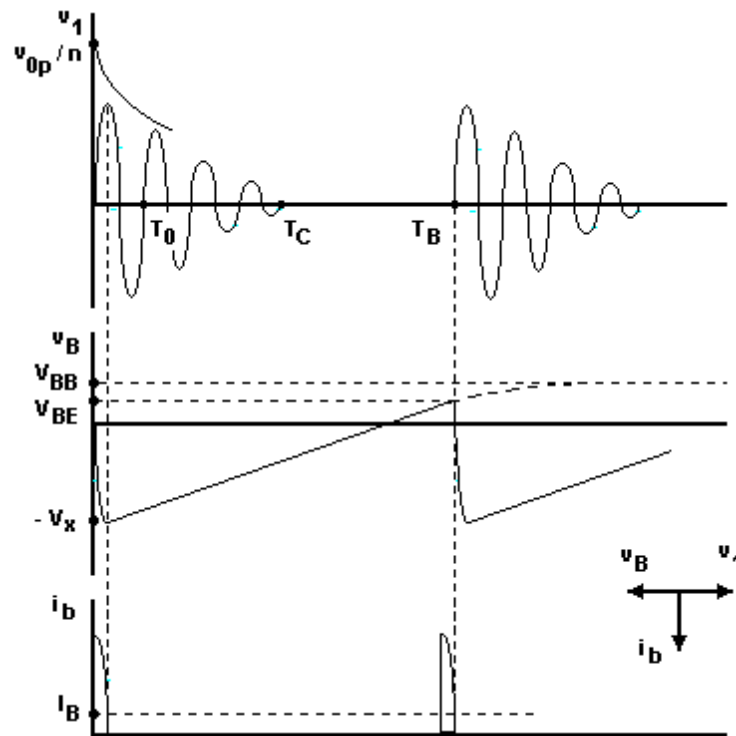
$$R_B \gg 1 / \omega_0 C_B$$

it will be possible to analyze the waves in their entirety. This way, the drawings show that  $C_B$  cannot lose its load and to maintain a damping in the base circuit because the diode base-emitter impedes it to him; this way the TBJ is cut (but observe you that same vec exists), and the condenser doesn't already continue more the variations of the sine wave, but rather losing its negative potential will try to arrive to that of the source  $V_{BB}$ . Concluding, for the idealized drawn wave forms (approximate) to the magnitude

$$T_0 \ll T_B$$

t corresponds him

$$v_B = -V_x + (V_{BB} + V_x) (1 - e^{-t/RBCB})$$



## Converters and Inverters

### Generalities

We call *convertors* to those circuits that convert a magnitude of DC to another magnitude of DC (generally higher, and they usually consist on an *inverter* and their corresponding rectification-filtrate), and *inverters* to those other circuits that transform it to AC (oscillators of power).

In these circuits that we study transformers they are used with magnetic nucleus that they offer behaviors astables. It takes advantage their saturation to cancel the magnetic inductance and with it their transformers properties. With the purpose of introducing us in the topic we abbreviate (see you the *inductors* chapter and *transformers of great value*)

$$L = \Delta\mu_{ef} N^2 S / l_{Fe} = (2 B_{SAT} / H_{SAT}) N^2 S / l_{Fe}$$

The losses for Foucault and hysteresis become considerable when working with waves squared by the great spectrum of their harmonic content. Consequently typical frequencies of operation are usually used

- ferrite of 1 at 20 [KHz]
- iron of 50 at 100 [Hz]

also, in a general way, the efficiency in the best cases is of the 90%

$$\eta = P_{SAL}(\text{en la carga RL}) / P_{ENT}(\text{a la entrada del transformador}) \sim 0,9$$



On the other hand, if is interested in knowing the effective resistance of these losses that we denominate  $R_0$ , we can outline if we call  $V_{SAL}$  to the effective voltage in the load

$$P_{R0} \sim V_{SAL}^2 / R_0$$

$$P_{SAL} = V_{SAL}^2 / n^2 R_L$$

of where

$$P_{SAL} = P_{SAL} + P_{R0}$$

$$P_{R0} = P_{SAL} (\eta^{-1} - 1)$$

$$R_0 = n^2 R_L / (\eta^{-1} - 1) \sim 9 n^2 R_L$$

For the inverters, it is enough many times the use of a condenser in parallel with the load in such a way that the square sign is a sine wave —they will filter harmonic. Other more sophisticated filters can also be used, as they are it those of filter impedance  $\pi$ , syntonized, etc. For these applications, clearing should be, the equations are no longer those that are presented.

If what we want is to manufacture a converter, then it will be enough to rectify and to filter with a condenser  $C_L$  the exit. For it will be enough the condition ( $f_0$  are the oscillation frequency)

$$R_L C_L \gg 1 / f_0$$

or to make a filter as it has been analyzed in the chapter of *power supply without stabilizing*. To find in these cases an esteem of the resistance that reflects a filter  $R_{Lef}$ , we equal the power that surrenders to the rectificador-filter with that of the load (we call «n» to the primary relationship of spires to secondary of the exit transformer)

$$f_0 \cdot \int_0^{1/2f_0} (V_{CC}^2 / n^2 R_{Lef}) \partial t \sim V_{CC}^2 / n^2 R_L$$

of where

$$R_{Lef} \sim R_L / 2$$

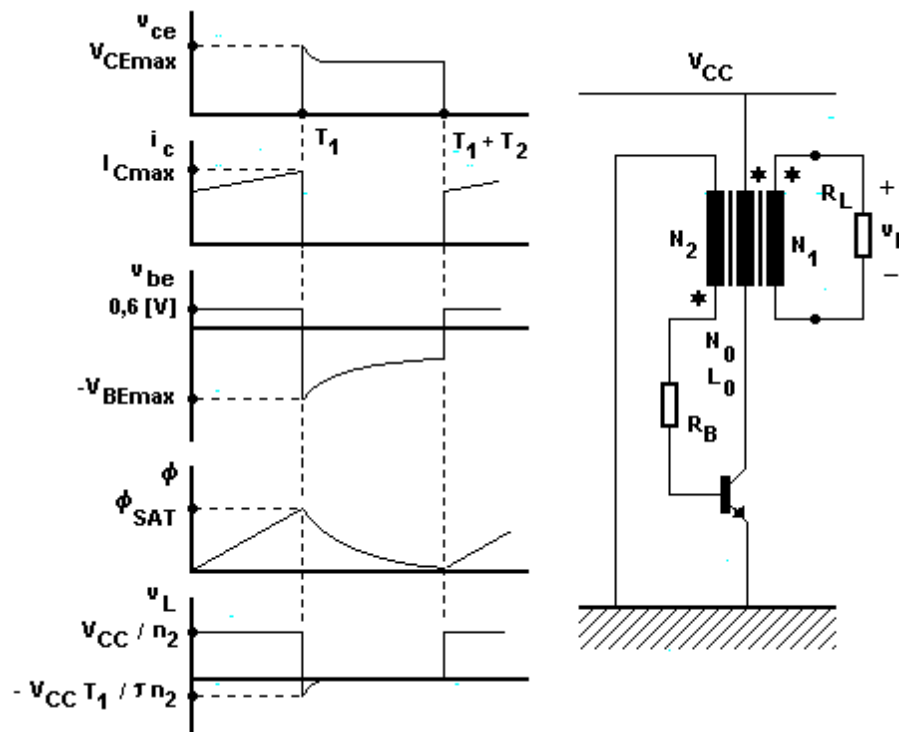
### Inverter of a TBJ and a transformer

When lighting the circuit their transitory one it will produce the saturation of the TBJ instantly (or a polarization will be added so that this happens) being

$$n_1 = N_1 / N_2$$

$$n_2 = N_1 / N_3$$

$$n_1^2 R_B \gg n_2^2 R_L \quad (\text{for not dissipating useless power in the base circuit})$$



There will be then in the ignition a continuous voltage  $V_{CC}$  applied on  $L_0$  that it will make have a lineal flow  $\phi$  in the time and that, when arriving to the saturation  $\phi_{SAT}$  the  $\Delta\mu_{ef}$  it will get lost eliminating at  $L_0$ . This results since in the cut of the TBJ in this instant properties transformers they won't exist. Immediately then the current of the primary one begins to diminish (the magnetic field is discharged) and it is regenerated  $L_0$  when existing  $\Delta\mu_{ef}$  again, maintaining in this way cut the TBJ. Once discharged the inductance, that is to say when their current is annulled, other transitory due to the distributed component it gives beginning to the oscillation again.

Analyzing the circuit, when the TBJ saturates we have

$$\begin{aligned} v_{L0} &= V_{CC} \\ v_{ce} &= V_{CC} - v_{L0} = 0 \\ v_{be} &= V_{BES} \sim 0,6 [V] \\ i_c &= I_{CS} = V_{CC} [ (n_2^2 R_L)^{-1} + t/L_0 ] \\ \phi &= (1/N_1) \int_0^t v_{L0} \partial t = V_{CC} t / N_1 \\ v_L &= v_{L0} / n_2 = V_{CC} / n_2 \end{aligned}$$

for the one which when reaching the saturation, then

$$T_1 = N_1 \phi_{SAT} / V_{CC}$$

Subsequently, during the cut of the TBJ

$$\begin{aligned} v_{L0} &= -\Delta V e^{-t/\tau} = -(V_{CC} T_1 / \tau) e^{-t/\tau} \\ \tau &= L_0 / n_2^2 R_L \\ v_{ce} &= V_{CC} - v_{L0} = V_{CC} [1 + (T_1 / \tau) e^{-t/\tau}] \end{aligned}$$

$$v_{be} = v_{L0} / n_1 = -(V_{CC} T_1 / \tau n_1) e^{-t/\tau}$$

$$i_c = 0$$

$$\phi = -v_{L0} L_0 / n_2^2 R_L = V_{CC} T_1 e^{-t/\tau}$$

$$v_L = v_{L0} / n_2 = (V_{CC} T_1 / \tau n_2) e^{-t/\tau}$$

and if we consider that it is discharged in approximately  $3\tau$

$$T_2 \approx 3\tau$$

For the admissibility of the TBJ, of the previous equations we obtain

$$I_{Cmax} = V_{CC} [(n_2^2 R_L)^{-1} + T_1 / L_0]$$

$$V_{CEmax} = V_{CC} [1 + (T_1 / \tau)]$$

$$-V_{BEmax} = V_{CC} T_1 / \tau n_1$$

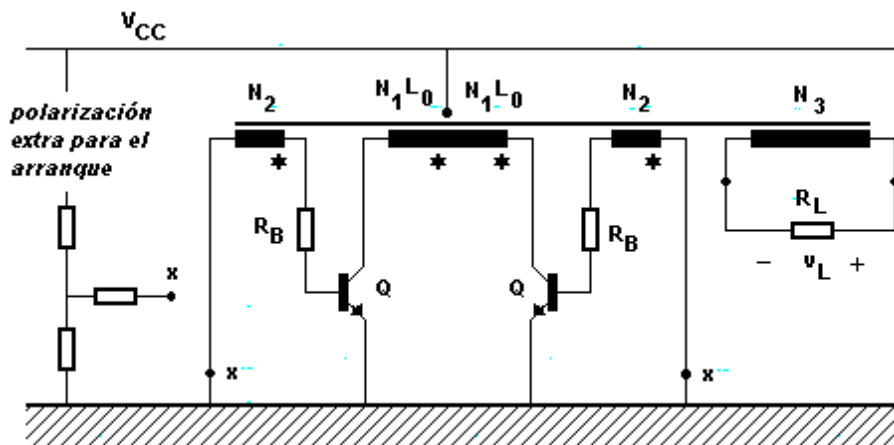
$$P_{CEmax} = (T_1 + T_2)^{-1} \int_0^{T_1} V_{CES} I_{CS} dt \approx V_{CC} V_{CES} [(n_2^2 R_L)^{-1} + T_1 / 2L_0]$$

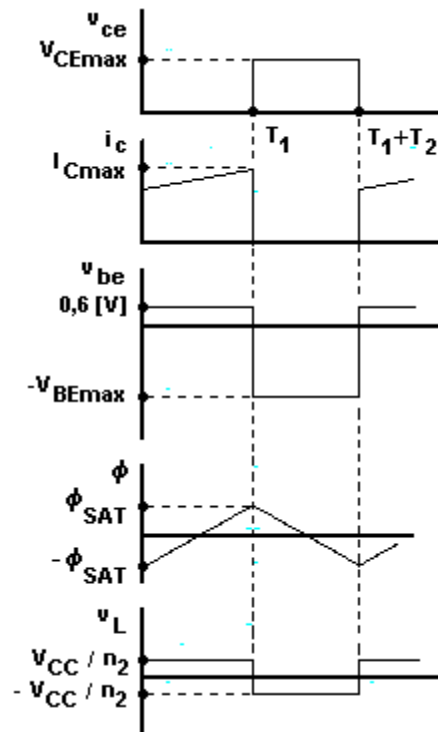
and finally in the load

$$P_{Lmax} \sim (T_1 + T_2)^{-1} \int_0^{T_1} (V_{CC}^2 / n_2^2 R_L) dt \sim V_{CC}^2 / 2 n_2^2 R_L$$

### Inverterr of two TBJ and a transformer

It is, in fact this circuit, a double version of the previous one where while a TBJ saturates the other one it goes to the cut, in such a way that the power on the load increases to twice as much. This way, for oneself useful power, the transistors are fewer demanded. In the drawing it is insinuated a possible additional polarization that should be used for if the circuit doesn't start up: the points rise «x» of the bases and they are connected to the resistors.





In the saturation hemicycle they are completed for each TBJ

$$v_{L0} = V_{CC}$$

$$v_{ce} = V_{CC} - v_{L0} = 0$$

$$v_{be} = V_{BES} \sim 0,6 [V]$$

$$i_c = I_{CS} = V_{CC} [ (n_2^2 R_L)^{-1} + t/L_0 ]$$

$$\phi = -\phi_{SAT} + (1/N_1) \int_0^t v_{L0} \partial t = -\phi_{SAT} + V_{CC} t / N_1$$

$$v_L = v_{L0} / n_2 = V_{CC} / n_2$$

for that that once passed  $T_1$  then the over-voltage of the cut will make the other TBJ drive and it will be the symmetry.

For the admissibility of the TBJ, of the previous equations we obtain

$$I_{Cmax} = V_{CC} [ (n_2^2 R_L)^{-1} + T_1/L_0 ]$$

$$V_{CEmax} = 2 V_{CC}$$

$$-V_{BEmax} = V_{CC} / n_1$$

$$P_{CEmax} = (T_1 + T_2)^{-1} \int_0^{T_1} V_{CES} I_{CS} \partial t \approx V_{CC} V_{CES} [ (n_2^2 R_L)^{-1} + T_1/2L_0 ] / 2$$

and finally in the load

$$P_{Lmax} \sim 2. (T_1 + T_2)^{-1} \int_0^{T_1} (V_{CC}^2 / n_2^2 R_L) \partial t \sim V_{CC}^2 / n_2^2 R$$

## Design

Be the data

$$R_L = \dots \quad f_0 = \dots \quad V_{Lmax} = \dots \quad (\text{maximum voltage in the load})$$

We adopt a power supply keeping in mind that it is convenient, to respect the equations that are worthless the collector-emitter voltages of saturations (that TBJ of a lot of power stops it usually arrives to the volt). This way, then, it is suggested

$$V_{CC} = \dots \gg 0,25 \text{ [V]}$$

We choose a lamination (or ferrite recipient, reason why will change the design), what will determine us (sees you the design in the *inductors* chapter and *transformers of great value*)

$$S = \dots \geq 0,00013 R_L$$

$$A = \dots$$

$$I_{Fe} = \dots$$

$$B_{SAT} = \dots$$

$$N_1 = V_{CC} / 4 f_0 S B_{SAT} = \dots$$

$$N_3 = N_1 V_{Lmax} / V_{CC} = \dots$$

$$H_{SAT} \approx N_3 V_{Lmax} / I_{Fe} R_L = \dots$$

$$L_0 = 2 N_1^2 S B_{SAT} / I_{Fe} H_{SAT} = \dots$$

where the experimentation of  $L_0$  is suggested after the armed one to obtain its correct value.

Subsequently, if we estimate a magnitude

$$N_2 = \dots$$

they are

$$n_1 = N_1 / N_2 = \dots$$

$$n_2 = N_1 / N_3 = \dots$$

$$R_{Lef} = R_L // R_0 / n_2^2 \sim 0,9 R_L = \dots$$

We obtain the admissibility of each TBJ

$$I_{Cmax} = V_{CC} [ (n_2^2 R_{Lef})^{-1} + 1/2 f_0 L_0 ] = \dots$$

$$V_{CEmax} = 2 V_{CC} = \dots$$

$$-V_{BEmax} = V_{CC} / n_1 = \dots$$

$$P_{CEmax} \approx V_{CC} / 8 f_0 L_0 = \dots$$

and of the manual

$$V_{CES} = \dots (\approx 0,1 \text{ [V]}) \ll V_{CC}$$

$$I_{CADM} = \dots < I_{Cmax}$$

$$V_{BES} = \dots (\approx 0,7 \text{ [V]})$$

$$\tau_{apag} = \dots \ll 1/2 f_0$$

$$\tau_{enc} = \dots \ll 1/2f_0$$

$$\beta = \dots$$

$$T_{JADM} = \dots$$

$$P_{CEADM} = \dots < P_{CEmax}$$

what will allow to find the thermal dissipator

$$surface = \dots$$

$$position = \dots$$

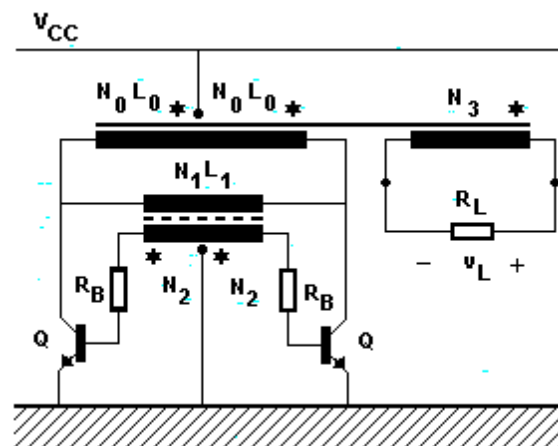
$$thickness = \dots$$

Subsequently we calculate

$$R_B = \beta (V_{CC}/n_1 - V_{BES}) / I_{Cmax} = \dots$$

### Inversor de dos TBJ y dos transformadores

This implementation is used for further powers (up to 500 [W]), since it consists on a great exit transformer that is commuted in this case by the circuit oscillator. This oscillator can be carried out with any other astable that determines the court-saturation of the TBJ. This way, the exit transformer doesn't determine the frequency but rather it only transmits the energy; it doesn't happen the same thing with that of the bases, since it will be saturated and it will offer the work cycle consequently —it recommends it to him of ferrite, while to that of iron exit.



If we observe the circuit with detail, we will see that it is not another thing that an investor like the one studied precedently of *two TBJ and a transformer*. For this reason the graphics and equations are the same ones, with the following exception

$$n_1 = N_1 / N_2$$

$$n_2 = N_0 / N_3$$

## Chap. 16 Makers of waves

### GENERATORS OF SAWTOOTH

#### Generalities

#### Generators of voltage

Type of voltage for simple ramp

Type of voltage for effect bootstrap

#### Design

Type of voltage for effect Miller

#### Design

#### Current generators

Current type for simple ramp

Current type for parallel efficiency

### DIGITAL SYNTHESIZER

#### Design

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### GENERATORS OF SAWTOOTH

#### Generalities

Used to generate electronic sweepings in the screens and monitors of the tubes of cathodic rays, they can be of two types according to the physical principle of deflection: for voltage (electric deflection in oscilographys) or for current (magnetic deflection in yokes).

The circuits consist, essentially, in a transfer of a single pole (inertia of the same one in  $1/\tau$ , being  $\tau$  their constant of time) and consequently, in excitements of the type step, they determine exponential to their exit (useful sweeping) in the way

$$x = X ( 1 - e^{-t/\tau} ) = X \{ 1 - [ 1 - (t/\tau) + (t/\tau)^2/2! + \dots ] \} = X [ (t/\tau) + (t^2/2\tau^2) + (t^3/6\tau^3) + \dots ]$$

that in the first part

$$x \sim X (t/\tau) [ 1 - (t/2\tau) ]$$

As it is of waiting, one will work preferably in the beginning of the exponential one for their lineality. This operability is applied better in the design with a factors that they specify the slope and

that next we detail

$$u(t) = \partial x / \partial t \quad \text{speed}$$

$$e_{v(t)} = [u_{(0)} - u_{(t)}] / u_{(0)} \quad \text{relative error of speed}$$

and in our transfer

$$u \sim \partial \{X(t/\tau) [1 - (t/2\tau)]\} / \partial t = X(1 - t/\tau) / \tau$$

$$e_v \sim t/\tau$$

which we will be able to stiller simplify if we are in the beginning of the exponential

$$t \ll 2\tau$$

$$x \sim X t / \tau$$

$$e_v \sim t/\tau \sim x / X$$

## Generators of voltage

### Type of voltage for simple ramp

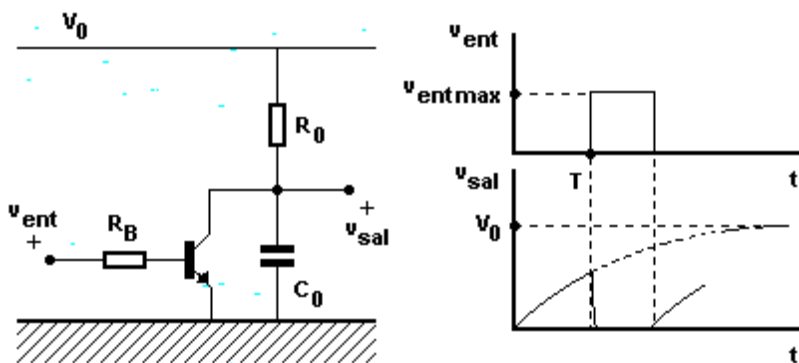
In this circuit a condenser will simply be loaded and after the period T will discharge it to him by means of a TBJ that will be saturated. The behavior equations are the following

$$v_{sal} = V_0 (1 - e^{-t/\tau_0})$$

$$\tau_0 = R_0 C_0$$

$$u(t) = V_0 e^{-t/\tau_0} / \tau_0$$

$$e_{v(T)} = 1 - e^{-T/\tau_0} \sim v_{sal(T)} / V_0$$

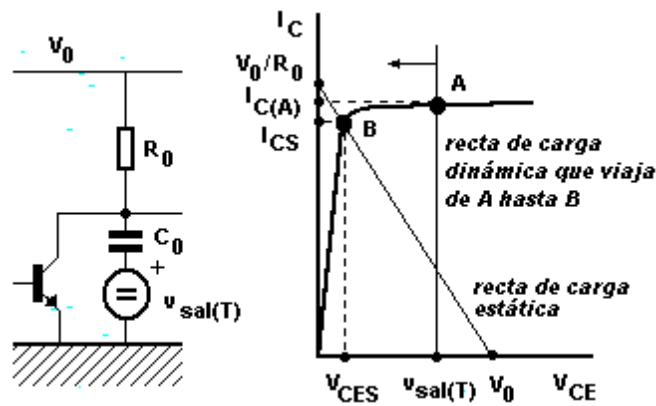


The base current will saturate to the dispositive taking it for an on the way to constant current

$$I_{BS} = (v_{entmax} - 0,6) / R_B$$

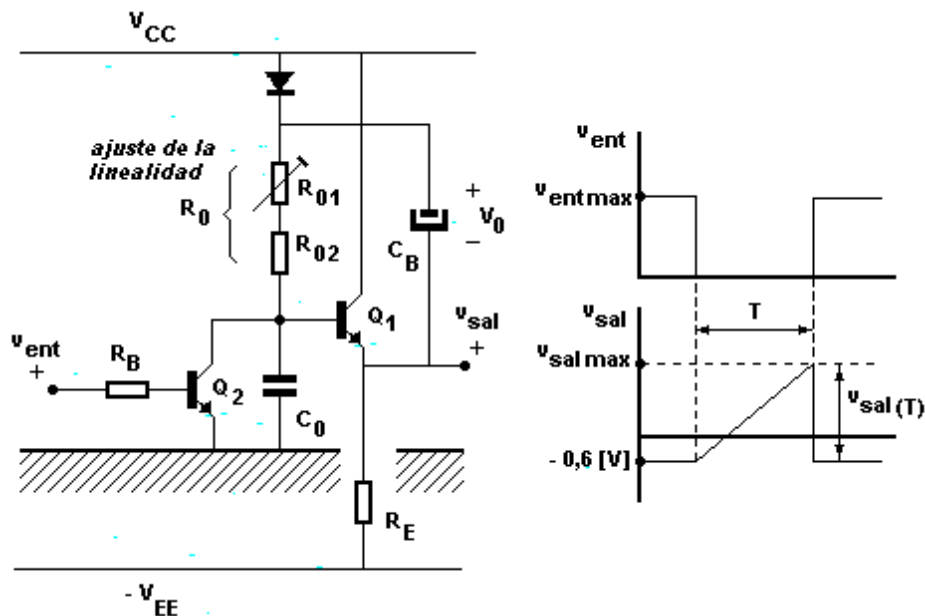
$$v_{sal} \sim v_{sal(T)} - C_0^{-1} \int_0^t i_c \partial t \sim v_{sal(T)} - \beta I_{BS} t / C_0$$





### Type of voltage for effect bootstrap

The circuit consists on making load the condenser  $C_0$  to constant current; or, said otherwise, with a great  $V_0$ . The following implementation makes work to the TBJ in its active area as amplifier follower (also being able to implement this with an AOV) for what will determine a practically generating voltage in its originator and that it will be opposed at  $V_0$ . Another form of thinking this is saying that the current for  $R_0$  practically won't change, but only in the small magnitudes of the trip that it has the diode base-emitter that, truly, it is worthless.



Their operation equations are the following ones. As for the TBJ switch  $Q_2$

$$v_{entmax} \sim I_{BS2} R_B + 0,6$$

and to the operation of the amplifier  $Q_1$

$$R_{ent1} = v_{b1} / i_{b1} \sim h_{11e1} + \beta_1 R_E$$

$$A_{v1} = v_{sal} / v_{b1} \sim \beta_1 R_E / (h_{11e1} + \beta_1 R_E) \sim 1$$

$$v_{salmax} \sim -0,6 + (V_0 - 0,6) T / \tau_0$$

$$\tau_0 = R_0 C_0$$

$$V_0 = I_{B1} R_0 + 0,6$$

On the other hand, if we outline the following equivalent circuit we will be able to find the error of speed

$$v_{sal} = A_{v1} v_{b1} = (A_{v1} V_0 / \tau_0) / s [s + (1 + R_0 / R_{ent1} - A_{v1}) / \tau_0] \rightarrow$$

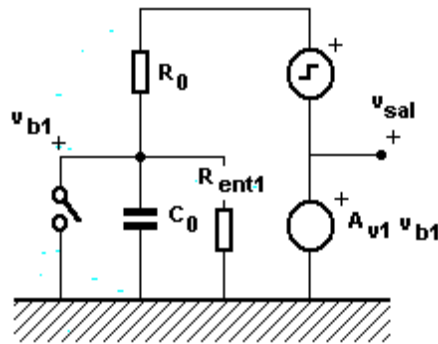
$$\rightarrow [V_0 \beta_1 R_E / (h_{11e1} + R_0)] (1 - e^{-t/\tau_B})$$

$$\tau_B = \tau_0 k$$

$$k = \beta_1 R_E / (h_{11e1} + R_0) \gg 1$$

$$u(t) = V_0 e^{-t/\tau_B} / \tau_0$$

$$e_{v(T)} = 1 - e^{-T/\tau_B} \sim [v_{sal(T)} / V_0] / k$$



being minimized the error in k times.

## Design

Be tha data

$$T = \dots \quad v_{entmax} = \dots \quad v_{salmax} = v_{sal(T)} - 0,6 = \dots \quad e_{v(T)} = \dots$$

Firstly we can choose two TBJ anyone and we obtain of the manual a polarization for  $Q_1$

$$V_{CE1} = \dots \quad I_{C1} = \dots \quad \beta_1 = \dots \quad h_{11e1} = \dots \quad I_{B1} = I_{C1} / \beta_2 = \dots$$

and another for  $Q_2$  knowing that it is discharged  $C_0$  to constant current  $I_{C2}$  from  $\sim v_{salmax}$  until being saturated

$$V_{CE2} \sim v_{salmax} = \dots \quad I_{C2} = \dots \quad \beta_2 = \dots \quad I_{B2} = I_{C2} / \beta_2 = \dots$$

As the maxim exit it is when  $Q_1$  almost saturate (the diode allows to lift the potential of its cathode above  $V_{CC}$ ), then we adopt

$$V_{CC} = \dots > v_{salmax}$$

and we continue calculating

$$R_B = (v_{entmax} - 0,6) / I_{B2} = \dots$$

$$R_0 = (V_0 - 0,6) / (I_{C2} + I_{B1}) = \dots$$

$$V_0 = I_{B1} R_0 + 0,6 = \dots$$

$$v_{sal(T)} = v_{salmax} + 0,6 = \dots$$

$$k = [ v_{sal(T)} / V_0 ] / e_{v(T)} = \dots$$

$$R_E = k (h_{11e1} + R_0) / \beta_1 = \dots$$

$$V_{EE} = I_{C1} R_E = \dots$$

$$C_0 \sim T ( V_0 - 0,6 ) / R_0 ( v_{salmax} + 0,6 ) = \dots$$

we verify the quick discharge of  $C_0$

$$C_0 ( v_{salmax} + 0,6 ) / I_{C2} = \dots \ll T$$

and the slow of the bootstrap  $C_B$

$$C_B = \dots \gg T / (h_{11e1} + R_0)$$

With regard to the adjustment of lineality  $R_{01}$  and the protection  $R_{02}$  will be optional.

## Type of voltage for effect Miller

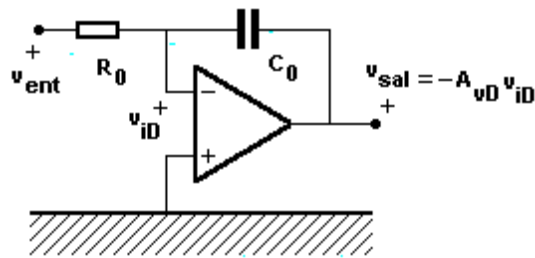
It has already been spoken of the effect Miller in the chapter of *amplifiers of radiofrequency class A in low level*. Here we will take advantage of those concepts like fictitious capacity; or said otherwise that magnify the voltage  $V_0$  virtually.

The implementation following sample the answer in frequency of an AOV with negative feedback. Let us suppose that it possesses, internally like it is of waiting, a dominant pole in  $w_1$  that diminishes the differential gain of continuous  $A_0$

$$A_{vD} = v_{sal} / v_{iD} = - A_0 / (1 + s/\omega_1)$$

$$A_v = v_{sal} / v_{ent} = - A_0 / [ (1 + s/\omega_0) (1 + s/\omega_1) + s A_0/\omega_0 ]$$

$$\omega_0 = 1 / \tau_0 = 1 / R_0 C_0$$

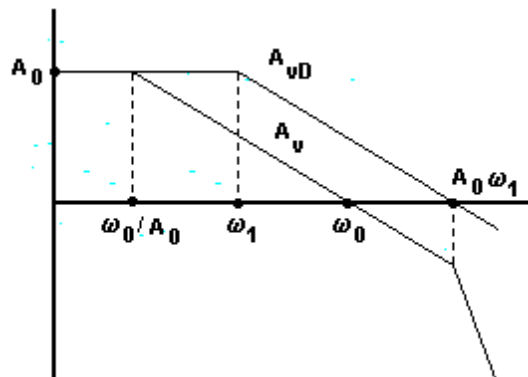


being in low

$$A_v \sim -A_0 / (1 + s A_0 / \omega_0)$$

and in high

$$A_v \sim -A_0 / [(1 + s/\omega_1) + s A_0 / \omega_0] = \omega_0 / s (1 + s/A_0 \omega_1)$$



what determines us an integrative almost perfect, because the Bode begins with a dominant pole of chain closed in  $\omega_0/A_0$  —  $\omega_1$  are of some few cycles (radians) per second.

Next we draw a typical circuit that we will design. The TBJ is taken charge of producing the discharge and the attenuator of synthesizing for Thevenin the necessary  $V_0$  and  $R_0$ . This way, the behavior equations are

$$R_0 = R_1 // R_2$$

$$V_0 = V_{CC} R_2 / (R_1 + R_2)$$

$$v_{entmax} = I_{BS} R_B + 0,6$$

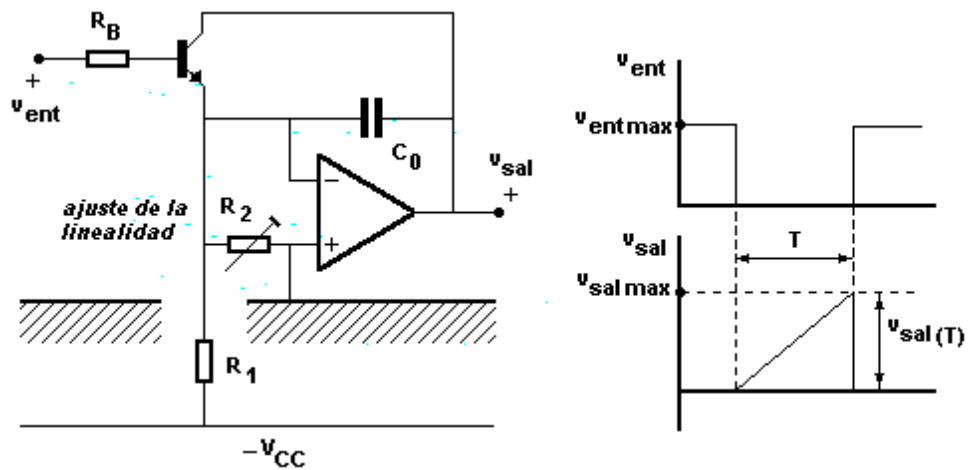
$$v_{sal} = A_{vD} V_0 (1 + R_0/R_{entD}) (1 - e^{-T/\tau_M})$$

$$\tau_M = A_{vD} C_0 R_0 / R_{entD} \sim \tau_0 k$$

$$k = A_{vD} = A_0 \gg 1$$

$$u(t) = V_0 e^{-t/\tau_M} / \tau_M$$

$$e_{v(T)} = 1 - e^{-T/\tau_M} \sim [v_{sal(T)} / V_0] / k$$



and conceptually

$$v_{sal} \sim C_0^{-1} \int_0^t (V_0/R_0) dt = (V_0/\tau_0) t$$

## Design

Be the data

$$T = \dots \quad v_{entmax} = \dots \quad v_{salmax} = \dots \quad e_{v(T)} = \dots$$

We choose an AOV with entrance to JFET and we obtain of the manual

$$\pm V_{CC} = \dots$$

$$A_0 = \dots$$

and a capacitor of low losses

$$C_0 = \dots$$

Of the precedent formulas we obtain then

$$V_0 = v_{salmax} / A_0 e_{v(T)} = \dots$$

$$R_0 = V_0 T / C_0 v_{salmax} = \dots$$

$$R_1 = R_0 V_{CC} / V_0 = \dots$$

$$R_2 = \dots \leq R_1 R_0 / (R_1 - R_0)$$

We adopt a TBJ with a collector current that discharges quickly to the condenser

$$I_{C2} = \dots \gg C_0 v_{salmax} / T$$

and of the manual we obtain

$$\beta = \dots$$

for that that

$$R_B \sim \beta (v_{entmax} - 0,6 - I_{C2} R_0 + V_0) / I_{C2} = \dots$$

## Current generators

### Current type for simple ramp

Calls of *simple ramp*, to the inductors when they are applied a continuous voltage they load their magnetism in an exponential way

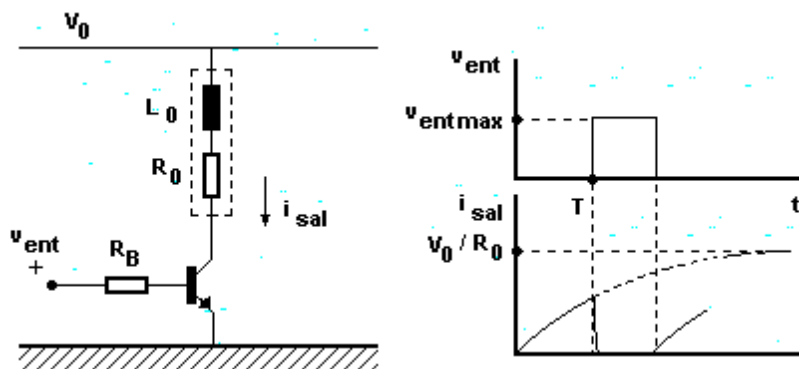
$$I_0 = V_0 / R_0 \text{ (application Norton to the Thevenin)}$$

$$i_{sal} = I_0 (1 - e^{-t/\tau_0})$$

$$\tau_0 = L_0 / C_0$$

$$u(t) = I_0 e^{-t/\tau_0} / \tau_0$$

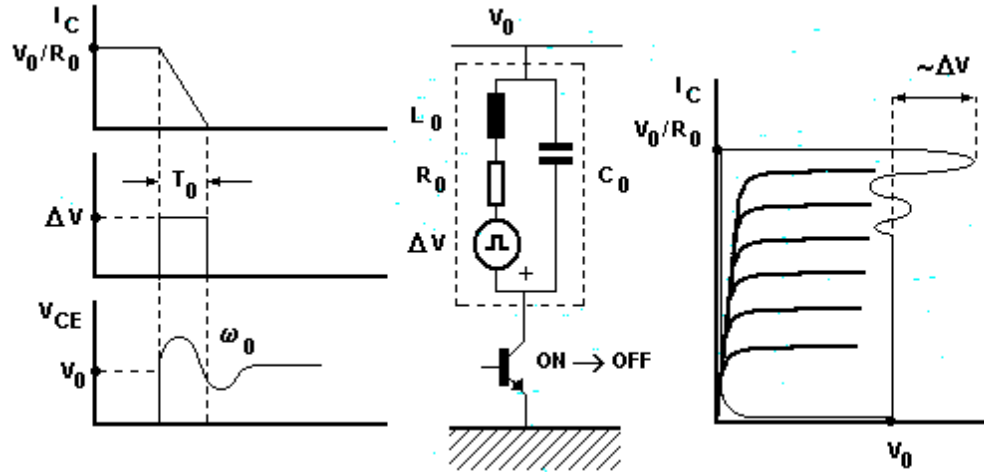
$$e_{v(T)} = 1 - e^{-T/\tau_0} \sim i_{sal(T)} / I_0$$



These behaviors are studied in the chapter of *relaxation oscillators* when seeing inverters and converters. What we will add here is their operation curve that, to be to constant current, they possess the form of the following figure and therefore, when being disconnected, they generate a voltage according to the law of Faraday

$$\Delta V = L \Delta I_C / \Delta t = L V_0 / R_0 T_0$$

$$\omega_0 \sim 1 / (L_0 C_0)^{1/2}$$



or, for their precise calculation, keeping in mind the properties of the oscillations, we should use Laplace

$$T_{(s)} = V_{C0} / \Delta V \rightarrow |T_{(\omega)}| e^{j\phi(\omega)}$$

$$V_{ent(s)} = T_{(s)} (\Delta V / s)$$

$$V_{ent(t)} = \mathcal{L}^{-1}[V_{ent(s)}] = k_1 \Delta V e^{-t/\tau} \sin(\omega_0 + \phi)t$$

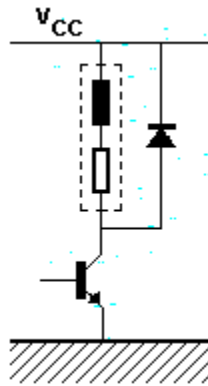
$$\omega_0 \sim 1 / (L_0 C_0)^{1/2}$$

$$\tau = k_2 L_0 / R_0$$

and that we omit their analysis, since besides being complex, it is not very practical because in the experiences it is always very variable their results due to the alineality and little precision of the parameters. It will be enough for the designer to take the worst case considering the protection of the TBJ like

$$V_{CEO} > V_0 + \Delta V$$

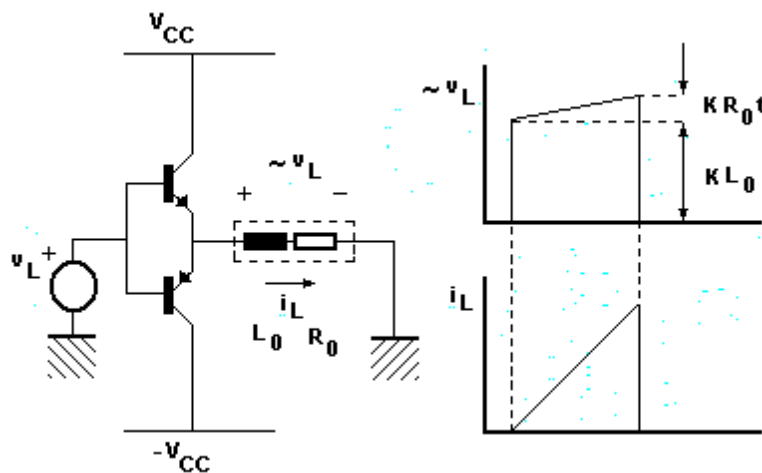
The following implementation eliminates the overvoltage, since the diode impedes with its conduction that the  $V_{CE}$  increases above  $V_{CC} + 0,6$ . On the other hand, in the conduction of the TBJ this rectifier it is in inverse and it doesn't affect to the circuit.



If what we look for is a ramp of magnetic flow  $\phi$ , that is to say of direct line of current for the coil, and having in all that the circuit in such a way has been designed that the surges don't affect, then we can observe that it has more than enough the same one the voltage it has the form of a continuous (due to  $L_0$ ) more a ramp (due to  $R_0$ ). The idea then, like sample the circuit that continues, is to synthesize this wave form in low level and to excite to the inductor of power with complementary exit. Calling  $v_L$  to the voltage on the inductor and  $i_L$  the current in ramp the one that circulates her, is

$$i_L = K t$$

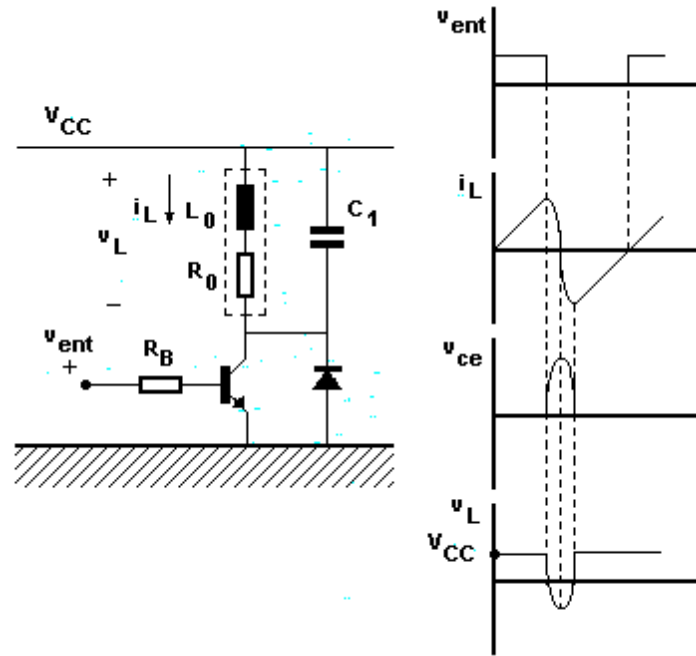
$$v_L = R_0 i_L + L_0 \partial i_L / \partial t = K R_0 t + K L_0 = K (R_0 t + L_0)$$



### Current type for parallel efficiency

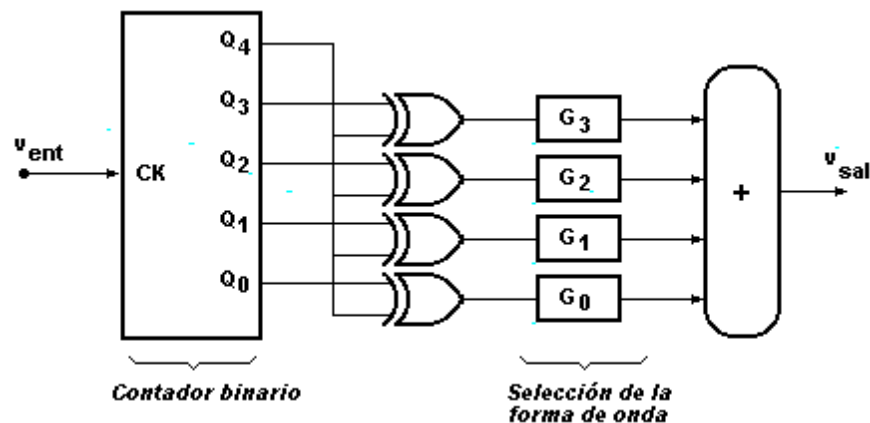
Applied for deflection circuits in television yokes, the configuration following sample an ingenious way (there are other forms, I eat that of efficiency series for example) of creating on the inductors a ramp current and hooked with the synchronism pulses. It takes advantage the own oscillation between  $L_0$  and  $C_1$ .





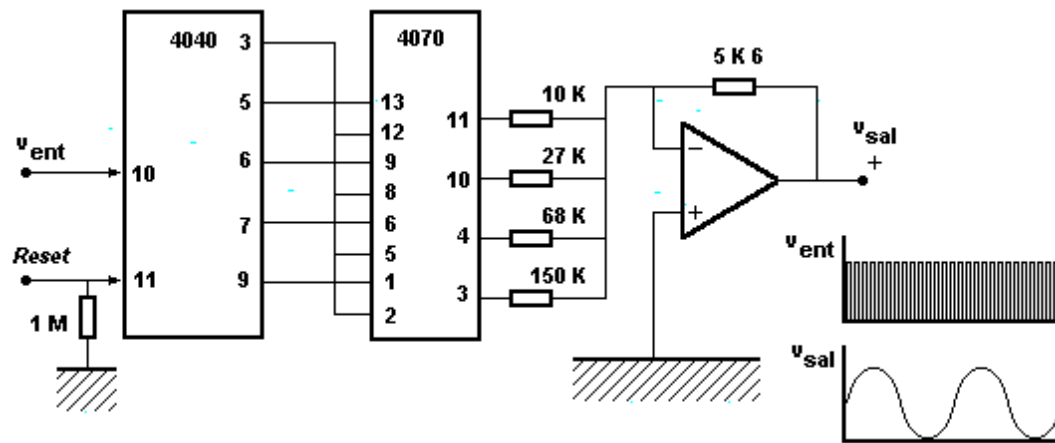
## DIGITAL SYNTHESIZER

At the moment the makers of waves are carried out satisfactorily by the digital implementations. The following outline shows a possible basic approach of synthesis. The entrance of frequency will provide exits that are pondered by the amplifiers  $G$  selecting the wave form that is wanted; the result will be an exit of frequency 32 times minor that that of the entrance and with 32 resolution levels.



## Design

An interesting implementation is the one that is shown next. It has designed it to him so that it makes a sine wave with 16 levels for hemicile, and feasible of to change their frequency or to also sweep very comfortably it, in the whole range of frequency that they allow the integrated circuits.



## Chap. 17 The Transistor in the commutation

The TBJ in continuous

The TBJ with pulses

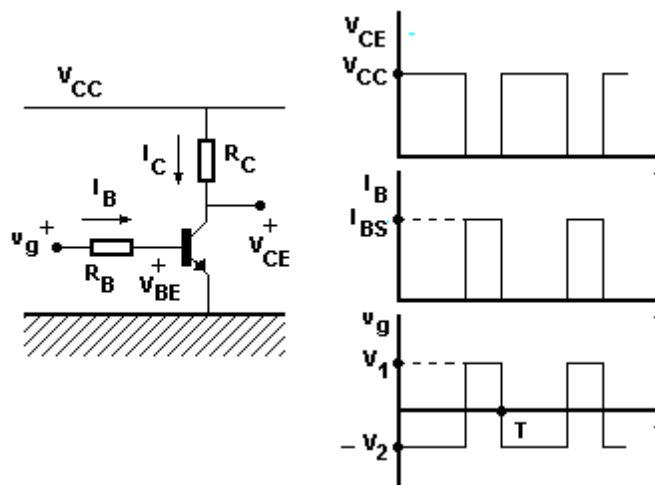
Times of ignition and off

Product gain for wide of band

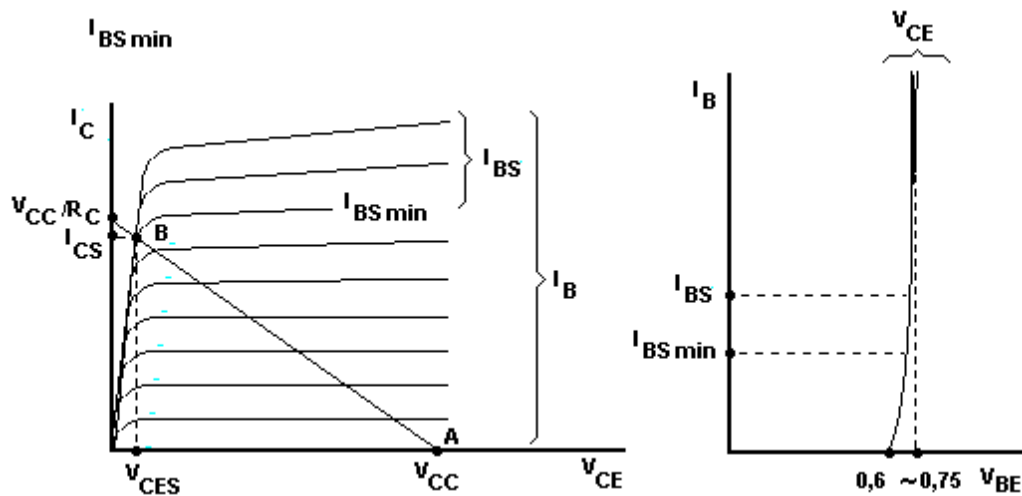
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The TBJ in continuous

Let us suppose to have a circuit like that of the figure



The following characteristics show their straight line of their operation when the TBJ is small and it commutes it to him of the cut A to the saturation B. When using bigger power will change the magnitudes, slopes and constants of the curves a little



The behavior equations are the following in the saturation

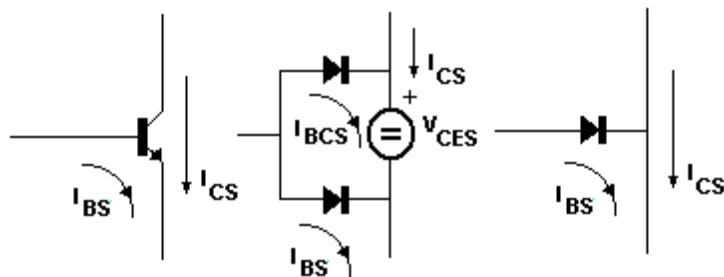
$$V_{BES} = V_{BCS} + V_{CES} \sim 0,7$$

$$V_{CES} \sim 0,1$$

$$V_{BCS} \sim 0,6$$

$$I_{BS} \sim (V_1 - V_{BES}) / R_B \geq I_{BSmin}$$

$$I_{CS} = (V_{CC} - V_{CES}) / R_C \sim V_{CC} / R_C \sim \beta I_{BSmin}$$



and where it can be observed that the TBJ in this state can be simplified to a simple diode (junctions base-emitter and base-collector in derivation), where it becomes independent the collector of its base and the  $\beta$  it is no longer more amplifier.

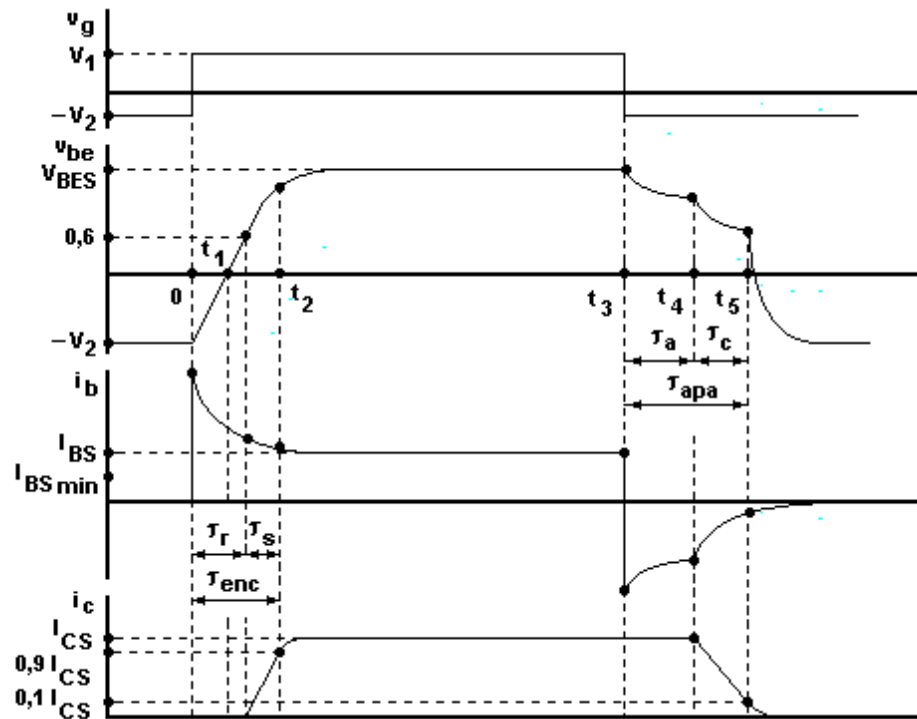
### The TBJ with pulses

#### Times of ignition and off

When the frequency of the pulses becomes big, that is to say of some KiloHertz in TBJ of power and newly after dozens of KiloHertz in those of low power, the capacitances characteristic of the junctions base-emitter and base-collector impede a correct pursuit of the sign. For that reason we study the *ignition* here and *off* of the transistor.

We will generalize the situation arming an experimental circuit as that of the previous figure and we will call *times of ignition* and *of to*

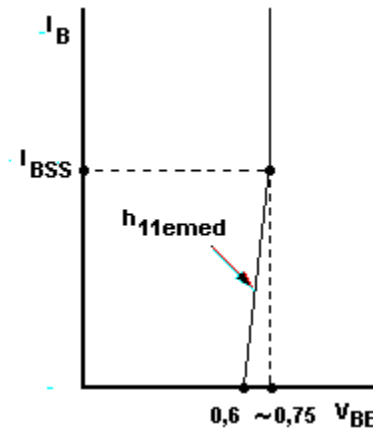
$\tau_{enc} = \tau_r + \tau_s$	time of ignition
$\tau_{apa} = \tau_a + \tau_c$	time of off
$\tau_r$	time of delay
$\tau_s$	time of ascent
$\tau_a$	time of storage
$\tau_c$	time of fall



We can have idea, and we stress this: only an *esteem* of these magnitudes, if we make certain simplifications. For example if we consider

- the idealization of the entrance curve as it is shown in the figure ( $r_{b'e} \sim h_{11med}$ )
- that the parameters  $C_{be}$ ,  $C_{bc}$  and  $\beta$  they don't change with the polarization point
- rejecting the  $C_{ce}$

in their defect, the dispositive will be experienced.



But if we accept them, then, during the time of delay  $\tau_r$  the equations are completed

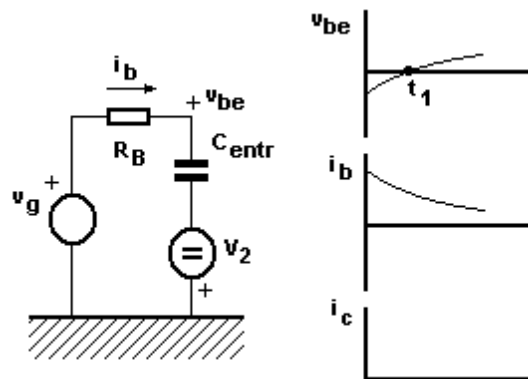
$$v_{be(t)} = -V_2 + (V_1 + V_2)(1 - e^{-t/\tau_{0r}}) < 0,6$$

$$i_c(t) = 0$$

$$i_b(t) = (v_g(t) - v_{be(t)}) / R_B = (V_1 + V_2) e^{-t/\tau_{0r}} / R_B$$

$$C_{entr} = C_{be} + C_{bc}$$

$$\tau_{0r} = R_B C_{entr}$$



what will allow to be defined

$$v_{be(\tau_r)} = 0,6$$

$$\tau_r = \tau_{0r} \ln [(V_1 + V_2) / (V_1 - 0,6)] \approx \tau_{0r} \ln (1 + V_2/V_1)$$

During the *time of ascent*  $\tau_s$  these other equations are completed (with to the help of the pattern  $\pi$ )

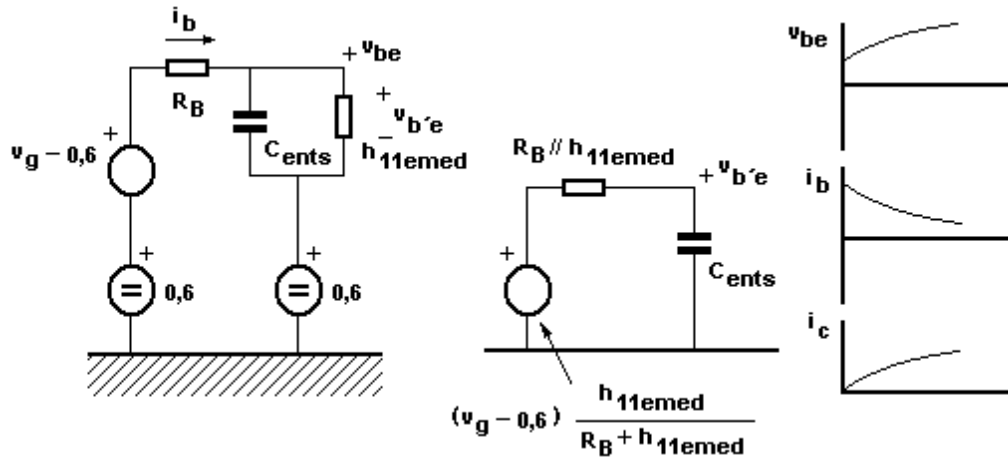
$$v_{be(t-\tau_r)} = v_{b'e(t-\tau_r)} + 0,6 = (V_1 - 0,6)(1 - e^{-(t-\tau_s)/\tau_{0s}}) / (1 + R_B/h_{11emed}) + 0,6 >$$

$$0,6 \quad i_{c(t-\tau_r)} = \beta v_{b'e(t-\tau_r)} / h_{11emed} = \beta (V_1 - 0,6)(1 - e^{-(t-\tau_s)/\tau_{0s}}) / (R_B + h_{11emed}) > 0$$

$$i_{b(t-\tau_r)} = [v_g(t-\tau_r) - v_{be(t-\tau_r)}] / R_B = (V_1 - 0,6) \{1 - [(1 - e^{-(t-\tau_s)/\tau_{0s}}) / (1 + R_B/h_{11emed})]\} / R_B$$

$$C_{ents} = C_{be} + C_{bc} (1 + \beta R_C / h_{11emed})$$

$$\tau_{0s} = R_B // h_{11emed} C_{ents}$$



what will allow to be defined

$$i_{c(t2-\tau_s)} = i_{c(\tau_s)} = \beta (V_1 - 0,6) (1 - e^{-\tau_s/\tau_{0s}}) / (R_B + h_{11emed}) = 0,9 I_{CS}$$

$$\tau_s = \tau_{0s} \ln \{ 1 + [0,9 I_{BSS} (R_B + h_{11emed}) / (0,6 - V_1)] \}^{-1} \approx \tau_{0s} \ln (1 - V_{CC} R_B / \beta V_1 R_C)^{-1}$$

Now, during the *time of storage*  $\tau_a$

$$v_{be(t-t3)} = v_{b'e(t-t3)} + 0,6 =$$

$$= V_{BES} - [(V_2 + 0,6)(1 + R_B/h_{11emed})^{-1} + V_{BES} - 0,6] [1 - e^{-(t-t3)/\tau_{0a}}] > 0,6$$

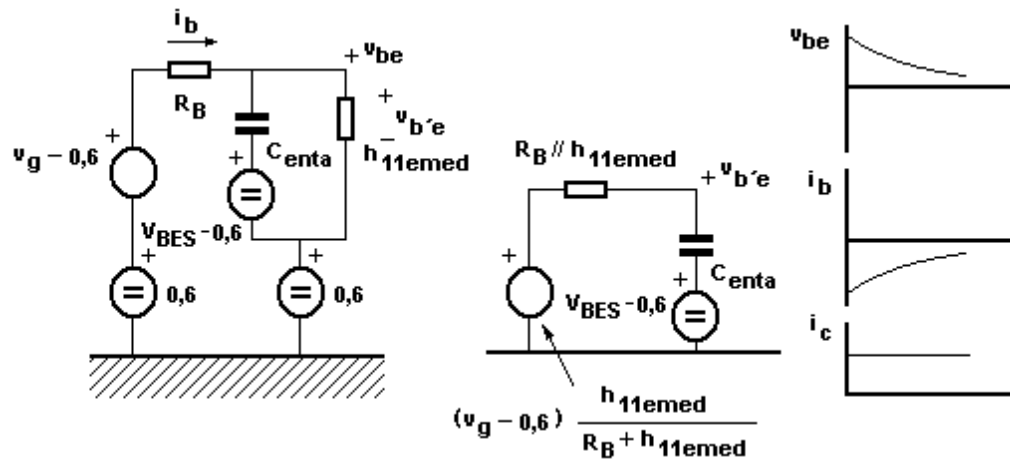
$$i_{c(t-t3)} = I_{CS} > 0$$

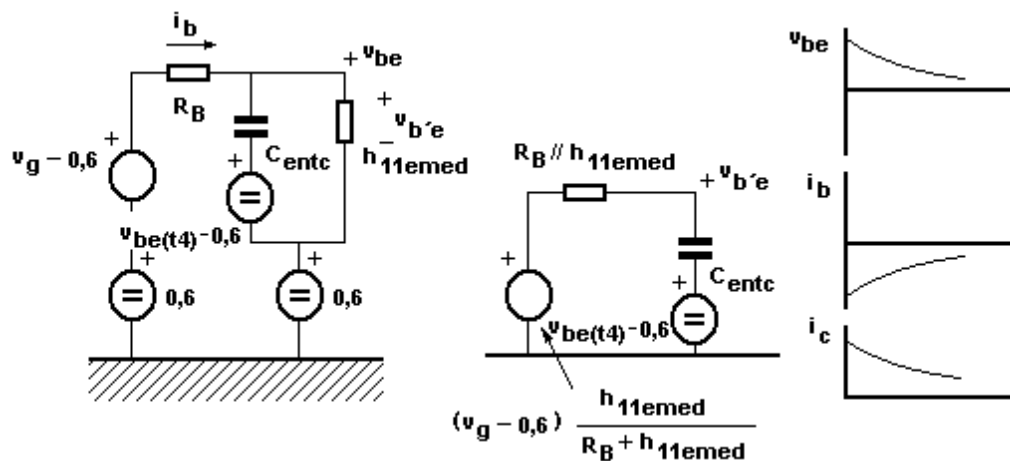
$$i_{b(t-t3)} = [v_g(t-t3) - v_{be(t-t3)}] / R_B =$$

$$= \{ [(V_2 + 0,6)(1 + R_B/h_{11emed})^{-1} + V_{BES} - 0,6] [1 - e^{-(t-t3)/\tau_{0a}}] - V_{BES} - V_2 \} / R_B$$

$$C_{enta} = C_{be} + C_{bc}$$

$$\tau_{0a} = R_B // h_{11emed} C_{enta}$$



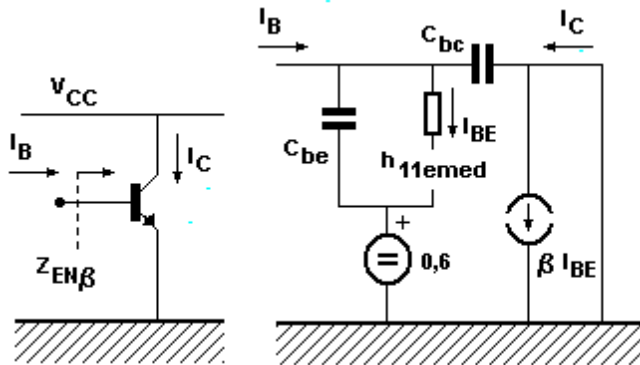




their *product gain for wide of band* PGB that is considered it in practically constant. The equations that they manifest these studies are the following in the common emitter

$$Z_{EN\beta} = 1 / [ h_{11\text{emed}}^{-1} + s (C_{be} + C_{bc}) ] = 1 / (C_{be} + C_{bc}) (s + \omega_{\beta}) \sim 1 / C_{be} (s + \omega_{\beta})$$

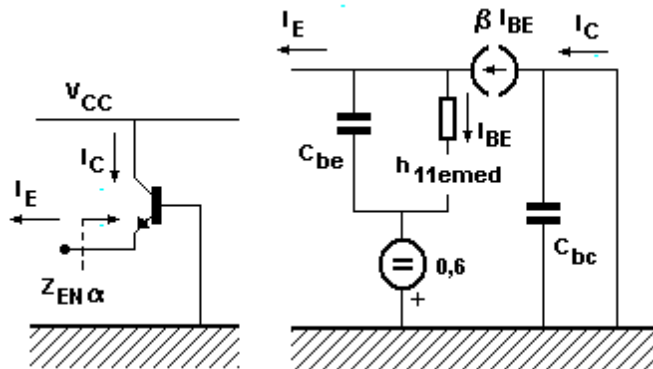
$$\omega_{\beta} = 1 / h_{11\text{emed}} (C_{be} + C_{bc}) \sim 1 / h_{11\text{emed}} C_{be}$$



and in common base

$$Z_{EN\alpha} = 1 / [ (1+\beta) h_{11\text{emed}}^{-1} + s C_{be} ] = 1 / C_{be} (s + \omega_{\alpha})$$

$$\omega_{\alpha} = (1+\beta) / h_{11\text{emed}} C_{be} \sim \beta / h_{11\text{emed}} C_{be}$$

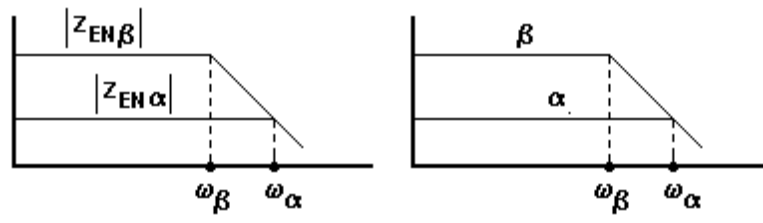


and being

$$\alpha = \beta / (1+\beta) \sim 1$$

it is finally (it calls *transition frequency* from the TBJ to  $\omega_{\tau}$ )

$$PGB = \omega_{\tau} = \beta \omega_{\beta} = \alpha \omega_{\alpha} \sim \omega_{\alpha}$$



## Chap. 18 Multivibrators

### GENERALITIES

### SCHMITT-TRIGGER

#### Generalities

#### With TBJ

#### Design

#### With AOV

#### Design

#### Con C-MOS

### BISTABLE

#### Generalities

#### With TBJ

#### Design

#### With C-MOS

### MONOSTABLE

#### Generalities

#### With TBJ

#### Design

#### With C-MOS

#### Design

#### With the CI 555

#### Design

### ASTABLE

#### Generalities

#### With TBJ

#### Design

#### With AOV

#### Design

#### With C-MOS

#### Design

#### OCV and FCV with the CI 555

#### Modulator of frequency (OCV)

#### Modulator of the width of the pulse (PCV)

#### OCV with the 4046

#### Design

## GENERALITIES

They are known with this name to four circuits with two active components that, while one of them saturates the other one it cuts himself, and that they are, that is: the *Schmitt-Trigger* (amplifier with positive feedback), the *bistable*, the *monostable* (timer) and the *astable* (unstable or oscillatory).

Their names have been given because in the electronic history to work with square waves, that is to say with a rich harmonic content or *vibrations*, they are circuits capable of "multi-vibrating" them.

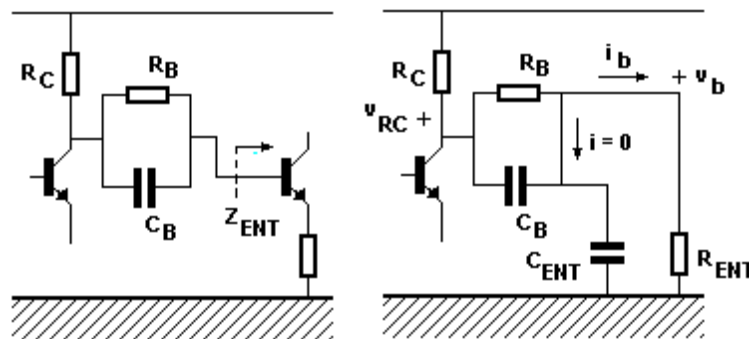
When a stage excites to the following one, speaking of the TBJ, small accelerating condensers  $C_B$  is usually incorporated for the flanks. Their design approach is the following —although the best thing will always be to experience it in each opportunity— giving balance to the bridge

$$R_B / sC_{ENT} = R_{ENT} / sC_B$$

then

$$C_B = C_{EN} R_{ENT} / C_{ENT}$$

$$v_b = v_{RC} / (1 + R_B / R_{ENT})$$



## SCHMITT-TRIGGER

### Generalities

As it was said, it consists on an amplifier with great positive feedback in such a way that their exit, obviously, it can only remain in a single state. When it gets excited it it will change forcibly but then when passing the sign it will return immediately to the rest.

It is characterized by not having to their exit oneself road in their changes, calling you to this "unconformity" like *hysteresis* always making the "good" use that had characterized to the words of the electronics.

### With TBJ

The following one is a possible implementation. It must be designed at  $Q_1$  cut and  $Q_2$  saturated it —is to say in rest. When the second it saturates the equations of the circuit they are

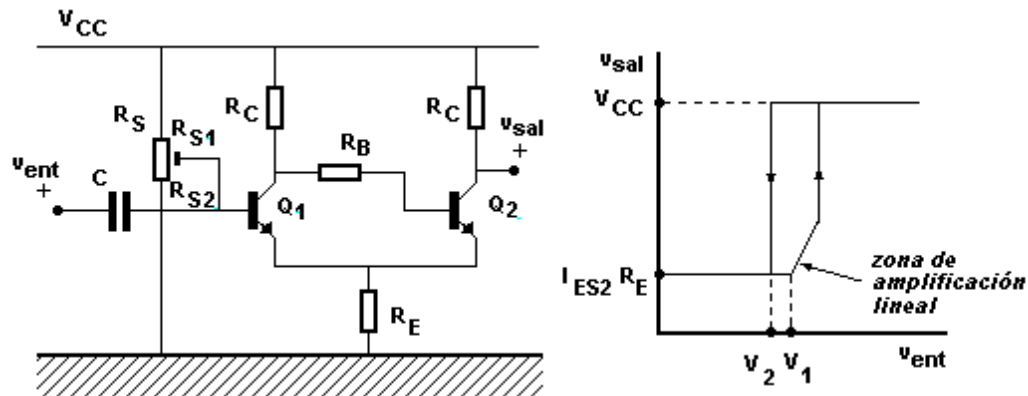
$$R_B \gg R_C$$

$$I_{ES2} \sim I_{CS2} \sim V_{CC} / (R_C + R_E)$$

$$I_{BS2} \sim (V_{CC} - 0,6 - I_{ES2} R_E) / R_B = [V_{CC} R_C - 0,6 (R_C + R_E)] / R_B (R_C + R_E)$$

$$V_{sal} = I_{ES2} R_E \sim V_{CC} / (1 + R_C / R_E)$$

$$V_1 = I_{ES2} R_E + 0,6$$



while when cutting it will be guaranteed himself the saturation of Q1. This way, if we apply Thevenin

$$k_1 = R_{S2} / (R_{S1} + R_{S2})$$

$$k_2 = R_E / (R_C + R_E)$$

$$R_{SS} = R_{S1} // R_{S2} = k_1 R_{S1}$$

$$R_{EE} = R_C // R_E = k_2 R_C$$

$$I_{BS1} = (k_1 V_{CC} - k_2 V_{CC} - 0,6) / (R_{SS} + R_{EE})$$

$$I_{CS1} = (V_{CC} - I_{BS1} R_E) / (R_C + R_E)$$

$$V_2 = I_{ES1} R_E + 0,6 = (I_{CS1} + I_{BS1}) R_E + 0,6$$

that we can simplify with the adoption

$$I_{BS1} = I_{CS1} / \beta$$

$$V_2 \sim I_{CS1} R_E + 0,6$$

## Design

Be the data

$$V_{CC} = \dots \quad R_C = \dots \quad V_1 = \dots \quad V_2 = \dots$$

We choose a TBJ anyone and of the manual we obtain

$$\beta = \dots$$

Subsequently, of the previous equations we find for the saturation of  $Q_2$

$$I_{CS2} = (V_1 - 0,6) / R_E = \dots$$

$$R_E = (V_{CC} / I_{CS2}) - R_C = \dots$$

$$I_{BS2} = \dots > I_{CS2} / \beta$$

$$R_B = [V_{CC} R_C - 0,6 (R_C + R_E)] / I_{BS2} (R_C + R_E) = \dots \gg R_C$$

and for the saturation of  $Q_1$

$$I_{ES1} = (V_2 - 0,6) / R_E = \dots$$

$$I_{BS1} = [I_{ES1} (R_C + R_E) - V_{CC}] / R_C = \dots$$

With the purpose of not increasing the equations, we adopt one pre-set that completes the conditions

$$I_{RS2} = \dots \gg I_{BS1}$$

$$R_S \sim V_{CC} / I_{RS2} = \dots$$

$$P_{RS} = V_{CC} I_{RS2} = \dots < 0,25$$

### With AOV

The circuit that continues allows an efficient Schmitt-Trigger. As we see in the following equations, the hysteresis can be designed  $\Delta V$  and its half point  $V_0$  as positive, zero or negative. If we apply Thevenin we have left to the dividing resistive  $R_1$  and  $R_2$  a reference that we denominate  $V_{REF}$  in series with the parallel resistive  $R_1 // R_2$

$$V_{REF} = V_{CC} (R_2 - R_1) / (R_2 + R_1)$$

being

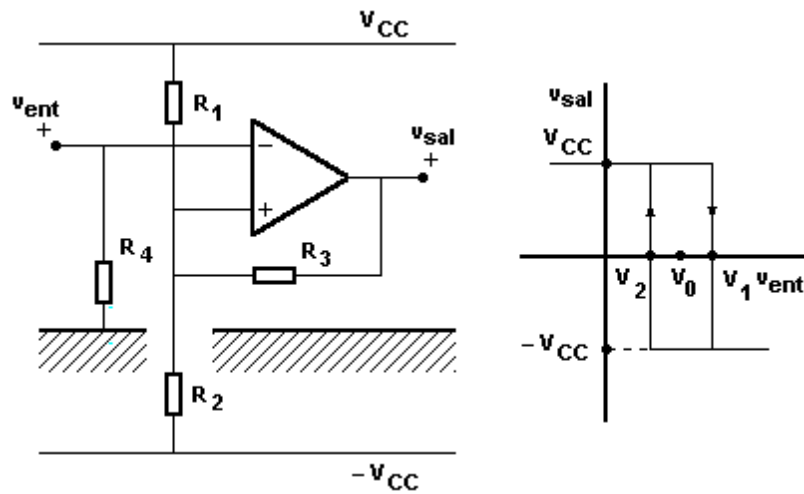
$$k = R_1 // R_2 / (R_3 + R_1 // R_2)$$

$$V_1 = k (V_{CC} - V_{REF}) + V_{REF}$$

$$V_2 = -k (V_{CC} + V_{REF}) + V_{REF}$$

$$V_0 = (V_1 + V_2) / 2 = V_{REF} (1 - k)$$

$$\Delta V = V_1 - V_2 = 2 k V_{CC}$$



## Design

Be the data

$$\pm V_{CC} = \dots \quad V_1 = \dots > = < 0 \quad V_2 = \dots > = < 0$$

We choose an AOV anyone and if for example we adopt

$$R_1 = \dots$$

$$R_4 = \dots$$

we will be able to calculate

$$k = (V_1 - V_2) / 2 V_{CC} = \dots$$

$$V_{REF} = (V_1 - k V_{CC}) / (1 - k) = \dots$$

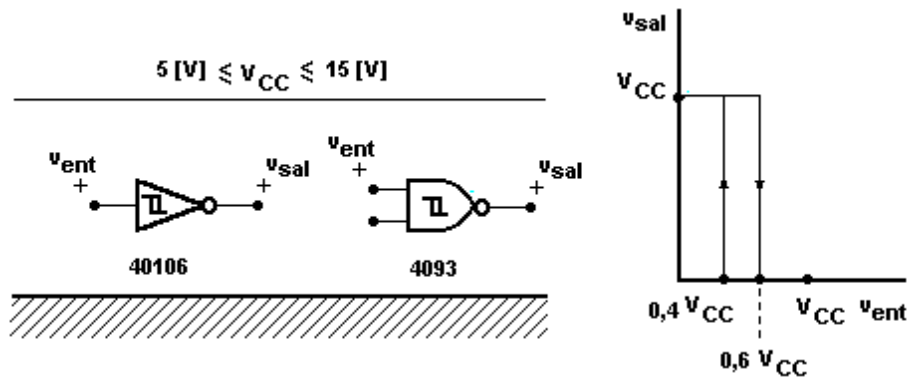
$$V_0 = V_{REF} (1 - k) = \dots$$

$$R_2 = R_1 (V_{CC} + V_{REF}) / (V_{CC} - V_{REF}) = \dots$$

$$R_3 = (R_1 // R_2) (1 - k) / k = \dots$$

## With C-MOS

These multivibrators is already designed. Clever to work with positive source among 5 to 15 [V], they present a characteristic as that of the figure



## BISTABLE

### Generalities

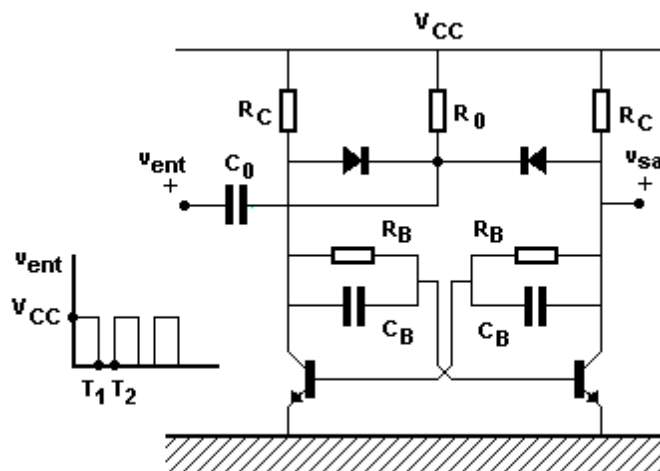
It is denominated this way to this multivibrator to have two stable states; that is to say that when it is commuted one of the dispositive the state it is retained, being able to be reverted then. It is a double Schmitt-Trigger autogenerated.

It is the fundamental circuit of all the Flip-Flop, and to be excited symmetrically it also denominates it to him as *RS* for their two entrances: one eats *reset* (to put it to zero) and another of *set* (to put it again to one). When it uses it to him with asynchronous excitement it responds to the operation of the Flip-Flop T.

It designs it to him in saturation state to each dispositive. To par excellence be a symmetrical circuit, one cannot know which active element it will go to the conduction first leaving to the other to the cut.

### With TBJ

A typical synchronous implementation (as Flip-Flop T) we see it in the following figure. The small condensers in the base resistances are necessary, I don't only with accelerators of flanks, but as the most important thing, that is: to collaborate with the transition with their loads cutting the one that was saturated.





## Design

Be the data

$$V_{CC} = \dots \quad R_C = \dots \quad T_1 = \dots \quad T_2 = \dots$$

We choose the transistors and with the fact

$$I_{CS} = V_{CC} / R_C = \dots$$

we obtain of the manual

$$\beta = \dots$$

what will determine us

$$R_B = \dots > \beta (V_{CC} - 0,6) / I_{CS}$$

So that the managing circuit of shot doesn't affect the calculations we design

$$R_0 = \dots \gg R_C$$

and we verify that the one couples it discharges their load in the hemicycle (the sign of having entered ago to commute that is to say for court of the TBJ that is saturated, with the descending flank of the entrance voltage), because if it is very big it can shoot it several times when entering the line of the discharge in the next cycle, and if it is very low it won't shoot it because the distributed capacitancis will absorb the transitory

$$C_0 = \dots < 3 (T_2 - T_1) / R_C$$

although the best thing is their experimentation, the same as those of the bases  $C_B$ .

## With C-MOS

We will study it as Flip-Flop RS.

## MONOSTABLE

### Generalities

Having a single stable state, it commutes for a period of time that we denominate T and, consequently, it is simply a circuit timer.

## With TBJ

The following implementation responds to a monostable coupled by collector. Here  $Q_1$  are cut and  $Q_2$  saturated. When it is shot and it is saturated by a moment to the first one the previous load of the base condenser  $C_B$  it will take to the cut a second until it is discharged and, in fact that time, is that of the monostable.

The behavior equations for this design are, when being in rest

$$R_B \gg R_C$$

$$I_{CS} \sim V_{CC} / R_C$$

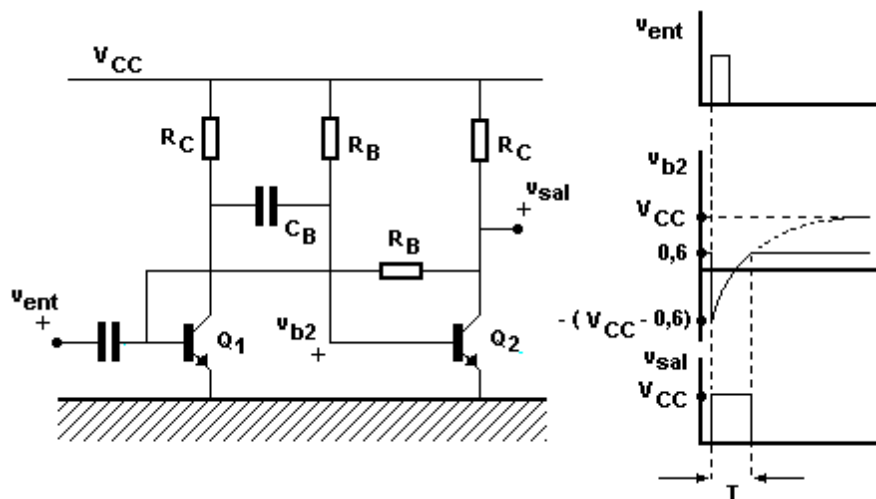
$$I_{BS} \sim (V_{CC} - 0,6) / R_B$$

and when it cuts  $Q_2$

$$v_{b2} = - (V_{CC} - 0,6) + (2V_{CC} - 0,6) (1 + e^{-t/RBCB})$$

in the transition

$$0,6 = - (V_{CC} - 0,6) + (2V_{CC} - 0,6) (1 + e^{-T/RBCB})$$



## Design

Be the data

$$V_{CC} = \dots \quad R_C = \dots \quad T = \dots$$

We choose the transistors and with the fact

$$I_{CS} = V_{CC} / R_C = \dots$$

we obtain of the manual

$$\beta = \dots$$

what will determine us

$$R_B = \dots > \beta (V_{CC} - 0,6) / I_{CS}$$

We calculate the condenser finally

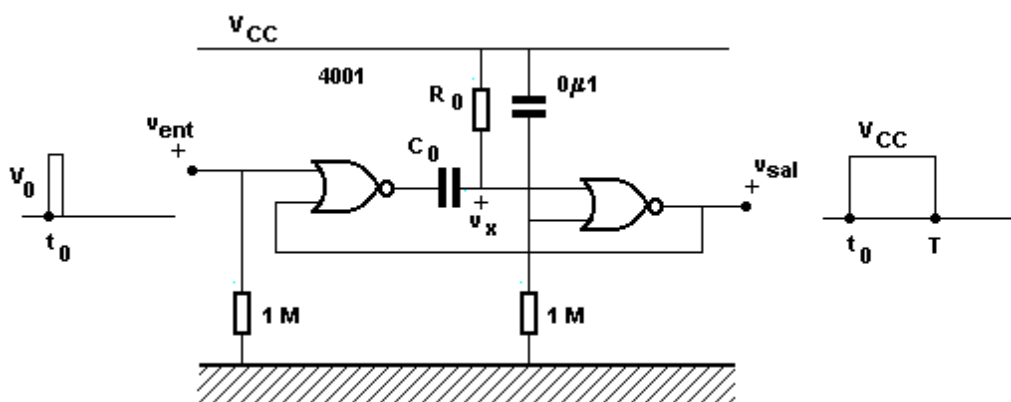
$$C_B = T / R_B \ln [(2V_{CC} - 0,6) / (V_{CC} - 0,6)] = \dots$$

### With C-MOS

There are already integrated circuits C-MOS dedicated to such an end. The following, discreet, shows a possible timer monostable just by half *chip*. It will be, logically, more precise if it implemented it to him with C-MOS of the type Schmitt-Trigger changing the NOR for NAND and the polarity of the shot. It has incorporated a previous reset for the capacitor of 100 [nF] and the resistor of 1 [M $\Omega$ ].

When shooting the circuit, that is to say when it is achieved that the exit of the NOR pre-excitatory falls to zero, the condenser will take to the cut to the second NOR maintaining this state until, "seeking" the condenser to arrive to  $V_{CC}$ , in the threshold of conduction of this second NOR this will drive and everything will return to the rest. The equation that it manifests then the period is

$$v_x = V_{CC} (1 - e^{-t/RC})$$



of where

$$0,7 V_{CC} \sim V_{CC} (1 - e^{-T/RC})$$

$$T \sim RC$$

### Design

Be

$$T = \dots < 2000 [\text{seg}] \quad V_0 = \dots$$

We simply choose the condenser (for electrolytic to use of grateful mark because the normal losses of a MegaOhm or less) and a source according to the width of the entrance pulse

$$C_0 = \dots$$

$$15 \geq V_{CC} = \dots \geq V_0 / 0,7$$

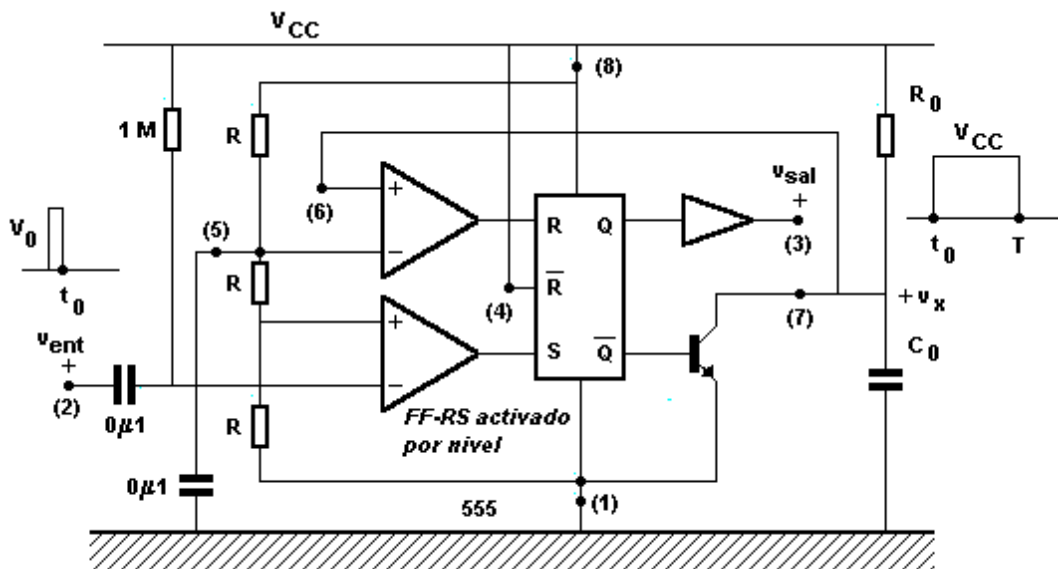
and with it (to avoid resistors above the MegaOhm if we don't want to keep in mind the losses of the condenser)

$$R_0 \sim T / C_0 = \dots < 20 [\text{M}\Omega]$$

### With the CI 555

The integrated circuit 555 possess a structure that allows, among other, the simple and efficient implementation of a multivibraor monostable. Subsequently we draw their circuit. In him the logical combinational of the Flip-Flop RS activated by level determines, before the openings of the the AOV, the load of  $C_0$ . Then it is canceled being discharged to constant current by the TBJ. Their operation equation is the following one

$$v_x = V_{CC} (1 - e^{-t/R_0 C_0})$$



$$2 V_{CC} / 3 = V_{CC} (1 - e^{-T/RC})$$

$$T \sim 1,1 R_0 C_0$$

## Design

Be

$$T = \dots < 2000 \text{ [seg]}$$

We simply choose the condenser (for electrolytic to use of grateful mark because the normal losses of a MegaOhm or less) and a source according to the width of the entrance pulse

$$C_0 = \dots$$

$$15 \geq V_{CC} = \dots \geq V_0 / 0,7$$

and with it (to avoid resisters above the MegaOhm if we don't want to keep in mind the losses of the condenser)

$$R_0 \sim 0,91 T / C_0 = \dots < 20 \text{ [M}\Omega\text{]}$$

## ASTABLE

### Generalities

Being their unstable state, it consists on an oscillator of pulses. It designs it to him with two amplifiers inverters and two nets, usually RC, that will determine a relaxation.

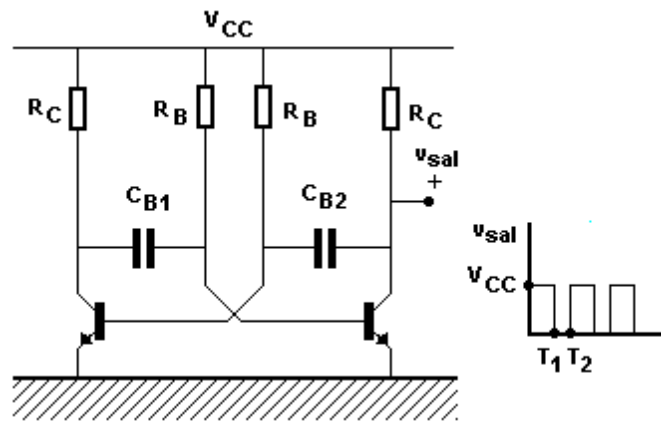
### With TBJ

The circuit shows a typical multivibrator astable coupled by collector. The graphs and operation equations are the same ones that the monostable studied previously. This way, for rest

$$R_B \gg R_C$$

$$I_{CS} \sim V_{CC} / R_C$$

$$I_{BS} \sim (V_{CC} - 0,6) / R_B$$



and when one of them is cut

$$v_b = - (V_{CC} - 0,6) + (2V_{CC} - 0,6) (1 + e^{-t/RBCB})$$

and in the transition

$$0,6 = - (V_{CC} - 0,6) + (2V_{CC} - 0,6) (1 + e^{-T/RBCB})$$

## Design

Be tha data

$$V_{CC} = \dots \quad R_C = \dots \quad T_1 = \dots \quad T_2 = \dots$$

We choose the transistors and with the fact

$$I_{CS} = V_{CC} / R_C = \dots$$

we obtain of the manual

$$\beta = \dots$$

what will determine us

$$R_B = \dots > \beta (V_{CC} - 0,6) / I_{CS}$$

We calculate the condensers finally

$$C_{B1} = T_1 / R_B \ln [(2V_{CC} - 0,6) / (V_{CC} - 0,6)] = \dots$$

$$C_{B2} = (T_2 - T_1) / R_B \ln [(2V_{CC} - 0,6) / (V_{CC} - 0,6)] = \dots$$

## With AOV

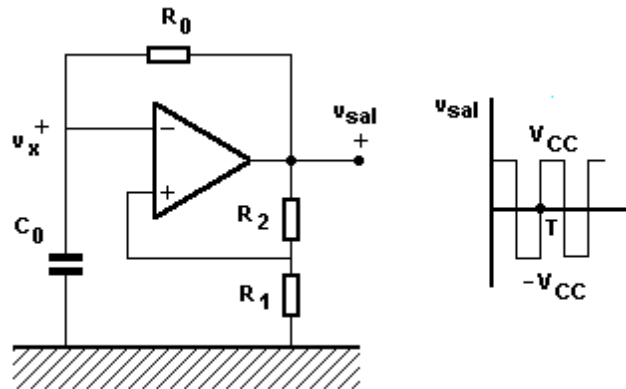
A simple implementation is that of the figure. It consists on a Schmitt-Trigger with reference null  $V_{REF}$ . The capacitor is loaded to the voltage of exit of the AOV, that is to say  $\pm V_{CC}$ , but it commutes when arriving respectively at  $V_1$  or  $V_2$ . Their fundamental equation is the load and discharge of the condenser

$$v_x = -V_2 + (V_{CC} + V_2) (1 + e^{-t/R_0 C_0})$$

of where

$$V_1 = -V_2 + (V_{CC} + V_2) [1 + e^{-(T/2)/R_0 C_0}]$$

$$T = 2R_0 C_0 \ln [(V_{CC} + V_2) / (V_{CC} - V_1)] = 2R_0 C_0 \ln [(1+k) / (1-k)]$$



## Design

Be the data

$$\pm V_{CC} = \dots \quad T = \dots$$

We choose an AOV anyone and we adopt

$$C_0 = \dots$$

$$R_1 = \dots$$

$$k = \dots < 1 \quad (it \text{ is suggested } 0,1)$$

and we calculate

$$R_2 = R_1 (1 - k) / k = \dots$$

$$R_0 = T / 2C_0 \ln [(1+k) / (1-k)] = \dots$$

## With C-MOS

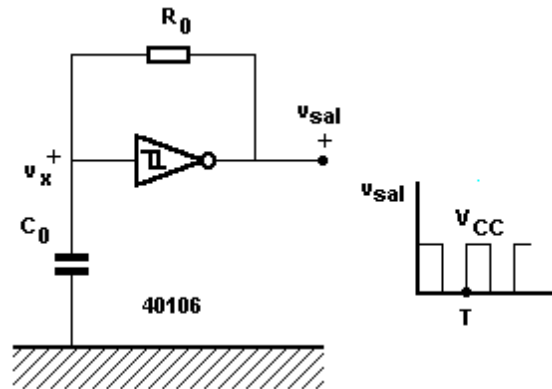
The same as the circuit with AOV, takes advantage a gate Schmitt-Trigger. Their equation is

$$v_x \sim 0,6V_{CC} + (1 + e^{-t/R_0C_0})$$

of where

$$0,2V_{CC} \sim 0,6V_{CC} + (1 + e^{-T/2R_0C_0})$$

$$T \sim 0,4 R_0C_0$$



## Design

Be the data

$T = \dots$

We choose a 40106 or 4093 uniting the two entrances and we adopt

$C_0 = \dots$

what will allow us to calculate

$R_0 = 2,5 T / C_0 = \dots$

## OCV and FCV with the CI 555

Returning to the integrated circuit 555, this allows us to control their work frequency with the configuration astable; in a similar way we can make with the width of their pulses.

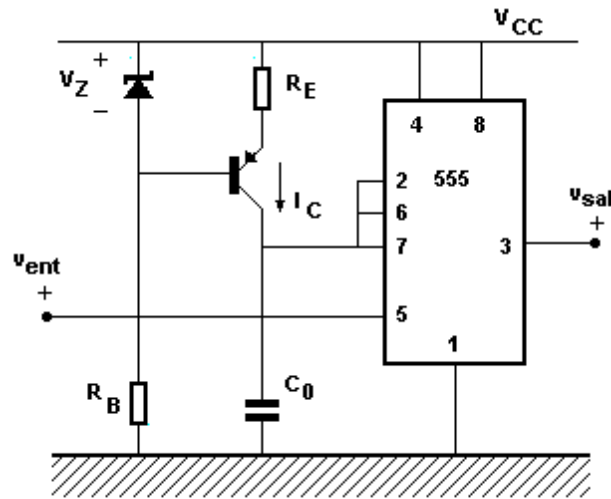
## Modulator of frequency (OCV)

The following circuit represents a possible implementation like Controlled Oscillator for Tension (OCV) continuous variable. The diode Zener feeds the TBJ producing a constant  $I_C$  and with it a ramp in  $C_0$  of voltage that it will be discharged generating the cycle to rhythm of the continuous



$v_{ent}$  quickly. Their period is given for

$$T = [ R_E C_0 / 2 (V_Z - 0,6) ] v_{ent}$$

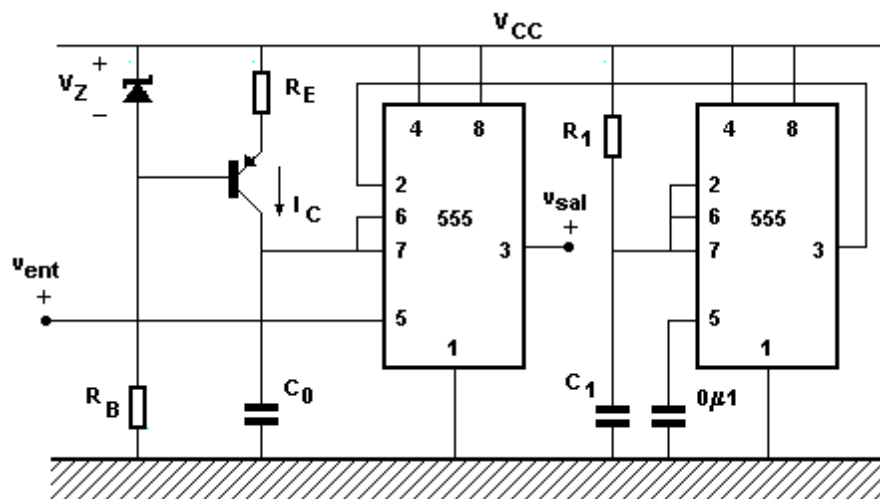


### Modulator of the width of the pulse (PCV)

The following circuit represents a possible implementation like Controlled Phase for Voltage (FCV) continuous variable. In a similar way that the previous circuit, the 555 facilitate this operation. If we call  $\Delta T$  to the width of the pulse in the period  $T$ , they are

$$T = 0,7 R_1 C_2$$

$$\Delta T = [ R_E C_0 / 2 (V_Z - 0,6) ] v_{ent}$$



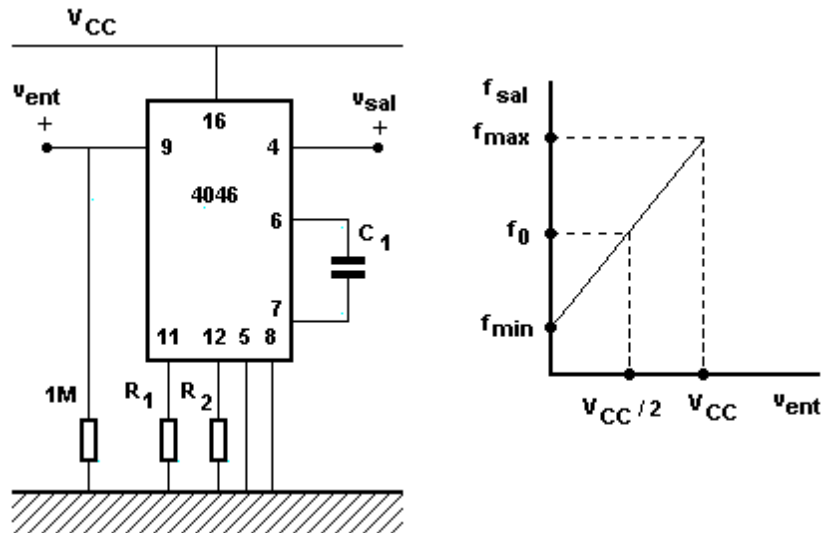
### OCV with the 4046

As all C-MOS, their alimentation will be understood between 5 and 15 [V] for its correct

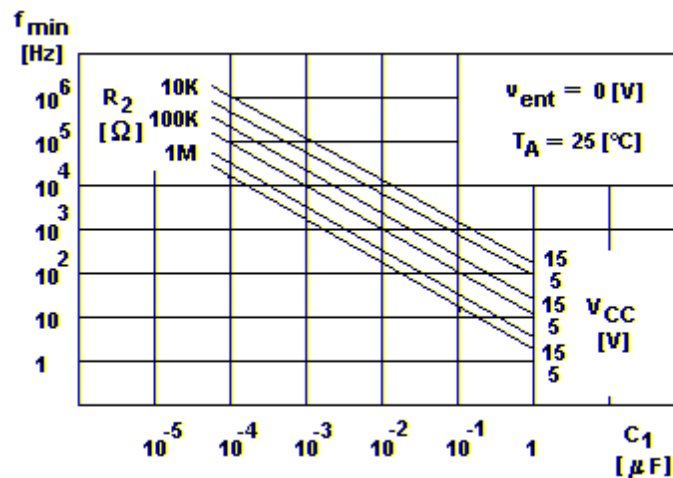
operation. This integrated circuit possesses other properties more than as OCV, but due to its low cost, versatility and efficiency has used it to him in this application. Their basic equation is a straight line

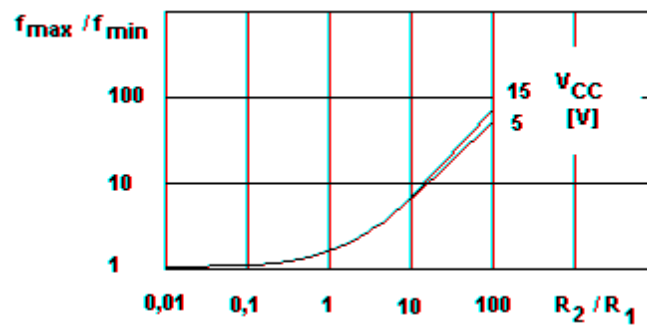
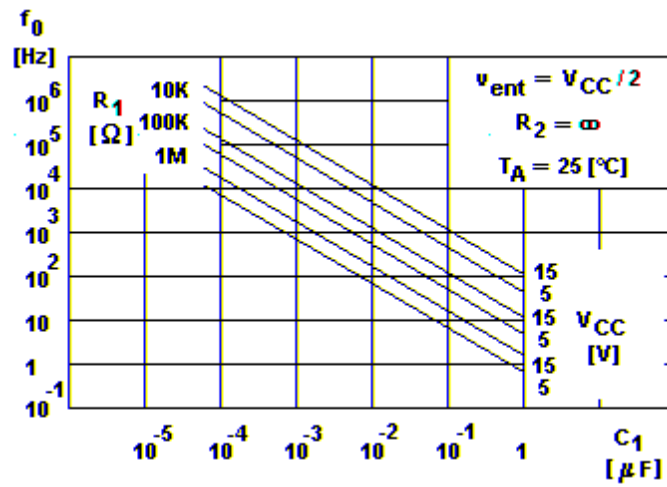
$$f_{sal} = f_{min} + 2 (f_0 - f_{min}) v_{ent} / V_{CC}$$

$$f_0 = (f_{max} + f_{min}) / 2$$



For their design the typical curves are attached that the maker offers. A first of *rest* (zero) in  $f_{min}$ , another of *gain* at  $f_0$  and a third of *polarization*





## Design

Be the data

$$f_{max} = \dots \quad f_{min} = \dots$$

We choose a polarization with the abacus of *rest*

$$V_{CC} = \dots$$

$$R_2 = \dots$$

$$C_1 = \dots$$

and then with the third

$$f_{max} / f_{min} = \dots$$

$$(R_2 / R_1) = \dots$$

$$R_1 = R_2 / (R_2 / R_1) = \dots$$

## Cap. 19    Combinationals and Sequentials

### GENERALITIES

#### FLIP-FLOP

##### Generalities

##### Activated Flip-Flop for Level

##### Flip-Flop RS

##### Flip-Flop JK

##### Flip-Flop T

##### Flip-Flop D

##### Flip-Flop Master-Slave

##### Accessories of the Flip-Flop

### COUNTERS OF PULSES

#### Generalities

#### Example of Design

### DIVIDERS OF FREQUENCY

#### Generalities

#### Asynchronous

#### Synchronous

#### Example of Design

### MULTIPLIERS OF FREQUENCY

#### Generalities

#### Example of Design

### DIGITAL COMPARATORS

### REGISTER OF DISPLACEMENTS

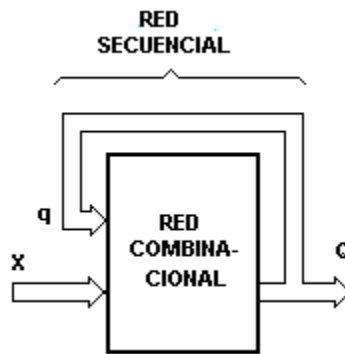
### MULTIPLEXER AND DI-MULTIPLEXER

#### Design of Combinationals Nets with Multiplexer

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### GENERALITIES

A net combinational is that that "combines" gates AND, OR, Negators and of the 3<sup>o</sup> State. A sequential one is this same one but with feedback. In the exits we will prefer to call to the states previous with small letter (q) to differentiate them of the present ones that it will be made with a capital (Q), and those of the entrance with a capital because being present, neither they changed during the transition ( $x = X$ ).



## FLIP-FLOP

### Generalities

Being the Flip-Flop the basic units of all the sequential systems, four types exist: the *RS*, the *JK*, the *T* and the *D*. AND the last ones three are implemented then of the first —we can with anyone of the results to project the remaining ones.

All they can be of two types, that is: *Flip-Flop activated by level* (FF-AN) or *Flip-Flop master-slave* (FF-ME). The first one receives their name to only act with the "levels" of amplitudes 0-1, on the other hand the second it consists on two combined FF-AN in such a way that one "controls" to the other.

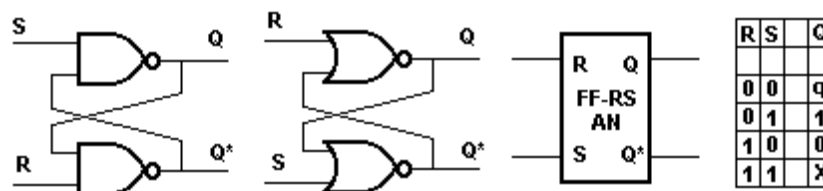
### Activated Flip-Flop for Level

#### Flip-Flop RS

Their basic unit (with gates NAND or NOR) it is drawn next and, like it acts for "levels" of amplitudes (0-1), it receives name *Flip-Flop RS activated by level* (FF-RS-AN). When this detail is not specified it is of the type *Flip-Flop RS master-slave* (FF-RS-ME). Their equations and operation table are

$$Q = S + q R^*$$

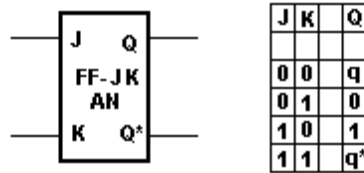
$$R S = 0$$



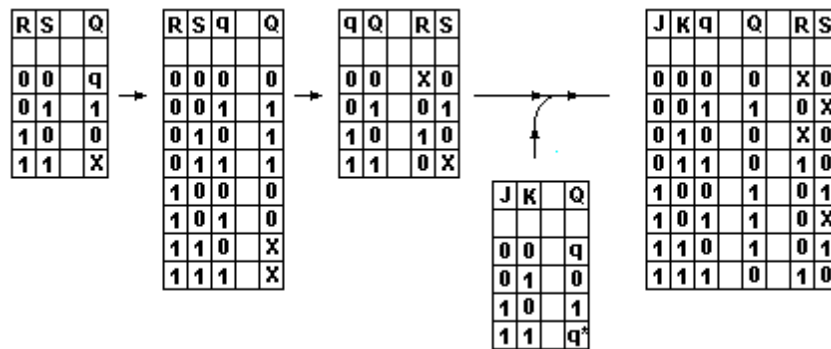
#### Flip-Flop JK

Their basic unit it is drawn next and, like it acts for "levels" of amplitudes (0-1), it receives name *Flip-Flop JK activated by level* (FF-JK-AN). When this detail is not specified it is of the type *Flip-Flop JK master-slave* (FF-JK-ME). Their equations and operation table are

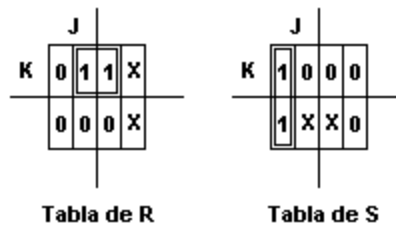
$$Q = J q^* + K^* q$$



Detail of its logical making is given starting from the FF-RS-AN.



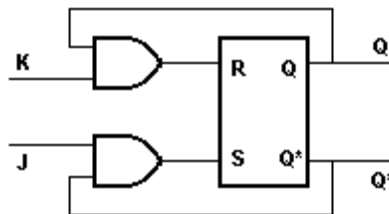
and if we simplify using Veich-Karnaugh for example



$$R = K q$$

$$S = J q^*$$

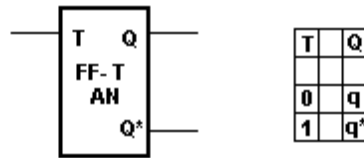
it is the circuit



## Flip-Flop T

Their basic unit it is drawn next and, like it acts for "levels" of amplitudes (0-1), it receives name *Flip-Flop T activated by level* (FF-T-AN). When this detail is not specified it is of the type *Flip-Flop T master-slave* (FF-T-ME). Their equations and operation table are

$$Q = T \oplus q$$

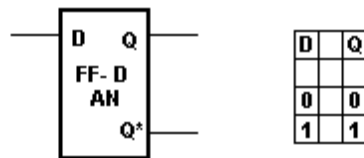


Starting from the FF-RS-AN this FF-T-AN can be designed following the steps shown previously, but it doesn't have coherence since when being activated by level it doesn't have utility.

## Flip-Flop D

Their basic unit it is drawn next and, like it acts for "levels" of amplitudes (0-1), it receives name *Flip-Flop D activated by level* (FF-D-AN). When this detail is not specified it is of the type *Flip-Flop D master-slave* (FF-D-ME) commonly also denominated *Latch*. Their equations and operation table are

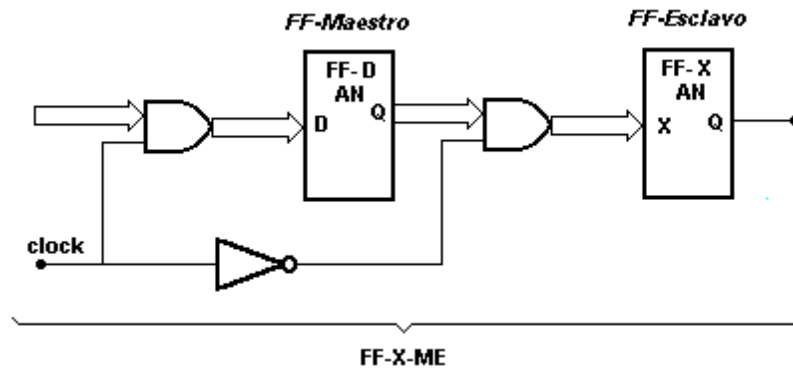
$$Q = D$$



Starting from the FF-RS-AN this FF-D-AN can be designed following the steps shown previously, but it doesn't have coherence since when being activated by level it doesn't have utility.

## Flip-Flop Master-Slave

All the four FF-AN can be implemented following the orders from a FF-D-AN to their entrance as sample the drawing. The FF-D makes of latch. Each pulse in the clock will make that the sign enters to the system (as exit of the FF-D-AN) and leave at the end following the FF slave's table of truth. This way, if the slave is a FF-X-AN, the whole group behaves as a FF-X-ME —here X it can be a FF or a complex sequential system.



### Accessories of the Flip-Flop

The Flip-Flop, usually and if another detail is not specified, they are always Master-Slave, and they usually bring other terminal like accessories. We name the following ones:

- Reset      *puts to 0 to Q*
- Set        *puts at 1 to Q*
- Clock
- Inhibition    *inhibits (it doesn't allow to happen) the sign entrance*

### COUNTERS OF PULSES

#### Generalities

They are systems of FF in cascade and related with combinationals nets in such a way that count, with a binary code anyone predetermined (binary pure, BCD, Jhonson, etc., or another invented by oneself) the pulses that enter to the clock of the system. This way, if all the clocks are connected in parallel or not, the accountants are denominated, respectively

- synchronous
- asynchronous

and we will study to the first ones.

The quantity «M» of pulses to count (including the corresponding rest) it is related with the number «n» of FF to use by means of the ecuation

$$2^{n-1} < M \leq 2^n$$

#### Example of Design

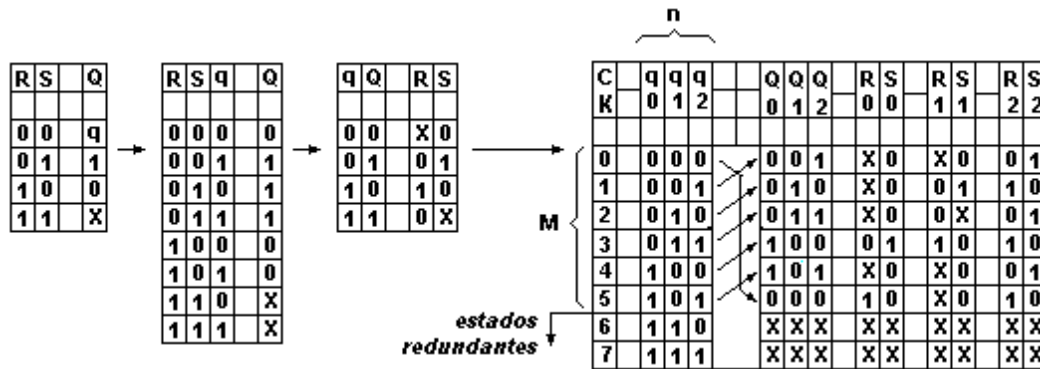
We want to count the pulses of a code, for example the binary one natural until the number 5; that is to say that starting from the pulse 6 the count will be restarted (auto-reset). Indeed, we can choose the minimum quantity of FF to use (and that therefore they will be used)



$$M = 6$$

$$2^{n-1} < M \leq 2^n \Rightarrow n = 3$$

We adopt the type of FF that we have, subsequently for example the RS.  
Now we complete the design tables



We simplify the results, for example for Veich-Karnaugh

$$R_0 = q_1 \cdot q_2$$

$$S_0 = q_1 \cdot q_2$$

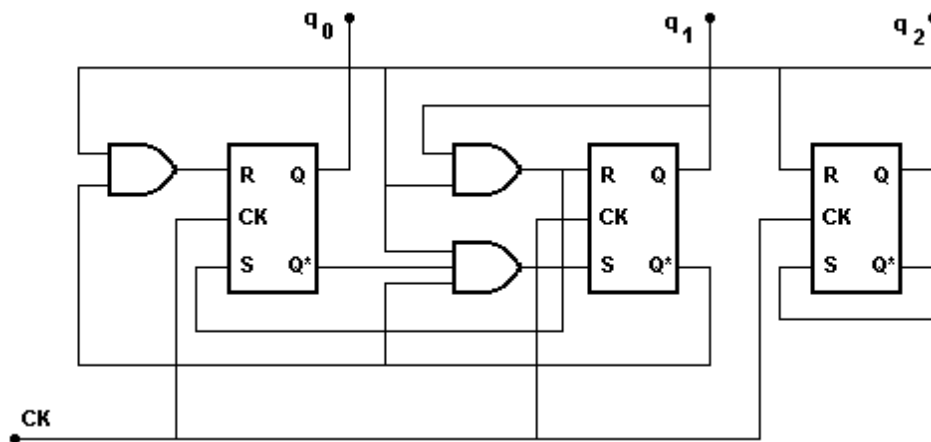
$$R_1 = q_1 \cdot q_2$$

$$S_1 = q_0 \cdot q_1 \cdot q_2$$

$$R_2 = q_2$$

$$S_2 = q_2^*$$

and we arm finally with her the circuit



## DIVIDERS OF FREQUENCY

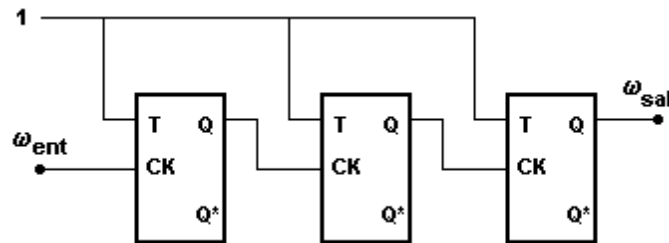
### Generalities

They can be made with asynchronous or synchronous counters.

## Asynchronous

Subsequently we see an asynchronous divider of frequency manufactured with a FF-T (remembers you that a FF can be manufactured starting from any other FF) that possess the property of taking out a pulse for each two of entrance. For it the last division is

$$\omega_{sal} = \omega_{ent} 2^n$$



## Synchronous

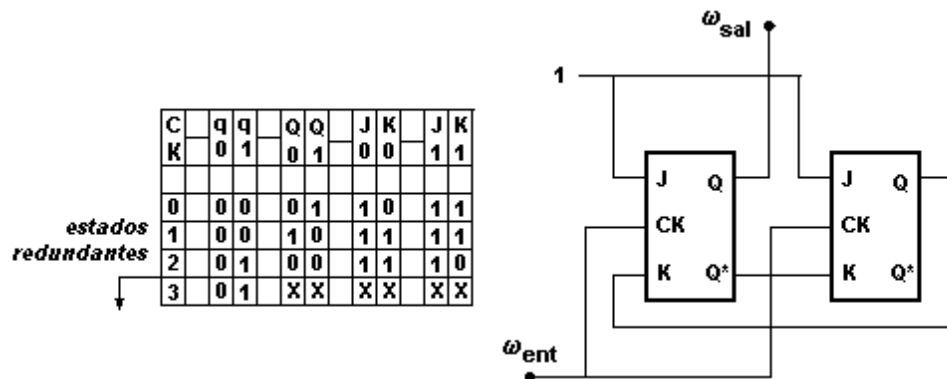
### Example of Design

Now then, let us suppose that we don't want to divide for a number  $2^n$  but for any other. For we use it the synchronous counter. When the quantity of pulses arrives to the quantity M, it will be designed the last FF in such a way that changes the state detecting this way with this the division. Following the project steps as newly it has been exposed when designing an counter anyone synchronous, we can achieve our objective.

Let us suppose that our fact is to divide for 3. We adopt, for example a FF-JK and then, with the previous approach, we design it in the following way

$$M = 3$$

$$2^{n-1} < M \leq 2^n \Rightarrow n = 2$$

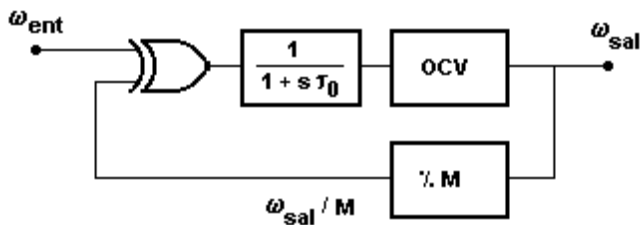


## MULTIPLIERS OF FREQUENCY

### Generalities

They can be made with a Phase Lock Loop (LFF) and a divider for M that is in the feedback —M is the accountant's pulses like it was seen precedently. Being *hooked* and *maintained* the LFF, the internal OCV will maintain the  $\omega_{ent}$  multiplied by M. This way then , the output frequency will be a multiple M of that of the entrance

$$\omega_{sal} = \omega_{OCV} = M \omega_{ent}$$



### Example of Design

Let us suppose that one has an entrance frequency that varies between a maximum  $f_{entmax}$  and a minimum  $f_{entmin}$ , and it wants it to him to multiply M times

$$f_{entmax} = \dots$$

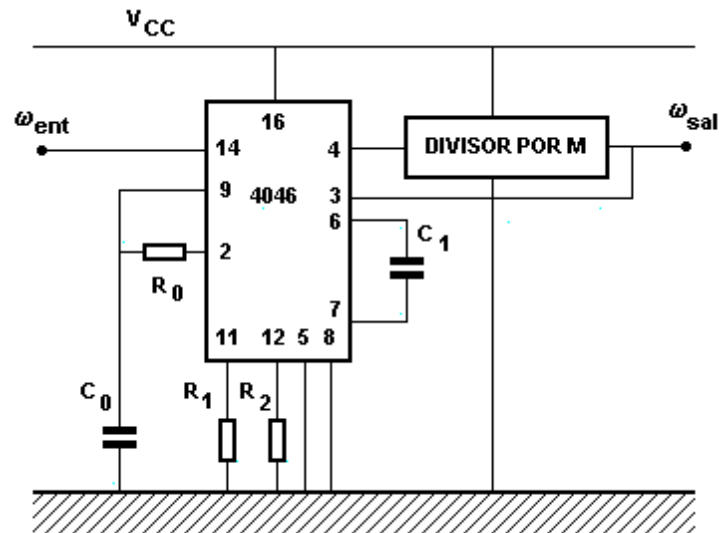
$$f_{entmin} = \dots$$

$$M = \dots$$

The circuit following sample a possible implementation. To design the OCV it should be appealed to the *multivibrators* chapter with the data

$$f_{max} = \dots > f_{entmax}$$

$$f_{min} = \dots < f_{entmin}$$



The net  $R_0C_0$  of the filter is suggested that it is experimental, although it can be considered its constant of time in such a way that filters the detected pulses

$$\tau_0 = R_0C_0 = \dots \gg 2 T_{\text{entmax}} = 4 / f_{\text{entmin}}$$

The *maintenance range* RM will be

$$\text{RM [Hz]} = M (f_{\text{max}} - f_{\text{min}}) = \dots > f_{\text{entmax}} - f_{\text{entmin}}$$

## DIGITAL COMPARATORS

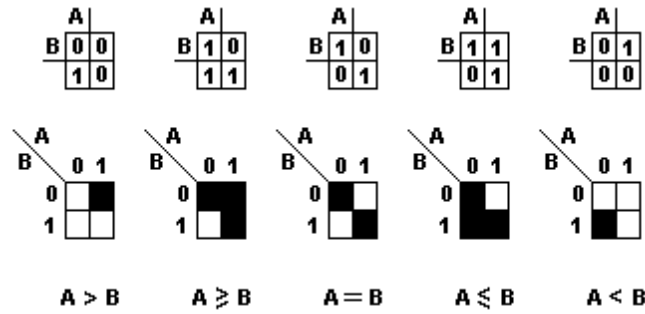
Two digital words (bytes) will be compared A and B of «m» bits each one of them according to the classification

$$A = A_m \dots A_1 A_0$$

$$B = B_m \dots B_1 B_0$$

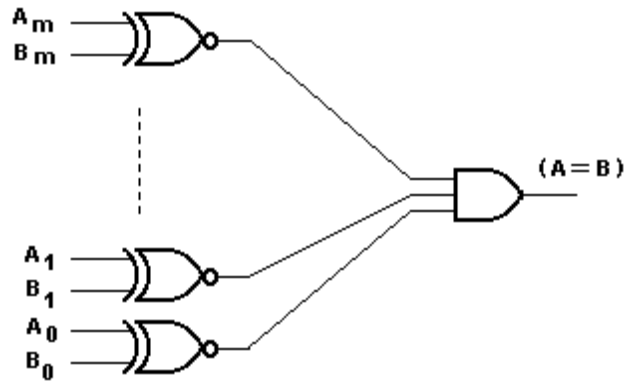
with «m» the bit of more weight

$A > B$	$\rightarrow$	$A B^*$
$A \geq B$	$\rightarrow$	$A + B^*$
$A = B$	$\rightarrow$	$(A \oplus B)^*$
$A \leq B$	$\rightarrow$	$A^* + B$
$A < B$	$\rightarrow$	$A^* B$



Indeed, to determine the case of *equality* it will be enough to compare each one of the bits respectively with gates OR-Exclusive

$$(A = B) = (A_m \oplus B_m)^* \dots (A_1 \oplus B_1)^* (A_0 \oplus B_0)^*$$



To explain the detection of the *difference* in excess or deficit we will use an example. Be  $m = 2$  and being  $A > B$ ; then just by that the bit of more weight is it it will be enough

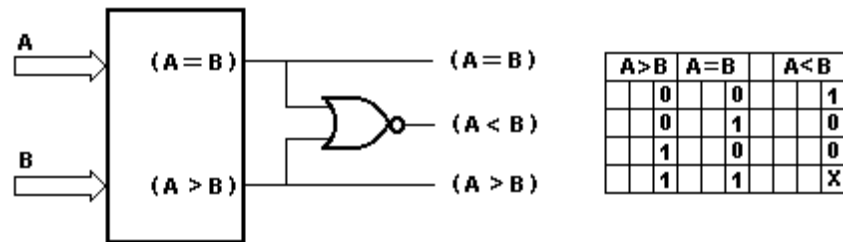
$$A_2 > B_2$$

or

$$\begin{aligned} A_2 = B_2 \quad \text{y} \quad A_1 > B_1 \\ A_2 = B_2 \quad \text{y} \quad A_1 = B_1 \quad \text{y} \quad A_0 > B_0 \end{aligned}$$

what will allow to arm the net following

$$\begin{aligned} (A > B) &= (A_2 > B_2) + (A_2 = B_2) [(A_1 > B_1) + (A_1 = B_1) (A_0 > B_0)] \rightarrow \\ &\rightarrow A_2 B_2^* + (A_2 \oplus B_2)^* [A_1 B_1^* + (A_1 \oplus B_1)^* + A_0 B_0^*] \end{aligned}$$

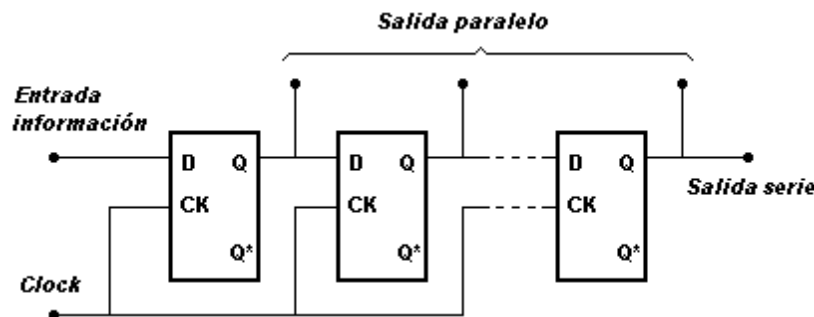


and of the table

$$(A < B) = (A > B)^* (A = B)^* = [(A > B) + (A = B)]^*$$

## REGISTER OF DISPLACEMENTS

They are chains of FF-D in cascade fed synchronously, in such a way that for each pulse in clock the digital information goes moving of FF in FF without suffering alteration—to remember that the table of truth of the FF-D it allows it. Their output can be series or parallel.

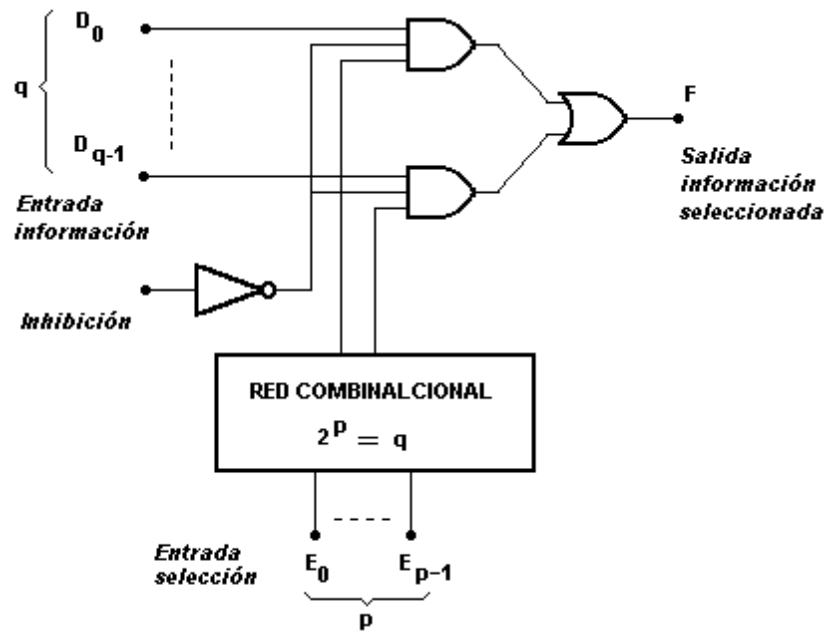


## MULTIPLEXER AND DI-MULTIPLEXER

It consists on a digital key and, for this, it can be to *select* (multiplexer) or reverse (di-multiplexer).

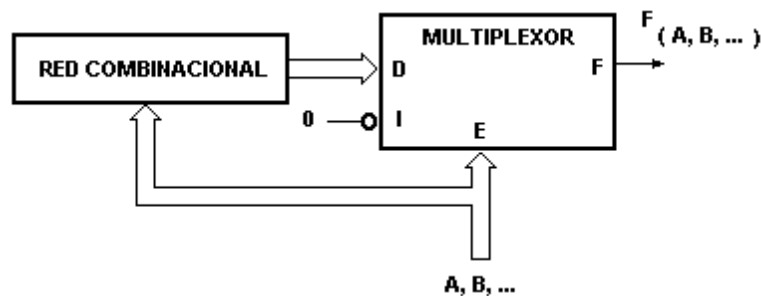
Their diagram like multiplexer offers in the drawing, where we have called with «q» to the *number of channels* and «p» to the *number of selection entrances*—combinations that will select them. It will be completed then that

$$2^p = q$$



### Design of Combinational Nets with Multiplexer

It is useful the design this way and not in discrete form because many gates and complications are saved in the circuit. But clearly will be that same they are inside the sophistication integrated by the maker the multiplexer.

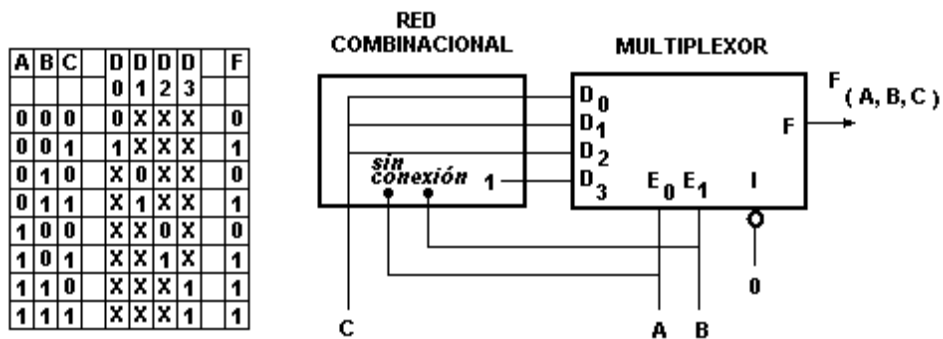


Let us suppose like fact to have a function anyone  $F_{(A,B,C)}$  (chosen at random) like sample the following table that we will design.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Firstly we choose a multiplexer of the biggest quantity in possible channels because this will

diminish the additional gates. Let us suppose that we have obtained one of 2 selections ( $p = 2$ ) that will be enough for this example. Subsequently we arm the table like it continues and then we simplify their result for Veich-Karnaugh.





## Chap. 20 Passive networks as adapters of impedance

### Generalities

#### Parameters of impedance and parameters of propagation

#### Characteristic impedance and iterative impedance

#### Adaptation of impedances

#### Function of the propagation

### Generalities

#### Symmetrical and disadapted network

#### Asymmetrical and adapted network

#### Adapting network of impedances, disphased and attenuator

#### Design attenuator

#### Design disphasator

#### Design attenuator and disphasator

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### Generalities

Let us have present in this whole chapter that, although the theoretical developments and their designs are for a single work frequency, it will also be able to approximately to become extensive to an entire spectrum if one works in short band; that is to say, if it is since the relationship among the half frequency divided by the band width is much bigger that the unit. On the other hand, the inductances and capacitances calculated in the designs presuppose not to be inductors and capacitors, that which will mean that, for the work frequencies their factors of merit reactivate  $Q_{ef}$  they are always much bigger that the unit.

#### Parameters of impedance and parameters of propagation

It is defined the parameters of *impedance*  $Z$  from a network to the following system of equations

$$\begin{aligned}V_{ent} &= i_{ent} Z_{11} + i_{sal} Z_{12} \\V_{sal} &= i_{ent} Z_{21} + i_{sal} Z_{22}\end{aligned}$$

those of *admittance*  $Y$

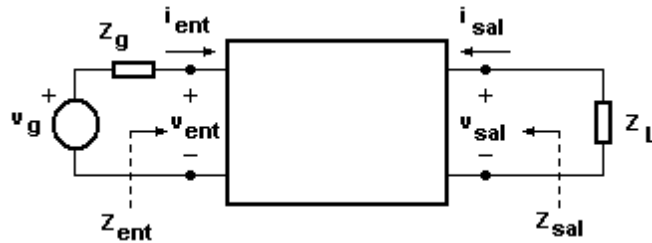
$$i_{ent} = V_{ent} Y_{11} + V_{sal} Y_{12}$$

$$i_{sal} = v_{ent} Y_{21} + v_{sal} Y_{22}$$

and those of *propagation* (or *transmission*  $\gamma$ )

$$v_{ent} = v_{sal} \gamma_{11} - i_{sal} \gamma_{12}$$

$$i_{ent} = v_{sal} \gamma_{21} - i_{sal} \gamma_{22}$$



that if we interpret to the same one as configuration T

$$Z_{11} = Z_1 + Z_3$$

$$Z_{12} = Z_3$$

$$Z_{21} = Z_3$$

$$Z_{22} = Z_2 + Z_3$$

$$Y_{11} = (Z_2 + Z_3) / (Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)$$

$$Y_{12} = -Z_3 / (Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)$$

$$Y_{21} = -Z_3 / (Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)$$

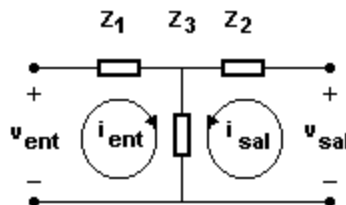
$$Y_{22} = (Z_1 + Z_3) / (Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)$$

$$\gamma_{11} = (Z_1 + Z_3) / Z_3 = Z_{11} / Z_{21}$$

$$\gamma_{12} = (Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3) / Z_3 = (Z_{11} Z_{22} - Z_{12} Z_{21}) / Z_{21}$$

$$\gamma_{21} = 1 / Z_3 = 1 / Z_{21}$$

$$\gamma_{22} = (Z_2 + Z_3) / Z_3 = Z_{22} / Z_{21}$$



and where one has the property

$$[\gamma] = -\gamma_{11}\gamma_{22} + \gamma_{12}\gamma_{21} = -1$$

## Characteristic impedance and iterative impedance

Of the previous network we obtain

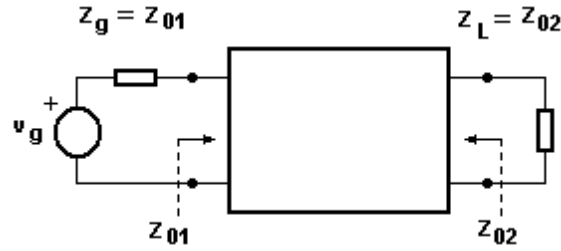
$$Z_{ent} = v_{ent} / i_{ent} = (v_{sal}\gamma_{11} - i_{sal}\gamma_{12}) / (v_{sal}\gamma_{21} - i_{sal}\gamma_{22}) = (Z_L\gamma_{11} + \gamma_{12}) / (Z_L\gamma_{21} + \gamma_{22})$$

$$Z_{sal} = v_{sal} / i_{sal} = (Z_g\gamma_{22} + \gamma_{12}) / (Z_g\gamma_{21} + \gamma_{11})$$

and we define *characteristic impedances of input*  $Z_{01}$  and *output*  $Z_{02}$  to the network to the following

$$Z_{01} = (Z_{02}\gamma_{11} + \gamma_{12}) / (Z_{02}\gamma_{21} + \gamma_{22})$$

$$Z_{02} = (Z_{01}\gamma_{22} + \gamma_{12}) / (Z_{01}\gamma_{21} + \gamma_{11})$$



that working them with the previous parameters is

$$Z_{01} = (\gamma_{11}\gamma_{12} + \gamma_{21}\gamma_{22})^{1/2} = (Z_{11} / Y_{11})^{1/2} = (Z_{entcc} Z_{entca})^{1/2}$$

$$Z_{02} = (\gamma_{22}\gamma_{12} + \gamma_{21}\gamma_{11})^{1/2} = (Z_{22} / Y_{22})^{1/2} = (Z_{salcc} Z_{salca})^{1/2}$$

where

$$Z_{entcc} = Z_{ent} \text{ con } Z_L = 0 + j 0$$

$$Z_{entca} = Z_{ent} \text{ con } Z_L = \infty + j 0$$

$$Z_{salcc} = Z_{sal} \text{ con } Z_g = 0 + j 0$$

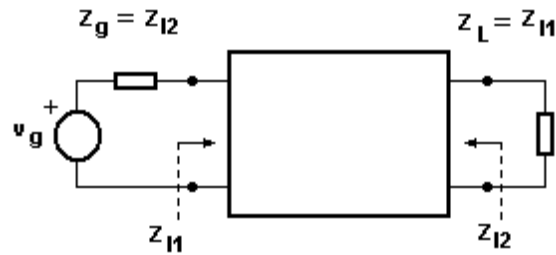
$$Z_{salca} = Z_{sal} \text{ con } Z_g = \infty + j 0$$

In summary, if we have a symmetrical network ( $Z_0 = Z_{01} = Z_{02}$ ), like it can be a transmission line, we call *characteristic impedance* from this line to that impedance that, making it physics in their other end, it determines that the wave that travels for her always finds the same magnitude resistive as if it was infinite —without reflection. The equations show that we can find it if we measure the impedance to their entrance, making short circuit and opening their terminals of the other side.

When the configuration works in disaptation, we define *impedances iteratives of input*  $Z_{I1}$  and *output*  $Z_{I2}$  from the network to the following

$$Z_{I1} = (Z_{I1}\gamma_{11} + \gamma_{12}) / (Z_{I1}\gamma_{21} + \gamma_{22})$$

$$Z_{I2} = (Z_{I2}\gamma_{22} + \gamma_{12}) / (Z_{I2}\gamma_{21} + \gamma_{11})$$



that they become in

$$Z_{I1} = [(\gamma_{22} - \gamma_{11}) / 2\gamma_{21}] \{ 1 \pm [1 + [4\gamma_{12}\gamma_{21} / (\gamma_{22} - \gamma_{11})^2]]^{1/2} \}$$

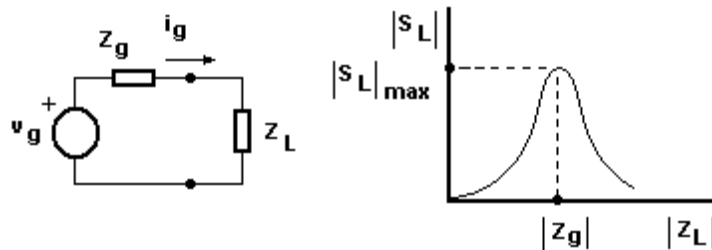
$$Z_{I2} = [(\gamma_{11} - \gamma_{22}) / 2\gamma_{21}] \{ 1 \pm [1 + [4\gamma_{12}\gamma_{21} / (\gamma_{11} - \gamma_{22})^2]]^{1/2} \}$$

### Adaptation of impedances

Remembering that in our nomenclature we call with S to the apparent power, P to the active one and W to it reactivates, we can find the maximum energy transfer for the following application

$$S_L = i_L^2 Z_L = v_g^2 Z_L / (Z_L + Z_g)^2$$

$$\partial |S_L| / \partial |Z_L| = |v_g|^2 [1 - 2|Z_L| / |Z_L + Z_g|] / |Z_L + Z_g|^2$$



expression that when being equaled to zero to obtain their *maximum*, it is the condition of more transfer of apparent power in

$$|Z_L| = |Z_g|$$

and for the active power

$$Z_L = Z_g^*$$

that is to say that will be made resonate the part it reactivates of the impedance eliminating it.

### Function of the propagation

#### Generalities

If the apparent power that surrenders to the entrance of the network gets lost something inside the same one, we will say that

$$S_{ent} = v_{ent} i_{ent} \neq S_{sal} = v_{sal} i_{sal}$$

and we will be able to define an energy efficiency that we define as *function of the propagation  $\gamma$*  in the network

$$\begin{aligned} e^{\gamma} &= (S_{ent} / S_{sal})^{1/2} = |(S_{ent} / S_{sal})^{1/2}| e^{j\beta} = \\ &= [(v_{ent}^2 / Z_{ent}) / (v_{sal}^2 / Z_L)]^{1/2} = (v_{ent} / v_{sal}) (Z_L / Z_{ent})^{1/2} \\ \gamma &= \gamma(\omega) = \alpha(\omega) [\text{Neper}] + j \beta(\omega) [\text{rad}] \end{aligned}$$

with

$$1 [\text{Neper}] \sim 8,686 [\text{dB}]$$

calling finally

$$\begin{aligned} \gamma & \quad \text{propagation function} \\ \alpha & \quad \text{attenuation function (apparent energy loss)} \\ \beta & \quad \text{phase function (displacement of phase of the input voltage)} \end{aligned}$$

If the network is adapted the equations they are

$$e^{\gamma} = (v_{ent} / v_{sal}) (Z_{02} / Z_{01})^{1/2} = (v_{ent} / v_{sal}) (Z_L / Z_g)^{1/2}$$

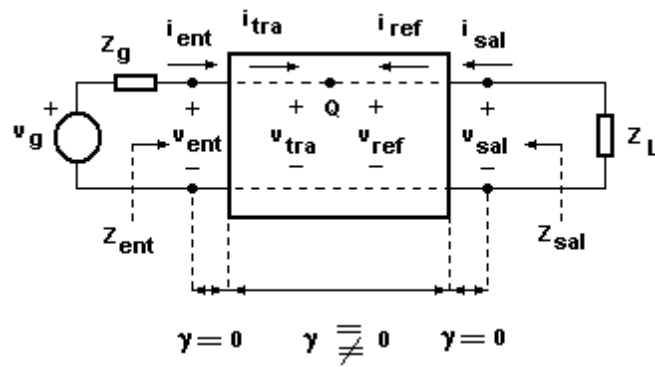
## Symmetrical and disadapted network

Let us suppose a symmetrical and disadapted network now

$$\begin{aligned} Z_0 &= Z_{01} = Z_{02} & \text{symmetry} \\ Z_0 &= Z_L \neq Z_g & \text{disadaptation to the output} \end{aligned}$$

and let us indicate in the drawing electric fields (proportional to voltages) that travel: one transmitted ( $v_{tra}$ ) and another reflected ( $v_{ref}$ ). In each point of the physical space of the network, here represented by Q, these waves generate an incident ( $v_{INC}$ ) and then salient ( $v_{SAL}$ ) of the point. This way then

$$\begin{aligned} v_{traSAL} &= v_{traINC} e^{-\gamma} \\ v_{refSAL} &= v_{refINC} e^{-\gamma} \end{aligned}$$



and for Kirchoff

$$\begin{aligned} i_{\text{traSAL}} &= i_{\text{traINC}} \\ i_{\text{refSAL}} &= i_{\text{refINC}} \end{aligned}$$

finding in this point Q at  $Z_0$  both waves

$$Z_0 = v_{\text{traINC}} / i_{\text{traINC}} = v_{\text{refINC}} / i_{\text{refINC}}$$

determining with it to the entrance of the network

$$Z_{\text{ent}} = v_{\text{ent}} / i_{\text{ent}} = Z_0 [ (e^{\gamma} + \rho_v e^{-\gamma}) / (e^{\gamma} - \rho_v e^{-\gamma}) ]$$

being denominated to  $\rho_v$  like *coefficient of reflection of the voltages*. Now, as

$$-i_{\text{sal}} = (v_{\text{traSAL}} + v_{\text{refSAL}}) / Z_L = (v_{\text{traSAL}} + v_{\text{refSAL}}) / Z_0$$

it is

$$\rho_v = v_{\text{refSAL}} / v_{\text{traSAL}} = (Z_L - Z_0) / (Z_L + Z_0)$$

consequently, working the equations

$$Z_{\text{ent}} = Z_0 [ (Z_L + Z_0)e^{\gamma} - (Z_L - Z_0)e^{-\gamma} ] / [ (Z_0 + Z_L)e^{\gamma} + (Z_0 - Z_L)e^{-\gamma} ]$$

that it shows us that

$$\begin{aligned} v_{\text{ent}} &= -i_{\text{sal}} [ (Z_L + Z_0)e^{\gamma} - (Z_L - Z_0)e^{-\gamma} ] / 2 = v_{\text{sal}} \text{ch} \gamma - i_{\text{sal}} Z_0 \text{sh} \gamma \\ i_{\text{ent}} &= -i_{\text{sal}} [ (Z_0 + Z_L)e^{\gamma} + (Z_0 - Z_L)e^{-\gamma} ] / 2Z_0 = (v_{\text{sal}} / Z_0) \text{sh} \gamma - i_{\text{sal}} \text{ch} \gamma \end{aligned}$$

being able to see here finally

$$\begin{aligned} \gamma_{11} &= \text{ch} \gamma \\ \gamma_{12} &= Z_0 \text{sh} \gamma \end{aligned}$$

$$\gamma_{21} = \text{sh}\gamma / Z_0$$

$$\gamma_{22} = \text{ch}\gamma$$

### Asymmetrical and adapted network

One can obtain a generalization of the previous case for an asymmetric and adapted network

$$Z_{01} \neq Z_{02} \quad \text{asymmetry}$$

$$Z_{01} = Z_g \quad \text{adaptation to the input}$$

$$Z_{02} = Z_L \quad \text{adaptation to the output}$$

To achieve this we take the system of equations of the propagation and let us divide

$$v_{\text{ent}} / v_{\text{sal}} = \gamma_{11} - i_{\text{sal}} \gamma_{12} / v_{\text{sal}} = \gamma_{11} - \gamma_{12} / Z_{02}$$

$$i_{\text{ent}} / (-i_{\text{sal}}) = v_{\text{sal}} \gamma_{21} / (-i_{\text{sal}}) - \gamma_{22} = Z_{02} \gamma_{21} - \gamma_{22}$$

of where (to remember that  $[\gamma] = -1$ )

$$e^{-\gamma} = (S_{\text{sal}} / S_{\text{ent}})^{1/2} = [ (v_{\text{sal}} (-i_{\text{sal}}) / (v_{\text{ent}} i_{\text{ent}})) ]^{1/2} =$$

$$= [ (\gamma_{11}\gamma_{22})^{1/2} - (\gamma_{12}\gamma_{21})^{1/2} ] / (\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}) = (\gamma_{11}\gamma_{22})^{1/2} - (\gamma_{12}\gamma_{21})^{1/2} =$$

$$= \text{ch}\gamma - \text{sh}\gamma$$

$$\text{ch}\gamma = (\gamma_{11}\gamma_{22})^{1/2}$$

$$\text{sh}\gamma = (\gamma_{12}\gamma_{21})^{1/2}$$

We can also deduce here for it previously seen

$$\text{th}\gamma = \text{sh}\gamma / \text{ch}\gamma = (Z_{\text{entCC}} / Z_{\text{entCA}})^{1/2} = (Z_{\text{salCC}} / Z_{\text{salCA}})^{1/2}$$

being obtained, either for the pattern T (star) or  $\pi$  (triangle), obviously same results

$$\text{sh}\gamma = (Z_{01}Z_{02})^{1/2} / Z_3 = Z_C / (Z_{01}Z_{02})^{1/2}$$

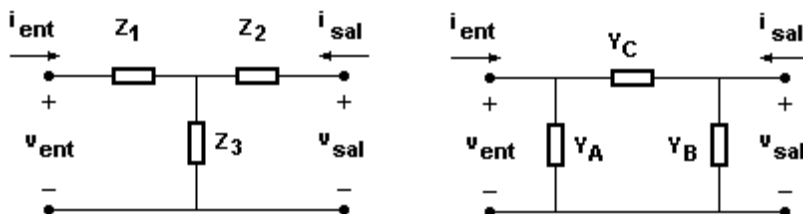
$$\text{th}\gamma = Z_{01} / (Z_1 + Z_3) = Z_{02} / (Z_2 + Z_3) = 1 / Z_{01} (Y_A + Y_C) = 1 / Z_{02} (Y_B + Y_C)$$

$$Z_{01} = [ (Z_1 + Z_3) (Z_1Z_2 + Z_1Z_3 + Z_2Z_3) / (Z_2 + Z_3) ]^{1/2} =$$

$$= 1 / [ (Y_A + Y_C) (Y_A Y_B + Y_A Y_C + Y_B Y_C) / (Y_B + Y_C) ]^{1/2}$$

$$Z_{02} = [ (Z_2 + Z_3) (Z_1Z_2 + Z_1Z_3 + Z_2Z_3) / (Z_1 + Z_3) ]^{1/2} =$$

$$= 1 / [ (Y_B + Y_C) (Y_A Y_B + Y_A Y_C + Y_B Y_C) / (Y_A + Y_C) ]^{-1/2}$$



## Adapting network of impedances, disphased and attenuator

Continuing with an asymmetric and adapted network had that

$$Z_1 = (Z_{01} / \text{th}\gamma) - Z_3$$

$$Z_2 = (Z_{02} / \text{th}\gamma) - Z_3$$

$$Z_3 = (Z_{01}Z_{02})^{1/2} / \text{sh}\gamma$$

of where the transmission of power through the adapting network will be

$$S_{\text{sal}} / S_{\text{ent}} = e^{-2\gamma} = e^{-2 \text{argsh} (Z_{01}Z_{02})/Z_3} = [Z_3 / [(Z_{01}Z_{02})^{1/2} + (Z_{01}Z_{02} + Z_3^2)^{1/2}]^2$$

Similarly it can demonstrate himself that

$$Y_A = (Y_{01} / \text{th}\gamma) - Y_C$$

$$Y_B = (Y_{02} / \text{th}\gamma) - Y_C$$

$$Y_C = (Y_{01}Y_{02})^{1/2} / \text{sh}\gamma$$

$$S_{\text{sal}} / S_{\text{ent}} = [Y_C / [(Y_{01}Y_{02})^{1/2} + (Y_{01}Y_{02} + Y_C^2)^{1/2}]^2$$

## Design attenuator

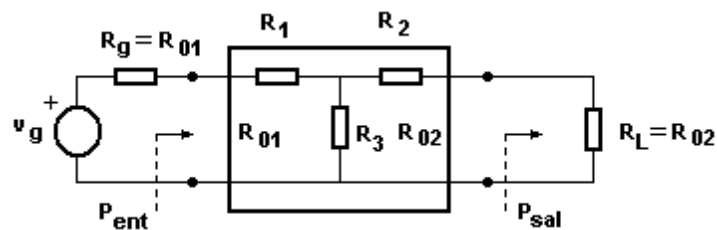
Be the data for an adapted and asymmetric network

$$\gamma = \alpha [\text{Neper}] + j \beta [\text{rad}] = \alpha [\text{Neper}] + j 0 \neq \alpha_{(\omega)}$$

$$S_{\text{sal}} / S_{\text{ent}} = P_{\text{sal}} / P_{\text{ent}} = \dots \leq 1 \neq S_{\text{sal}(\omega)} / S_{\text{ent}(\omega)}$$

$$Z_{01} = Z_{01} + j 0 = R_g = \dots$$

$$Z_{02} = Z_{02} + j 0 = R_L = \dots$$



The design can also be made with  $S_{\text{sal}}/S_{\text{ent}} > 1$ , but it will imply in the development some component negative resistive, indicating this that will have some internal amplification the network and already, then, it would not be passive.

We obtain the energy *attenuation* subsequently

$$\alpha = \ln (P_{\text{ent}} / P_{\text{sal}})^{1/2} = \dots$$



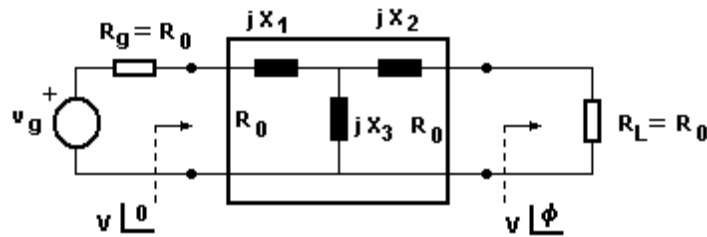
and with this

$$\begin{aligned} \operatorname{sh} \alpha &= (e^{\alpha} - e^{-\alpha}) / 2 = \dots \\ \operatorname{th} \alpha &= (e^{2\alpha} - 1) / (e^{2\alpha} + 1) = \dots \\ R_3 &= (R_g R_L)^{1/2} / \operatorname{sh} \alpha = \dots \\ R_1 &= (R_g / \operatorname{th} \alpha) - R_3 = \dots \\ R_2 &= (R_L / \operatorname{th} \alpha) - R_3 = \dots \end{aligned}$$

## Design disphasator

Be the data for an adapted and symmetrical network

$$\begin{aligned} \gamma &= \alpha [\text{Neper}] + j \beta [\text{rad}] = 0 + j \beta = \beta(\omega) \\ Z_0 &= R_0 + j 0 = Z_{01} = Z_{02} = R_g = R_L = \dots \\ S_{\text{sal}} / S_{\text{ent}} &= W_{\text{sal}} / W_{\text{ent}} = (v_{\text{sal}} / v_{\text{ent}})^2 Z_{01} / R_L = v_{\text{sal}} / v_{\text{ent}} = 1 e^{j\phi} \\ \beta &\neq \phi = \dots \geq \leq 0 \\ f &= \dots \end{aligned}$$



Of the precedent equations the *phase function* is calculated

$$\beta = -j \ln (W_{\text{ent}} / W_{\text{sal}})^{1/2} = -j (v_{\text{ent}} / v_{\text{sal}}) = -j \ln e^{-j\phi} = -\phi = \dots$$

what will determine us

$$\begin{aligned} X_3 &= -R_0 / \operatorname{sen} \beta = \dots \\ X_1 &= X_2 = -(R_0 / \operatorname{tg} \beta) + X_3 = \dots \end{aligned}$$

that it will determine for reactances positive inductors (of high  $Q_{\text{ef}}$  to the work frequency)

$$\begin{aligned} L_3 &= X_3 / \omega = \dots \\ L_1 &= L_2 = X_1 / \omega = \dots \end{aligned}$$

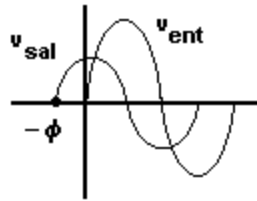
or for the negative capacitors

$$\begin{aligned} C_3 &= -1 / \omega X_3 = \dots \\ C_1 &= C_2 = -1 / \omega X_1 = \dots \end{aligned}$$

## Design attenuator and disphasator

Be an adapted and asymmetric network, where the design is the same as for the general precedent case where the component reactivities of the generator are canceled and of the load with  $X_{gg}$  and  $X_{LL}$

$$v_{sal}/v_{ent} = |v_{sal}/v_{ent}| e^{j\phi}$$

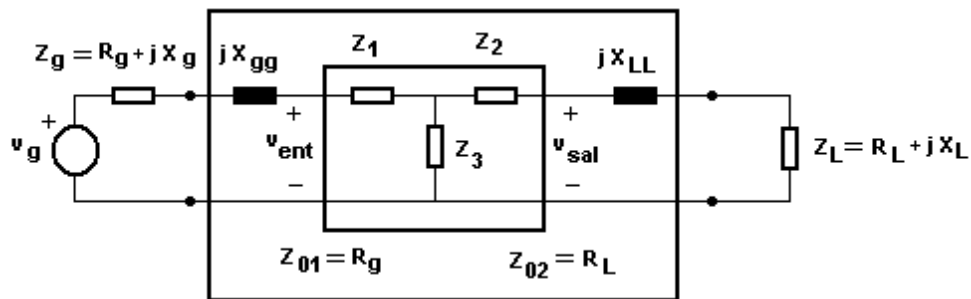


$$\begin{aligned} S_{sal} / S_{ent} &= S_{sal(\omega)} / S_{ent(\omega)} = W_{sal} / W_{ent} = (v_{sal}/v_{ent})^2 Z_{01}/Z_L = \\ &= (|v_{sal}/v_{ent}|^2 R_g/R_L) e^{j2\phi} \end{aligned}$$

$$Z_{01} = R_{01} = R_g$$

$$Z_{02} = R_{02} = R_L$$

$$\gamma = \gamma_{(\omega)} = \alpha_{(\omega)} [\text{Neper}] + j \beta_{(\omega)} [\text{rad}]$$



This way, with the data

$$Z_g = R_g + jX_g = \dots$$

$$Z_L = R_L + jX_L = \dots$$

$$f = \dots$$

$$|v_{sal}/v_{ent}| = \dots \geq \leq 1$$

$$\phi = \dots \geq \leq 0 \text{ (if the network puts back the phase then } \phi \text{ it is negative)}$$

we calculate (the inferior abacus can be used we want)

$$\alpha = \ln(|v_{ent}/v_{sal}|^2 R_L/R_g)^{1/2} = \dots$$

$$\beta = -\phi = \dots$$

$$Z_3 = (R_g R_L)^{1/2} / \text{sh} \gamma = (R_g R_L)^{1/2} / (\cos \beta \text{ sh} \alpha + j \text{sen} \beta \text{ ch} \alpha) = \dots$$

$$Z_1 = (R_g / \text{th}\gamma) - Z_3 = [R_g / (1 + j \text{th}\alpha \text{tg}\beta) / (\text{th}\alpha + j \text{tg}\beta)] - Z_3 = \dots$$

$$Z_2 = (R_L / \text{th}\gamma) - Z_3 = [R_L / (1 + j \text{th}\alpha \text{tg}\beta) / (\text{th}\alpha + j \text{tg}\beta)] - Z_3 = \dots$$

and of them their terms resistives

$$R_1 = \mathbf{R} [Z_1] = \dots$$

$$R_2 = \mathbf{R} [Z_2] = \dots$$

$$R_3 = \mathbf{R} [Z_3] = \dots$$

as well as reactives

$$X_1 = \mathbf{I} [Z_1] = \dots$$

$$X_2 = \mathbf{I} [Z_2] = \dots$$

$$X_3 = \mathbf{I} [Z_3] = \dots$$

Subsequently and like it was said, to neutralize the effects reagents of the generator and of the load we make

$$X_{gg} = -X_g = \dots$$

$$X_{LL} = -X_L = \dots$$

Finally we find the component reactives. If they give positive as inductors (with high  $Q_{ef}$ )

$$L_1 = X_1 / \omega = \dots$$

$$L_2 = X_2 / \omega = \dots$$

$$L_3 = X_3 / \omega = \dots$$

$$L_{gg} = X_{gg} / \omega = \dots$$

$$L_{LL} = X_{LL} / \omega = \dots$$

and if they are it negative as capacitors

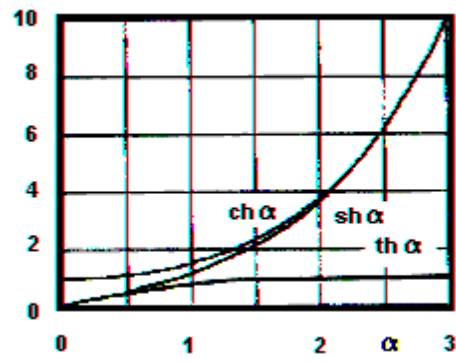
$$C_1 = -1 / \omega X_1 = \dots$$

$$C_2 = -1 / \omega X_2 = \dots$$

$$C_3 = -1 / \omega X_3 = \dots$$

$$C_{gg} = -1 / \omega X_{gg} = \dots$$

$$C_{LL} = -1 / \omega X_{LL} = \dots$$



## Chap. 21 Passive networks as filters of frequency (I Part)

### Generalities

#### Filter of product of constant reactances

Design low-pass

Design high-pass

Design band-pass

Design band-attenuate

#### Filter of product of constant reactances, derived "m" times

Design low-pass

Design high-pass

Design band-pass

Design band-attenuate

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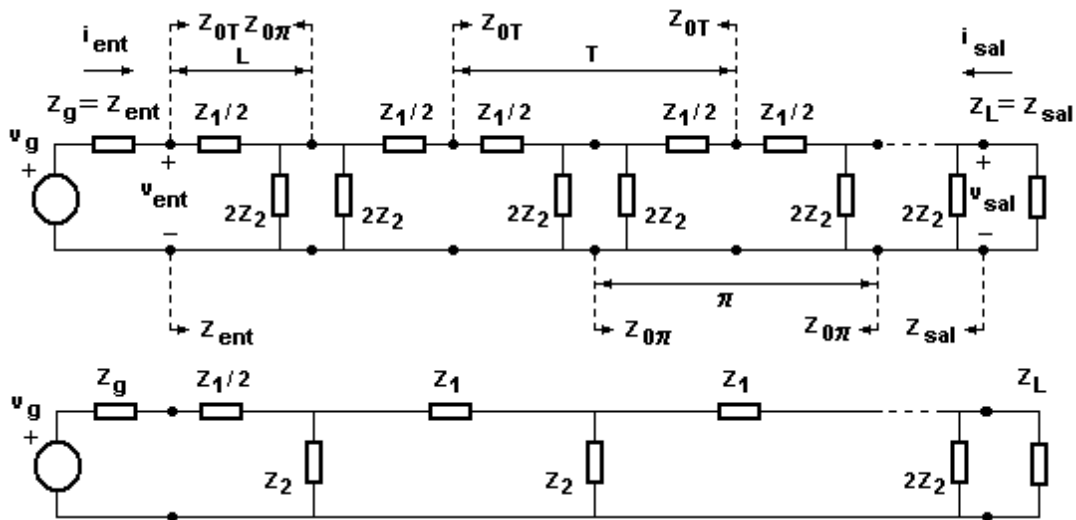
### Generalities

In the following figure we draw the three basic units that we will study and we will design, where

$$Z_{0T} = (Z_1 Z_2 + Z_1^2/4)^{1/2}$$

$$Z_{0\pi} = (Y_1 Y_2 + Y_2^2/4)^{1/2}$$

$$Z_{0T} Z_{0\pi} = Z_1 Z_2$$



If we want that in this network thermal energy (active) doesn't vanish, it will be completed that its impedances are it reactivate pure (in the practice with high  $Q_{ef}$ )

$$Z_1 = j X_1$$

$$Z_2 = j X_2$$

$$X_1 \geq \leq 0$$

$$X_2 \geq \leq 0$$

what will determine us

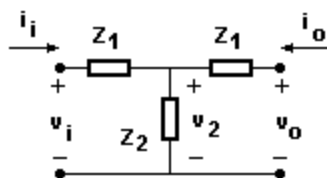
$$Z_{0T} = j (X_1 X_2 + X_1^2/4)^{1/2}$$

$$Z_{0\pi} = (B_1 B_2 + B_2^2/4)^{1/2}$$

$$Z_{0T} Z_{0\pi} = -X_1 X_2$$

On the other hand, in the cell following T observes that

$$i_i + i_o = (v_i - v_2) / Z_1 + (v_o - v_2) / Z_1 = v_2 / Z_2$$



and as we have seen in the chapter of *passive networks as adapters of impedance*, we have that the *propagation function* is here as

$$e^\gamma = v_i / v_2 = v_2 / v_o$$

$$\text{sh} (\gamma/2) = \text{sh} (\alpha/2) \cos (\beta/2) + j \text{ch} (\alpha/2) \text{sen} (\beta/2) = (X_1/4X_2)^{1/2}$$

When the reactances is of the same sign we have

$$Z_{0T} = j (X_1 X_2 + X_1^2/4)^{1/2} = j (|X_1 X_2| + |X_1|^2/4)^{1/2} = j X_{0T} \rightarrow \text{imaginaria}$$

what demonstrates that

$$\begin{aligned} \text{sh}(\alpha/2) \cos(\beta/2) &= (k/4)^{1/2} \\ k &= |X_1/X_2| \end{aligned}$$

and therefore

$$\begin{aligned} \alpha &= 2 \arg \text{sh} |k/4|^{1/2} \rightarrow \\ \text{attenuate band} \\ \beta &= 0 \end{aligned}$$

Now in the inverse case, that is to say when the reactances is of opposed sign

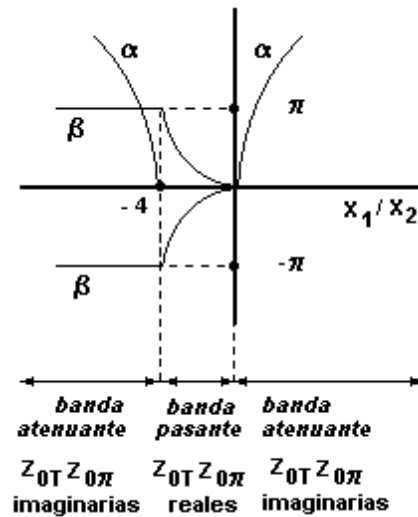
$$Z_{0T} = j (X_1 X_2 + X_1^2/4)^{1/2} = j [-|X_1 X_2| - (1 - k/4)]^{1/2}$$

being able to give that

$$\begin{aligned} -|k/4| &> -1 \Rightarrow k < 4 \\ Z_{0T} &= R_{0T} \\ \rightarrow \text{real} \\ \alpha &= 0 \\ \rightarrow \text{pass band} \\ \beta &= 2 \arcsin |k/4|^{1/2} \end{aligned}$$

or

$$\begin{aligned} -1 &> |k/4| > -\infty \Rightarrow k > 4 \\ Z_{0T} &= j X_{0T} \\ \rightarrow \text{imaginary} \\ \alpha &= 2 \arg \text{ch} |k/4|^{1/2} \rightarrow \\ \text{attenuate band} \\ \beta &= \pm \pi \end{aligned}$$



For the pattern  $\pi$  the analysis is similar.

### Filter of product of constant reactances

The filter is called with this name —with respect the frequency— when  $X_1$  and  $X_2$  are a capacitive and the other inductive, and to its product we call it  $R^2$

$$Z_1 Z_2 = L / C = R^2 \neq R^2_{(\omega)}$$

If we are inside the band pass we know that it is completed

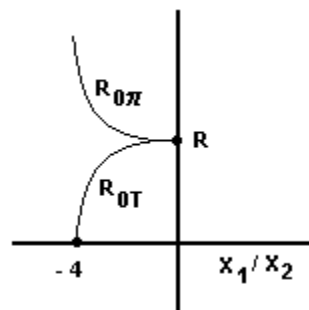
$$-1 < X_1/X_2 < 0$$

consequently

$$Z_{0T} = R (1 - k)^{1/2}$$

$$Z_{0\pi} = R / (1 - k)^{1/2}$$

$$Z_{0T} Z_{0\pi} = R_{0T} R_{0\pi} = R^2$$





When putting «n» stages in cascade the attenuation and the phase displacement obviously will increase. Returning to the drawing of the previous one, if we call  $A_v$  to the amplification (or attenuation, since it can have syntonies that make it) of the voltage in the cell

$$A_v = v_o / v_i = |A_v| e^{j\phi}$$

it is the propagation

$$e^\gamma = (v_{ent} / v_{sal}) (R_L / R_g)^{1/2} = A_v^{-n} (R_L / R_g)^{1/2} = |A_v|^{-n} (R_L / R_g)^{1/2} e^{jn\phi}$$

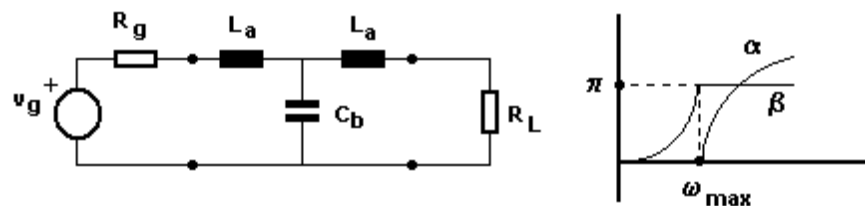
$$\alpha = \ln (|A_v|^{-n} (R_L / R_g)^{1/2})$$

$$\beta = -n\phi$$

### Design low-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\max} = \dots$$



Having present the equations and previous graph has

$$(X_1/X_2)_{\max} = (X_1^2/R^2)_{\max} = -4 \Rightarrow X_{1(\omega_{\max})} = 2R$$

$$R^2 = X_1 X_2 = L_1 / C_2$$

and we calculate finally

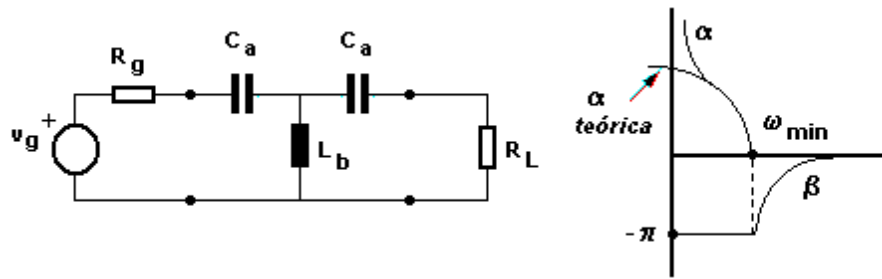
$$L_a = L_1 / 2 = (R_g R_L)^{1/2} / \omega_{\max} = \dots \quad (\text{alto } Q_{ef})$$

$$C_b = C_2 = 2 / (R_g R_L)^{1/2} \omega_{\max} = \dots$$

### Design high-pass

Be tha data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots$$



Having present the equations and previous graph has

$$(X_1/X_2)_{\min} = (X_1^2/R^2)_{\min} = -4 \Rightarrow X_{1(\omega_{\min})} = -2R$$

$$R^2 = X_1 X_2 = L_2 / C_1$$

and we calculate finally

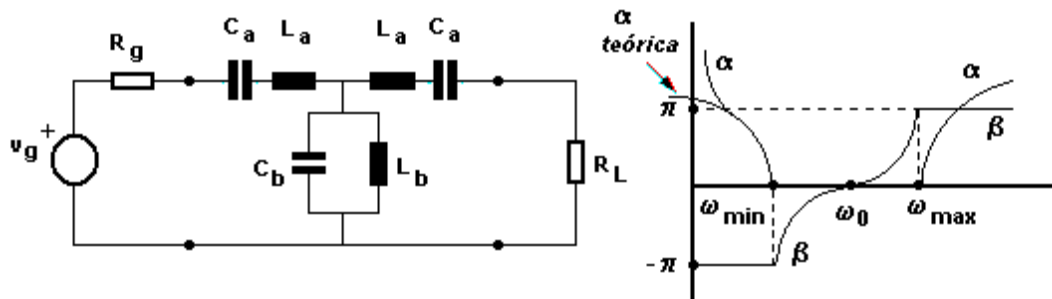
$$L_b = L_2 = (R_g R_L)^{1/2} / 2 \omega_{\min} = \dots \text{ (alto } Q_{ef} \text{)}$$

$$C_a = 2 C_1 = 2 / (R_g R_L)^{1/2} \omega_{\min} = \dots$$

## Design band-pass

Be tha data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



If we design

$$L_1 C_1 = L_2 C_2$$

having present the equations and previous graph has

$$R^2 = [ (\omega L_1 - 1/\omega C_1) / (\omega C_2 - 1/\omega L_2) ] = L_2 / C_1$$

$$\pm [ (X_1/X_2)_{\omega_{\max}; \omega_{\min}} / 4 ]^{1/2} = \pm [ (X_1^2/R^2)_{\omega_{\max}; \omega_{\min}} / 4 ]^{1/2} = \pm 1$$

$$\Rightarrow X_{1(\omega_{\max}; \omega_{\min})} = \pm 2R$$

$$X_{1(\omega_{\max})} = \omega_{\max} L_1 - 1/\omega_{\max} C_1 = 2 R$$

$$X_{1(\omega_{\min})} = \omega_{\min} L_1 - 1/\omega_{\min} C_1 = -2 R$$

and we calculate finally

$$L_a = L_1 / 2 = (R_g R_L)^{1/2} / (\omega_{\max} - \omega_{\min}) = \dots \text{ (alto } Q_{ef} \text{)}$$

$$C_a = 2 C_1 = (1/\omega_{\min} - 1/\omega_{\max}) / (R_g R_L)^{1/2} = \dots$$

$$L_b = L_2 = R_g R_L C_a / 2 = \dots \text{ (alto } Q_{ef} \text{)}$$

$$C_b = C_2 = L_a C_a / L_b = \dots$$

If we wanted to know the value of  $\omega_0$  we also make

$$\begin{aligned} (X_1/X_2)_{\omega_{\max};\omega_{\min}} &= [ - (\omega^2 L_2 C_1 - 1)^2 / \omega^2 L_2 C_1 ]_{\omega_{\max};\omega_{\min}} = \\ &= [ - (\omega^2 L_a C_a - 1)^2 / \omega^2 L_a (C_a/2) ]_{\omega_{\max};\omega_{\min}} = -4 \end{aligned}$$

what will determine

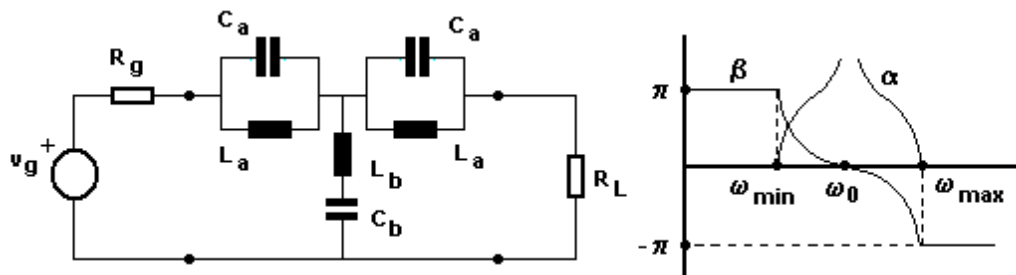
$$\omega_{\max};\omega_{\min} = \omega_0 \pm (2L_a C_b)^{-1/2}$$

$$\omega_0 = (\omega_{\max} + \omega_{\min}) / 2 = [ (1/C_a + 1/2C_b) / L_a ]^{1/2} = \dots$$

## Design band-attenuate

Be tha data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



If we design

$$L_1 C_1 = L_2 C_2$$

having present the equations and previous graph has

$$R^2 = [ (\omega L_2 - 1/\omega C_2) / (\omega C_1 - 1/\omega L_1) ] = L_1 / C_2$$

$$\pm [ (X_1/X_2)_{\omega_{\max};\omega_{\min}} / 4 ]^{1/2} = \pm [ (X_1^2/-R^2)_{\omega_{\max};\omega_{\min}} / 4 ]^{1/2} = \pm 1$$

$$\Rightarrow X_{1(\omega_{\max};\omega_{\min})} = \pm 2 R$$

$$X_{1(\omega_{\max})} = \omega_{\max} C_1 - 1/\omega_{\max} L_1 = 2 R$$

$$X_{1(\omega_{\min})} = \omega_{\min} C_1 - 1/\omega_{\min} L_1 = -2 R$$

and we calculate finally

$$L_a = L_1 / 2 = (R_g R_L)^{1/2} / (1/\omega_{\min} - 1/\omega_{\max}) = \dots \text{ (alto } Q_{ef} \text{)}$$

$$C_a = 2 C_1 = 1 / (R_g R_L)^{1/2} (\omega_{\max} - \omega_{\min}) = \dots$$

$$C_b = C_2 = 2 L_a / R_g R_L = \dots$$

$$L_b = L_2 = L_a C_a / C_b = \dots \text{ (alto } Q_{ef} \text{)}$$

### Filter of product of constant reactances, derived "m" times

I is defined this way to the networks like those that we study but with the following conditions

$$X_{1m} = m X_1$$

$$0 < m \leq 1$$

$$Z_{0Tm} = Z_{0T}$$

$$Z_{0\pi m} \neq Z_{0\pi}$$

of where it is

$$Z_{0Tm} = j (X_{1m} X_{2m} + X_{1m}^2 / 4)^{1/2}$$

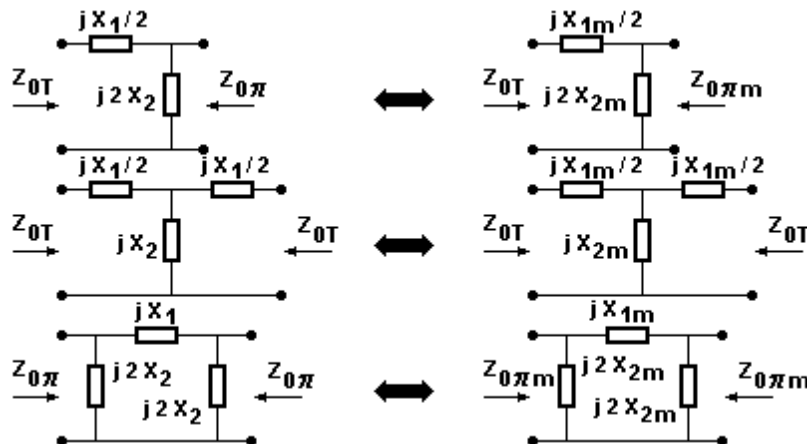
$$Z_{0\pi m} = -j (B_{1m} B_{2m} + B_{1m}^2 / 4)^{1/2}$$

$$Z_{0Tm} Z_{0\pi m} = -X_{1m} X_{2m} \neq Z_{0T} Z_{0\pi}$$

$$X_{2m} = (1 - m^2) X_1 / 4m + X_2 / m$$

If we keep in mind the precedent definitions, it is completed that

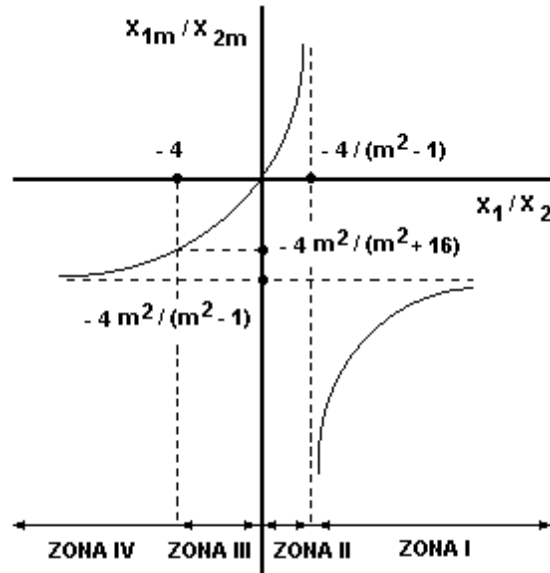
$$X_{1m} / X_{2m} = m^2 / [(1 + m^2)/4 + X_2 / X_1]$$



Now we outline the previous consideration again but it stops our derived network

$$\text{sh}(\gamma_m/2) = \text{sh}(\alpha_m/2) \cos(\beta_m/2) + j \text{ch}(\alpha_m/2) \text{sen}(\beta_m/2) = (X_{1m}/4X_{2m})^{1/2}$$

what determines that it stays the made analysis. We draw the graph then in function of  $X_1/X_2$  again



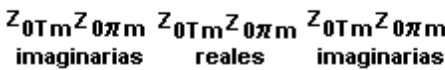
standing out in her four zones:

- ZONA I  $4 / (m^2 + 1) < X_1/X_2 < \infty$   
 $\Rightarrow -\infty < X_{1m}/X_{2m} < -4 m^2 / (m^2 + 16)$
- ZONA II  $0 < X_1/X_2 < 4 / (m^2 - 1)$   
 $\Rightarrow 0 < X_{1m}/X_{2m} < \infty$   
 $\alpha_m = 2 \arg \text{sh} [ m / | 1 - m^2 + 4X_2/X_1 |^{1/2} ] \rightarrow \text{attenuate band}$   
 $\beta_m = 0$   
 $Z_{0T} = j X_{0T} \rightarrow$

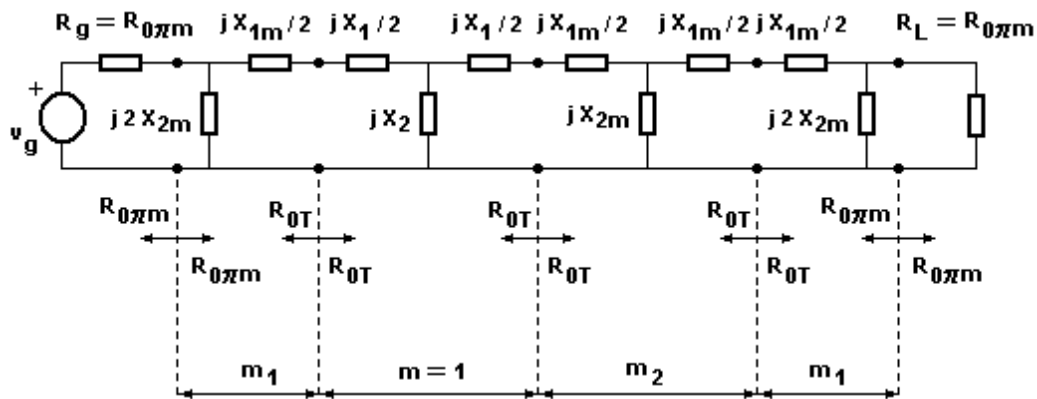
*imaginary*

- ZONA III  $-4 < X_1/X_2 < 0$   
 $\Rightarrow -4 m^2 / (m^2 + 16) < X_{1m}/X_{2m} < 0$   
 $\alpha_m = 0 \rightarrow \text{banda pasante}$   
 $\beta_m = 2 \arcsin [ m / | 1 - m^2 + 4X_2/X_1 |^{1/2} ]$   
 $Z_{0T} = R_{0T} \rightarrow \text{real}$
- ZONA IV  $-\infty < X_1/X_2 < -4$   
 $\Rightarrow -4 m^2 / (m^2 - 1) < X_{1m}/X_{2m} < -4 m^2 / (m^2 + 16)$   
 $\alpha_m = 2 \arg \text{ch} [ m / | 1 - m^2 + 4X_2/X_1 |^{1/2} ] \rightarrow \text{attenuate band}$   
 $\beta_m = \pm \pi$   
 $Z_{0T} = j X_{0T} \rightarrow$

*imaginary*



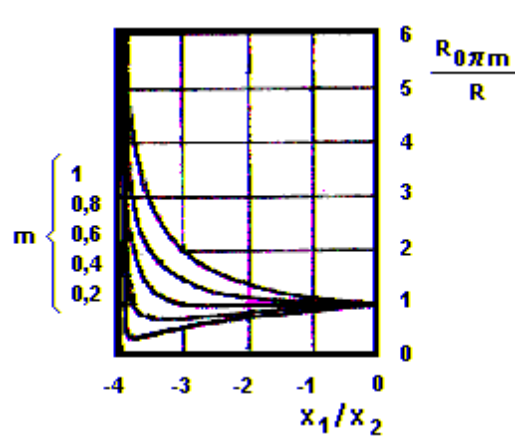
- adapting of impedances (it maintains to  $R_{0\pi}$  constant inside the band pass), derived  $m_1$  times of the prototype
  - properly this filter pass-band (prototype of  $m = 1$ )
  - filter of additional attenuation (it produces sharp selectivity flanks), derived  $m_2$  times of the prototype


$$Z_{0\pi m} = -X_{1m}X_{2m}/Z_{0Tm} = -X_{1m}X_{2m}/R_{0T} = \\ = R \{ 1 - [(1 - m^2)|X_1/X_2|/4] \} / \{ [1 - |X_1/X_2|/4] \}^{1/2} = R_{0\pi m}$$

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of  $R_{0\pi m}$  in passing in the band, it should be

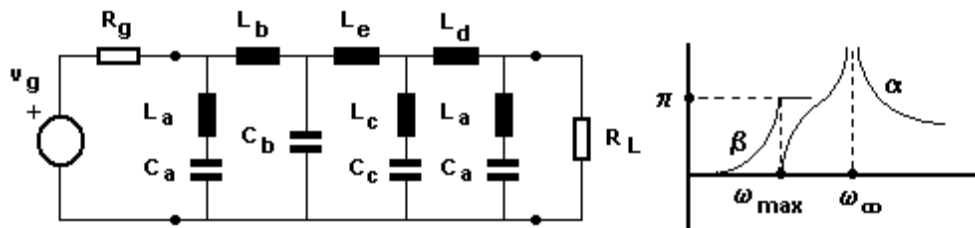
$$m_1 = 0,6$$



### Design low-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\max} = \dots$$



If to this circuit we replace it for the primitive one seen

$$L_a = 2L_{2m}$$

$$L_b = L_{1m}/2 + L_1/2$$

$$L_c = L_{2m}$$

$$L_d = L_{1m}/2 + L_{1m}/2$$

$$L_e = L_1/2 + L_{1m}/2$$

$$C_a = C_{2m}/2$$

$$C_b = C_2$$

$$C_c = C_{2m}$$

they are

$$L_1 = 2 R_L / \omega_{\max} = \dots$$

$$C_2 = 2 / R_L \omega_{\max} = \dots$$

and like we have chosen for maximum plane response

$$m_1 = 0,6$$

$$X_{1m} = \omega L_{1m} = m \omega L_1$$

$$X_{2m} = \omega L_{2m} - 1 / \omega C_{2m} = [(1 - m^2)\omega L_1 / 4m] + [(-1/\omega C_2) / m]$$

we can project

$$L_a = 2L_{2m} = (1 - m_1^2)L_1 / 2m_1 \sim 0,53 L_1 = \dots$$

$$L_b = L_{1m}/2 + L_1/2 = (1 + m_1^2)L_1 / 2 = 0,8 L_1 = \dots$$

$$C_a = C_{2m}/2 = m_1 C_2 / 2 = 0,3 C_2 = \dots$$

$$C_b = C_2 = \dots$$

Continuing, if we adopt an attenuation frequency the next thing possible to that of court to have a good selectivity

$$\omega_{\infty} = \dots > \sim \omega_{\max}$$

we will be able to propose

$$(X_1/X_2)_{\omega_{\infty}} = -\omega_{\infty}^2 L_1 C_2 = 4 / (m_2^2 - 1)$$

deducing finally with it

$$m_2 = [1 - (4 / \omega_{\infty}^2 L_1 C_2)]^{1/2} = [1 - (\omega_{\max}/\omega_{\infty})^2]^{1/2} = \dots$$

$$L_c = L_{2m} = (1 - m_2^2)L_1 / 4m_2 = \dots$$

$$L_d = L_{1m}/2 + L_1/2 = (m_1 + m_2)L_1 / 2 = (0,6 + m_2)L_1 / 2 = \dots$$

$$L_e = L_1/2 + L_{1m}/2 = (1 + m_2)L_1 / 2 = \dots$$

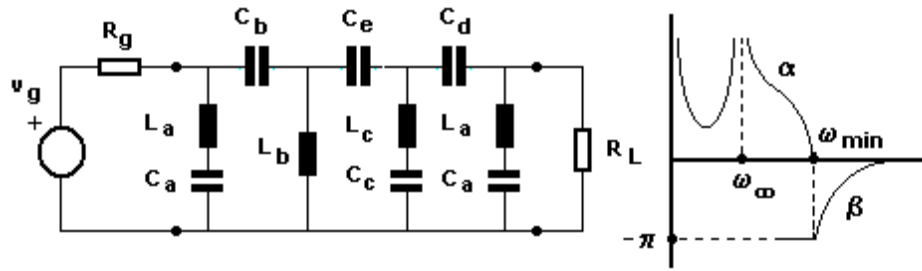
$$C_c = C_{2m} = m_2 C_2 = \dots$$

## Design high-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots$$





If to this circuit we replace it for the primitive one seen

$$L_a = 2L_{2m}$$

$$L_b = L_2$$

$$L_c = L_{2m}$$

$$C_a = C_{2m}/2$$

$$C_b = 2C_{1m}/2C_1$$

$$C_c = C_{2m}$$

$$C_d = 2C_{1m1}/2C_{1m2}$$

$$C_e = 2C_1/2C_{1m}$$

they are

$$L_2 = R_L / 2 \omega_{\min} = \dots$$

$$C_1 = 1 / 2 R_L \omega_{\min} = \dots$$

and like we have chosen for maximum plane response

$$m_1 = 0,6$$

$$X_{1m} = -1 / \omega C_{1m} = -m / \omega C_1$$

$$X_{2m} = \omega L_{2m} - 1 / \omega C_{2m} = [\omega L_2 / m] + [(1 - m^2)(-1/\omega C_1) / 4m]$$

we can project

$$L_a = 2L_{2m} = 2L_2 / m_1 \sim 3,33 L_2 = \dots$$

$$L_b = L_2 = \dots$$

$$C_a = C_{2m}/2 = 2m_1 C_1 / (1 - m_1^2) \sim 1,87 C_1 = \dots$$

$$C_b = 2C_{1m}/2C_1 = 2C_1 / (1 + m_1) = 1,25 C_1 = \dots$$

Continuing, if we adopt an attenuation frequency the next thing possible to that of court to have a good selectivity

$$\omega_{\infty} = \dots < \sim \omega_{\min}$$

we will be able to propose

$$(X_1/X_2)_{\omega_{\infty}} = -1 / \omega_{\infty}^2 L_2 C_1 = 4 / (m_2^2 - 1)$$

deducing finally with it

$$m_2 = [1 - (4 \omega_{\infty}^2 L_2 C_1)]^{1/2} = [1 - (\omega_{\infty}/\omega_{\min})^2]^{1/2} = \dots$$

$$L_c = L_{2m} = L_2 / m_2 = \dots$$

$$C_c = C_{2m} = 4m_2 C_1 / (1 - m_2^2) = \dots$$

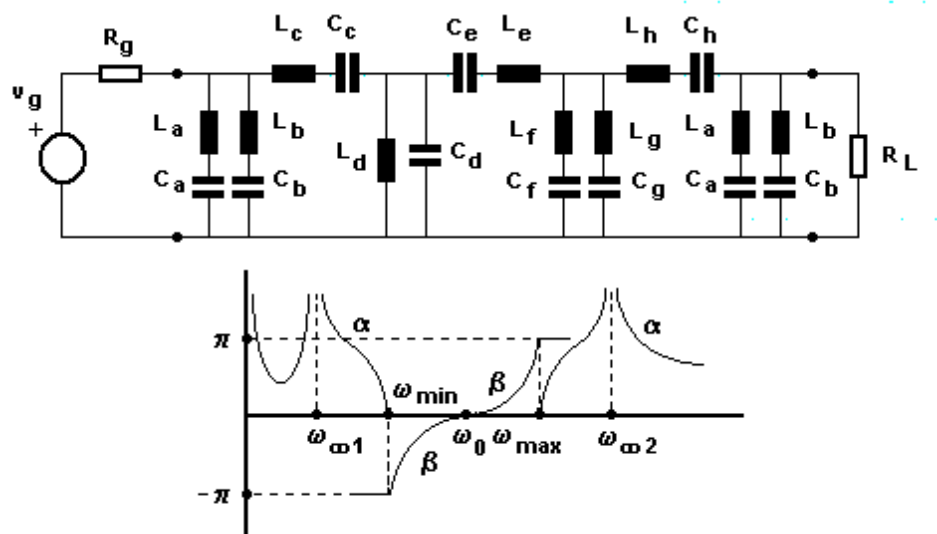
$$C_d = 2C_{1m1} // 2C_{1m2} = 2C_1 / (m_1 - m_2) = 2C_1 / (0,6 - m_2) = \dots$$

$$C_e = 2C_1 // 2C_{1m} = 2C_1 / (1 + m_2) = \dots$$

## Design band-pass

Be the data

$$R_L = R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



Continuing, if we adopt an attenuation frequency  $\omega_{\infty 2}$  the next thing possible to that of court to have a good selectivity

$$\omega_{\infty 2} = \dots > \sim \omega_{\max}$$

we will be able to use the following expression to verify a wanted position of  $\omega_{\infty 1}$  of in the system

$$\omega_{\infty 1} = \omega_{\max} \omega_{\min} / \omega_{\infty 2} = \dots < \sim \omega_{\min}$$

Now with the following equations

$$\omega_0 = (\omega_{\max}\omega_{\min})^{1/2} = \dots$$

$$m = \{ 1 - [ (\omega_{\max}/\omega_0 - \omega_0/\omega_{\max}) / (\omega_{\infty 2}/\omega_0 - \omega_0/\omega_{\infty 2}) ]^2 \}^{1/2} = \dots$$

$$A = (1 - m^2) / 4m = \dots$$

$$B = \omega_{\infty 2} / \omega_0 = \dots$$

$$L = 2R_L / (\omega_{\max} - \omega_{\min}) = \dots$$

$$C = (1/\omega_{\min} - 1/\omega_{\max}) / 2R_L = \dots$$

we will be able to calculate finally

$$L_a = 2LA(1 + B^{-2}) = \dots$$

$$L_b = 2LA(1 + B^2) = \dots$$

$$L_c = (1 + m)L / 2 = \dots$$

$$L_d = R_L^2 C = \dots$$

$$L_e = (1 + m)L / 2 = \dots$$

$$L_f = LA(1 + B^{-2}) = \dots$$

$$L_g = LA(1 + B^2) = \dots$$

$$L_h = mL / 2 = \dots$$

$$C_a = C / 2LA(1 + B^2) = \dots$$

$$C_b = C / 2LA(1 + B^{-2}) = \dots$$

$$C_c = 2C / (1 + m) = \dots$$

$$C_d = L / R_L^2 = \dots$$

$$C_e = 2C / (1 + m) = \dots$$

$$C_f = C / LA(1 + B^2) = \dots$$

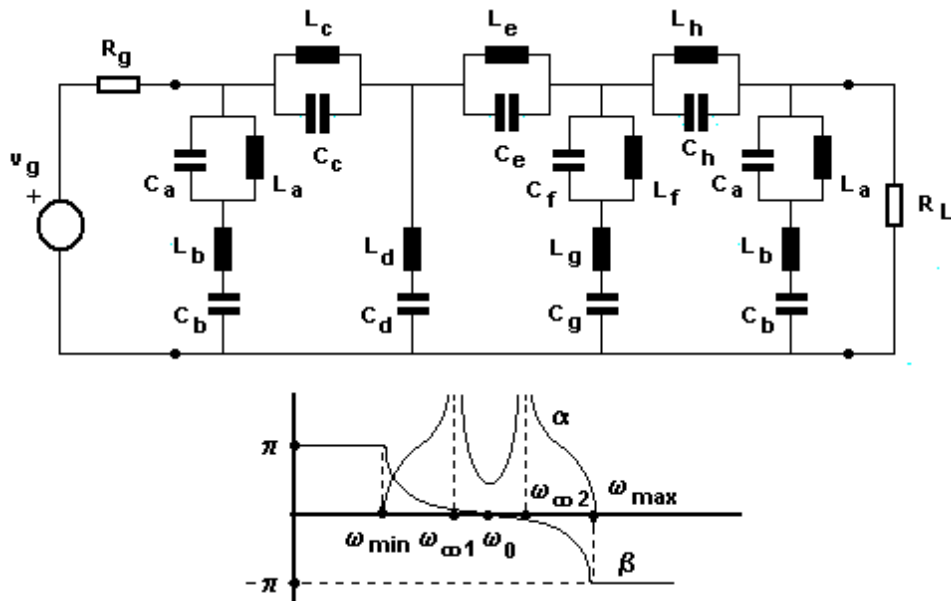
$$C_g = C / LA(1 + B^{-2}) = \dots$$

$$C_h = C / m = \dots$$

## Design band-attenuate

Be the data

$$R_L = R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



Continuing, if we adopt an attenuation frequency  $\omega_{\infty 2}$  the next thing possible to that of course to have a good selectivity

$$\omega_{\infty 2} = \dots < \sim \omega_{\max}$$

we will be able to use the following expression to verify a wanted position of  $\omega_{\infty 1}$  of in the system

$$\omega_{\infty 1} = \omega_{\max} \omega_{\min} / \omega_{\infty 2} = \dots > \sim \omega_{\min}$$

Now with the following equations

$$\omega_0 = (\omega_{\max} \omega_{\min})^{1/2} = \dots$$

$$m = \{ 1 - [ (\omega_{\infty 2} / \omega_0 - \omega_0 / \omega_{\infty 2}) / (\omega_{\max} / \omega_0 - \omega_0 / \omega_{\max}) ]^2 \}^{1/2} = \dots$$

$$A = (1 - m^2) / 4m = \dots$$

$$L = 2R_L (1/\omega_{\min} - 1/\omega_{\max}) = \dots$$

$$C = 1 / 2R_L (\omega_{\max} - \omega_{\min}) = \dots$$

we will be able to calculate finally

$$L_a = LA / 2 = \dots$$

$$L_b = 2CR_L^2 / m = \dots$$

$$L_c = (1 + m)L = \dots$$

$$L_d = R_L^2 C = \dots$$

$$L_e = (1 + m)L = \dots$$

$$L_f = LA = \dots$$

$$L_g = R_L^2 C / m = \dots$$

$$L_h = mL = \dots$$

$$C_a = C / 2A = \dots$$

$$C_b = mL / 2R_L^2 = \dots$$

$$C_c = C / (1 + m) = \dots$$

$$C_d = L / R_L^2 = \dots$$

$$C_e = C / (1 + m) = \dots$$

$$C_f = C / A = \dots$$

$$C_g = mL / R_L^2 = \dots$$

$$C_h = C / m = \dots$$

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## Chap. 22 Passive networks as filters of frequency (II Part)

### Crossed filters

#### Generalites

#### Design double band-pass

#### Filters RC

#### Generalites

#### Design low-pass

#### Design high-pass

#### Design band-pass

#### Design band-attenuate

### Crossed filters

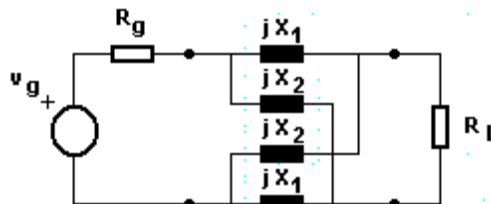
#### Generalites

In the following figure a symmetrical net is shown not dissipative of heat. Their characteristic impedance is

$$Z_0 = Z_{01} = Z_{02} = (jX_1 jX_2)^{1/2} = (-X_1 X_2)^{1/2}$$

$$X_1 \geq 0 \leq X_2$$

$$\begin{aligned} \text{th}(\gamma/2) &= [(\text{ch}\gamma - 1) / (\text{ch}\gamma + 1)]^{1/2} = (jX_1 / jX_2)^{1/2} = (X_1 / X_2)^{1/2} = \\ &= [\text{th}(\alpha/2) + j \text{tg}(\beta/2)] / [1 + j \text{th}(\alpha/2) \text{tg}(\beta/2)] \end{aligned}$$



When the sign of the reactances is different it happens

$$Z_0 = (-X_1 X_2)^{1/2} = |X_1 X_2|^{1/2} = R_0 \rightarrow \text{real pure}$$

$$\text{th}(\gamma/2) = (X_1 / X_2)^{1/2} = j |X_1 / X_2|^{1/2}$$

$$\alpha = 0$$

$$\beta = \text{arc tg } |X_1 / X_2|^{1/2}$$

→ imaginary pure

→ pass band

and when they are same

$$Z_0 = (-X_1 X_2)^{1/2} = j |X_1 X_2|^{1/2} = j X_0$$

→ imaginary pure

$$\text{th}(\gamma/2) = (X_1 / X_2)^{1/2} = |X_1 / X_2|^{1/2}$$

→ real pure

$$\text{th}(\alpha/2) < 1$$

here being been able to give two possible things

$$1) \quad |X_1 X_2| < 1$$

$$\alpha = 2 \arg \text{th} |X_1 / X_2|^{1/2}$$

→ attenuate band

$$\beta = 0$$

$$2) \quad |X_1 X_2| > 1$$

$$\alpha = 2 \arg \text{th} |X_2 / X_1|^{1/2}$$

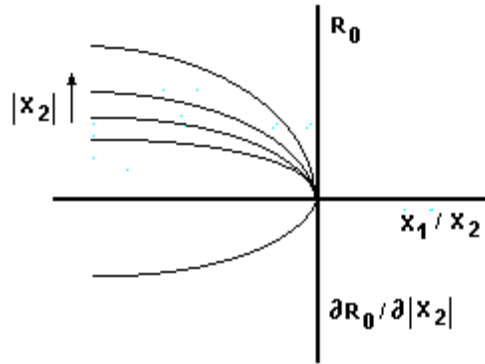
→ attenuate band

$$\beta = \pm \pi$$

Subsequently we make the graph of this result  $Z_0 = R_0$  in the band pass

$$Z_0 = R_0 = |X_1 X_2|^{1/2} = |X_2| |X_1 / X_2|^{1/2}$$

$$\partial R_0 / \partial |X_2| = |X_1 / X_2|^{1/2} / 2$$



Although these filters have a good selectivity, the variation of  $R_0$  with the frequency brings its little use. It can be believed that this would be solved if it is designed to the such reactances that their product is independent of the frequency (f.ex.: an inductance and a capacitance) and with it  $R_0$  that it is constant inside the band in passing, but however this is not possible because it will bring a negative product and then the band pass would be infinite.

With the purpose of designing these filters, we will use the equations of Foster

$$X(s) = H [ s (s^2 + \omega_b^2) (s^2 + \omega_b^2) \dots ] / [ (s^2 + \omega_a^2) (s^2 + \omega_c^2) \dots ] =$$

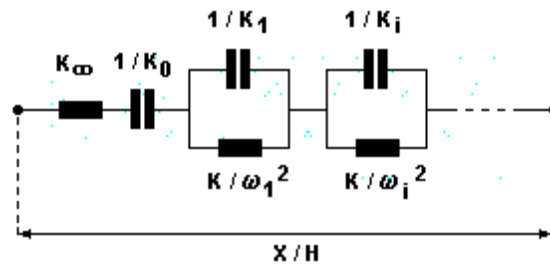
$$= H [ s K_\infty + K_0/s + \sum_i K_i s / (s^2 + \omega_i^2) ]$$

$$H \neq H_{(s)}$$

$$K_{\infty} = X_{(s=j\infty)} / s$$

$$K_0 = s X_{(s=j0)}$$

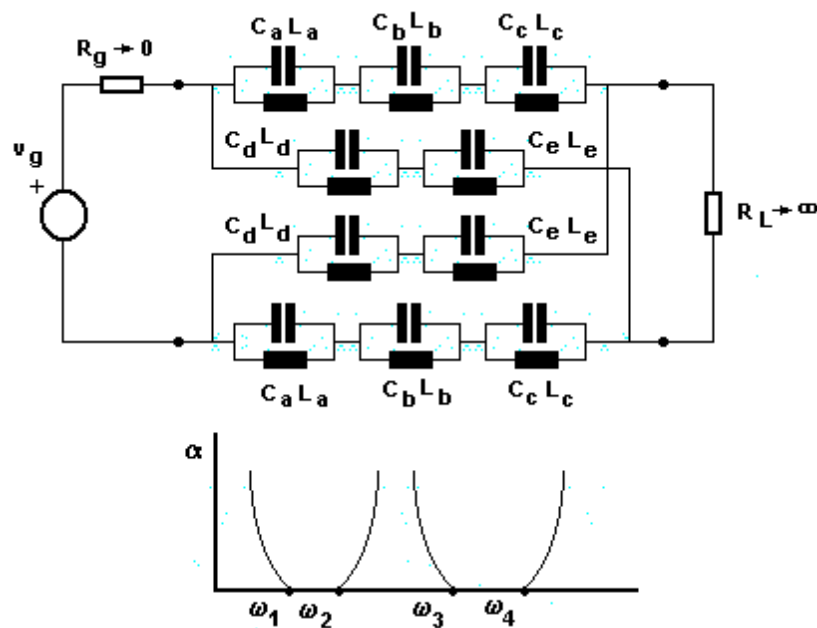
$$K_i = (s^2 + \omega_i^2) X_{(s=j\omega_i)} / s$$



### Design double band-pass

Be the data for the filter crossed network

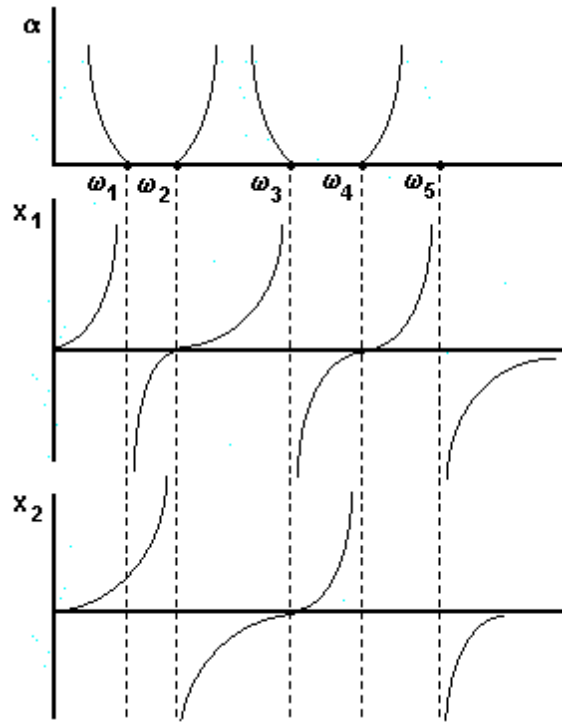
$$f_1 = \dots f_2 = \dots f_3 = \dots f_4 = \dots$$



We outline a system that their reactances is of different sign (system LC) inside the band pass. Indeed, we choose

$$\omega_5 = \dots > \omega_4$$





consequently

$$X_1 = H_1 [ s (s^2 + \omega_2^2) (s^2 + \omega_4^2) ] / [ (s^2 + \omega_1^2) (s^2 + \omega_3^2) (s^2 + \omega_5^2) ]$$

$$X_2 = H_1 [ s (s^2 + \omega_3^2) ] / [ (s^2 + \omega_2^2) (s^2 + \omega_5^2) ]$$

and for  $X_1$

$$K_\infty = 0$$

$$K_0 = 0$$

$$K_1 = [ (\omega_2^2 - \omega_1^2) (\omega_4^2 - \omega_1^2) ] / [ (\omega_3^2 - \omega_1^2) (\omega_5^2 - \omega_1^2) ]$$

$$\Rightarrow C_a = 1 / K_1 = \dots$$

$$L_a = K_1 / \omega_1^2 = \dots$$

$$K_3 = [ (\omega_2^2 - \omega_3^2) (\omega_4^2 - \omega_3^2) ] / [ (\omega_1^2 - \omega_3^2) (\omega_5^2 - \omega_3^2) ]$$

$$\Rightarrow C_b = 1 / K_3 = \dots$$

$$L_b = K_3 / \omega_3^2 = \dots$$

$$K_5 = [ (\omega_2^2 - \omega_5^2) (\omega_4^2 - \omega_5^2) ] / [ (\omega_1^2 - \omega_5^2) (\omega_3^2 - \omega_5^2) ]$$

$$\Rightarrow C_c = 1 / K_5 = \dots$$

$$L_c = K_5 / \omega_5^2 = \dots$$

and now for  $X_2$

$$K_\infty = 0$$

$$K_0 = 0$$

$$K_2 = [ (\omega_3^2 - \omega_2^2) ] / [ (\omega_5^2 - \omega_2^2) ]$$

$$\Rightarrow C_d = 1 / K_2 = \dots$$

$$L_d = K_2 / \omega_2^2 = \dots$$

$$K_5 = [ (\omega_3^2 - \omega_5^2) ] / [ (\omega_2^2 - \omega_5^2) ]$$

$$\Rightarrow C_e = 1 / K_5 = \dots$$

$$L_e = K_5 / \omega_5^2 = \dots$$

## Filters RC

### Generalites

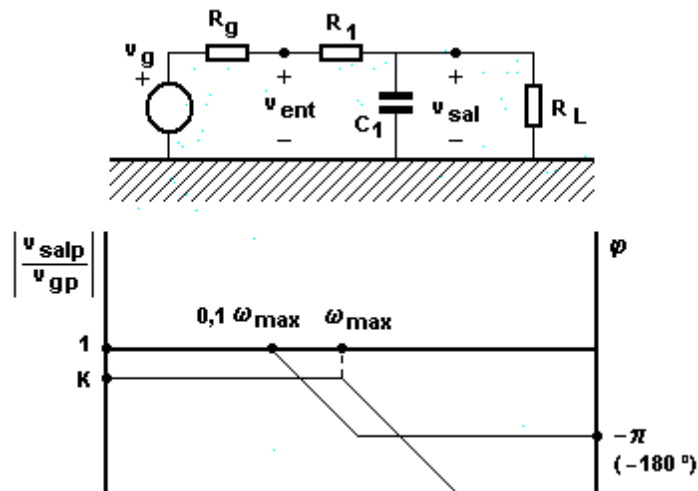
These filters are not of complex analysis in their *characteristic impedance and propagation function* because they usually work desadaptates and they are then of easy calculation. The reason is that to the use being for low frequencies the distributed capacitances is not necessary to eliminate with syntonies, and the amplifiers also possess enough gain like to allow us these advantages.

On the other hand, we clarify that in the graphics of the next designs we will obviate, for simplicity, the real curved. One will have present that, for each pole or zero, the power half happens to some approximate ones 3 [dB] and in phase at about 6 [°].

### Design low-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\max} = \dots \quad K = |v_{\text{salp}}/v_{\text{gp}}| = \dots < 1$$



We outline the equations

$$K = R_L / (R_g + R_1 + R_L)$$

$$\omega_{\max} = 1 / C_1 R_L / (R_g + R_1)$$

and we design clearing of them

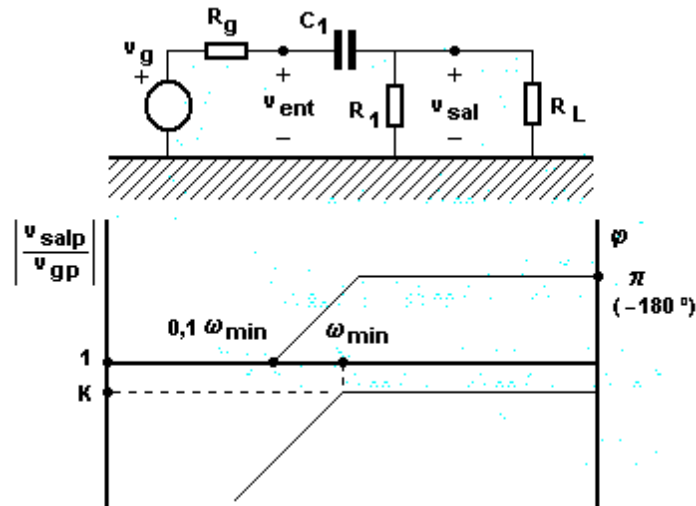
$$R_1 = R_L (K^{-1} - 1) - R_g = \dots$$

$$C_1 = 1 / \omega_{\max} R_L // (R_g + R_1) = \dots$$

## Design high-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots \quad K = |v_{\text{salp}}/v_{\text{gp}}| = \dots < 1$$



We outline the equations

$$K = R_1 // R_L / (R_g + R_1 // R_L)$$

$$\omega_{\min} = 1 / C_1 (R_g + R_1 // R_L)$$

and we design clearing of them

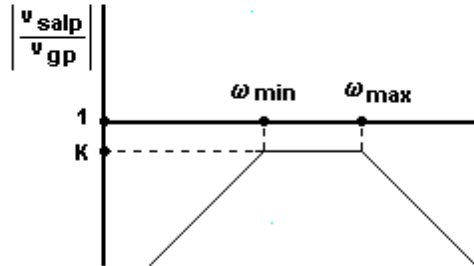
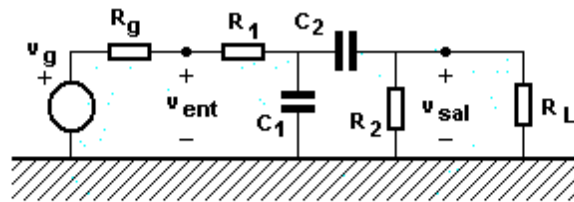
$$R_1 = 1 / [(K^{-1} - 1)/R_g - 1/R_L] = \dots$$

$$C_1 = 1 / \omega_{\min} (R_g + R_1 // R_L) = \dots$$

## Design band-pass

Be the data

$$R_L = \dots \quad R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots \quad K = |v_{\text{salp}}/v_{\text{gp}}| = \dots < 1$$



The circuit will design it with the two cells seen up to now and, so that this is feasible, the second won't load to the first one; that means that

$$1 / \omega C_1 \ll 1 / \omega C_2$$

If for example we adopt

$$C_1 = \dots$$

we will be able to design with it

$$C_2 = \dots \ll C_1$$

$$R_1 = 1 / \omega_{\max} C_1 - R_g = \dots$$

$$R_2 = 1 / (\omega_{\min} C_2 - 1 / R_g) = \dots$$

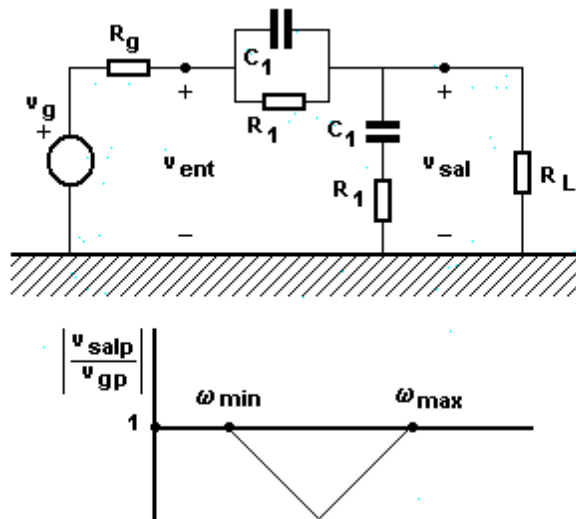
and we verify the attenuation in passing in the band

$$R_2 // R_L / (R_1 + R_g + R_2 // R_L) = \dots \geq K$$

### Design band-attenuate

Be the data

$$R_L = \dots \gg R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



If to simplify we design  $R_g$  and  $R_L$  that are worthless

$$R_g \ll R_1$$

$$R_L \gg R_1$$

the transfer is

$$v_{salp}/v_{gp} \sim (s + \omega_0)^2 / (s + \omega_{min})(s + \omega_{max})$$

$$\omega_0 = (\omega_{min}\omega_{max})^{1/2} = 1 / R_1 C_1$$

$$\omega_{min}; \omega_{max} \sim 1,5 (1 \pm 0,745) / R_1 C_1$$

then, if we adopt

$$C_1 = \dots$$

we will be able to calculate and to verify

$$R_g \ll R_1 = 1 / (\omega_{min}\omega_{max})^{1/2} C_1 = \dots \ll R_L$$

## Chap. 23 Active networks as filters of frequency and displaced of phase (I Part)

### GENERALITIES

#### FILTERS WITH NEGATIVE COMPONENTS

Design tone-pass

#### FILTERS WITH POSITIVE COMPONENTS

##### Slopes of first order (+20 [dB/DEC])

Generalities

Design low-pass

Design high-pass

Design band-pass

Design band-attenuate

##### Slopes of second order with limited plane response (+40 [dB/DEC])

Generalities

Design low-pass

Design high-pass

Design band-pass (and/or tone-pass)

Design band-attenuate (and/or tone-pass)

---

### GENERALITIES

The advantages of the use of active dispositives in the filters, as the AOV, are the following

- bigger easiness to design filters in cascade without they are loaded
- mayor facilidad para diseñar filtros en cascada sin que se carguen
- gain and/or attenuation adjustable
- possibility to synthesize "pathological" circuits (negatives or invertess)

but their limitation is given for

- reach of frequencies

#### FILTERS WITH NEGATIVE COMPONENTS

The transfer for the following circuit is (if  $R_g \ll R_1$  then  $v_{sal}/v_{ent}$  is similar to  $v_{sal}/v_g$ )

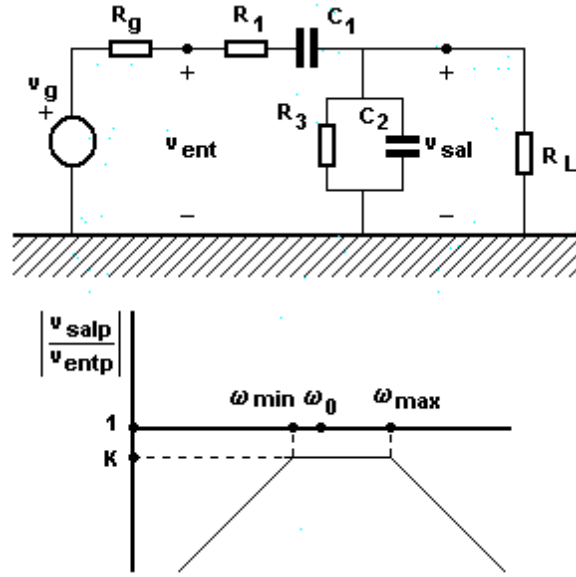
$$v_{sal} / v_{ent} = (1/R_1 C_2) s / (s^2 + s 1/\tau + \omega_0^2) = (1/R_1 C_2) s / (s + \omega_{min})(s + \omega_{max})$$

$$\omega_{min} ; \omega_{max} = \{ 1 \pm [1 - (2\tau\omega_0)^2]^{1/2} \}$$

$$\tau = R_1 R_2 C_1 C_2 / (R_1 C_1 + R_2 C_2 + R_2 C_1)$$

$$\omega_0 = (\omega_{min} \omega_{max})^{1/2} = 1 / (R_1 R_2 C_1 C_2)$$

$$R_2 = R_3 // R_L$$

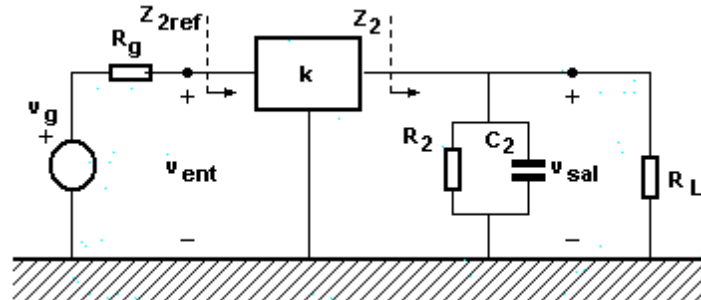


If now we define a *factor of voltage over-gain* (we have demonstrated in the chapter of *amplifiers of RF, § filter impedance*, that this factor is similar to the factor reactivates  $Q$  of a syntonized circuit)

$$\xi = \tau \omega_0 = (R_1 R_2 C_1 C_2)^{1/2} / (R_1 C_1 + R_2 C_2 + R_2 C_1)$$

and we observe that if we add a stage according to the following outline, it is reflected

$$Z_{2ref} = k Z_2 = 1 / (1/k R_2 + s C_2/k)$$



and if now we design

$$R_1 = R_2$$

$$C_1 = C_2$$

$$\omega_0 = 1 / R_1 C_1 = 1 / R_2 C_2$$

it is

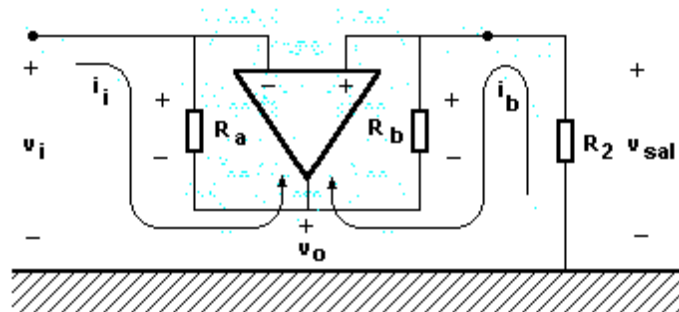
$$\xi = 1 / (2 + k)$$

$$k \geq \leq 0$$

and this way we can control the width of band of the filter adjusting «k». A circuit that achieves this is the *negative convertor of impedances in current (CINI)* of the following figure; to implement it we remember that the differential voltage of the AOV, for outputs delimited in its supply, is practically null. Then

$$Z_{2ref} = v_i / i_i = v_{sal} / (i_b R_b / R_a) = - R_a / R_b = k Z_2$$

$$k \leq 0$$



As the system it is with positive feedback, it will be necessary to verify the condition of their stability

$$V_{INV} > V_{NO-INV}$$

$$v_o R_1 / (R_1 + R_a) > v_o R_2 / (R_2 + R_b)$$

or

$$|k| < R_1 / R_2$$

that, of not being completed, then the terminals of input of the AOV will be invested. For this new case it will be now the condition

$$|k| > R_1 / R_2$$

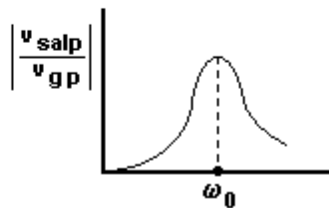
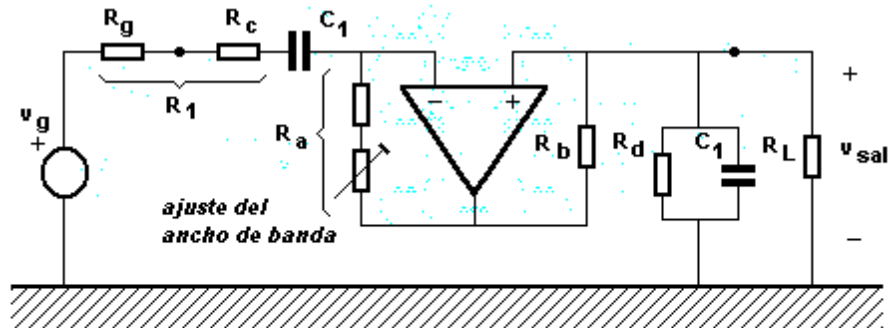
## Design tone-pass



Be the data

$$R_g = \dots \quad f_0 = \dots$$

$Q = \dots$  (similar to  $\xi$ , we remember that this magnitude cannot be very high to work in low frequencies, that is to say, with a band width not very small with regard to  $f_0$ )



Firstly we can adopt

$$C_1 = \dots$$

$$R_b = \dots$$

and we find of the conditions and precedent formulas

$$R_1 = 1 / \omega_0 C_1 = \dots$$

$$R_c = R_1 - R_g = \dots$$

$$R_d = R_1 R_L / (R_1 - R_3) = \dots$$

$$k = (1 / Q) - 2 = \dots$$

$$R_a = -k R_b = \dots$$

It is suggested after the experimentation to achieve the stability changing, or leaving, the terminals of entrance of the AOV and adjusting the syntonny wanted with the pre-set.

## FILTERS WITH POSITIVE COMPONENTS

### Slopes of first order (+20 [dB/DEC])

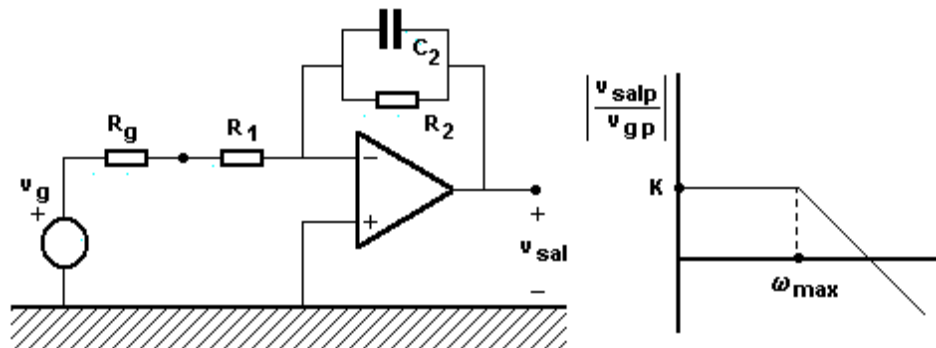
### Generalities

Here the attenuations to the court frequencies are of approximately 3 [dB].

### Design low-pass

Be the data

$$R_g = \dots \quad f_{\max} = \dots \quad K = \dots \geq \leq 1$$



Of the I outline of the transfer we obtain

$$v_{sal} / v_g = [-1 / C_2 (R_1 + R_g)] / (s + \omega_{\max})$$

$$\omega_{\max} = 1 / R_2 C_2$$

$$K = R_2 / (R_1 + R_g)$$

we adopt

$$C_2 = \dots$$

and we find

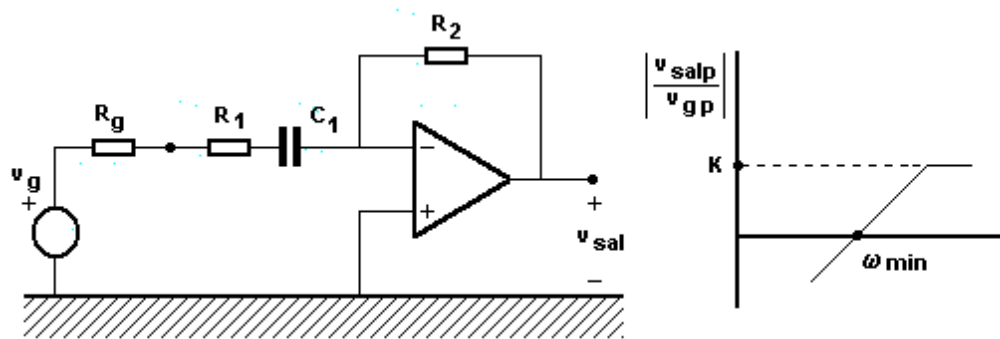
$$R_2 = 1 / \omega_{\max} C_1 = \dots$$

$$R_1 = K (R_1 + R_g) = \dots$$

### Design high-pass

Be the data

$$R_g = \dots \quad f_{\min} = \dots \quad K = \dots \geq \leq 1$$



Of the I outline of the transfer we obtain

$$v_{sal} / v_g = [- R_2 / (R_1 + R_g)] s / (s + \omega_{min})$$

$$\omega_{min} = 1 / (R_1 + R_g) C_1$$

$$K = R_2 / (R_1 + R_g)$$

we adopt

$$C_1 = \dots$$

and we find

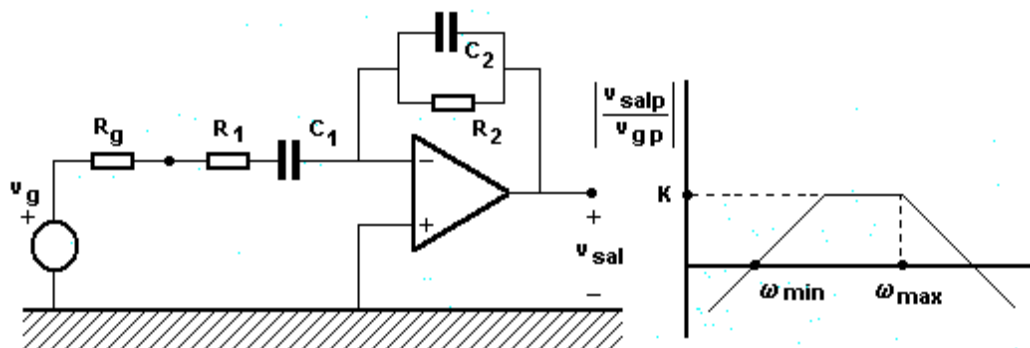
$$R_1 = (1 / \omega_{min} C_1) - R_g = \dots$$

$$R_2 = K (R_1 + R_g) = \dots$$

### Design band-pass

Be the data

$$R_g = \dots \quad f_{min} = \dots \quad f_{max} = \dots \quad K = \dots \geq 1$$



Of the I outline of the transfer we obtain

$$v_{sal} / v_g = [- 1 / (R_1 + R_g) C_2] s / (s + \omega_{min}) (s + \omega_{max})$$

$$\omega_{\min} = 1 / (R_1 + R_g)C_1$$

$$\omega_{\max} = 1 / R_2 C_2$$

$$K = R_2 / (R_1 + R_g)$$

we adopt

$$C_2 = \dots$$

and we find

$$R_2 = 1 / \omega_{\max} C_2 = \dots$$

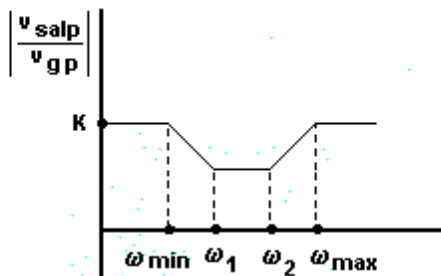
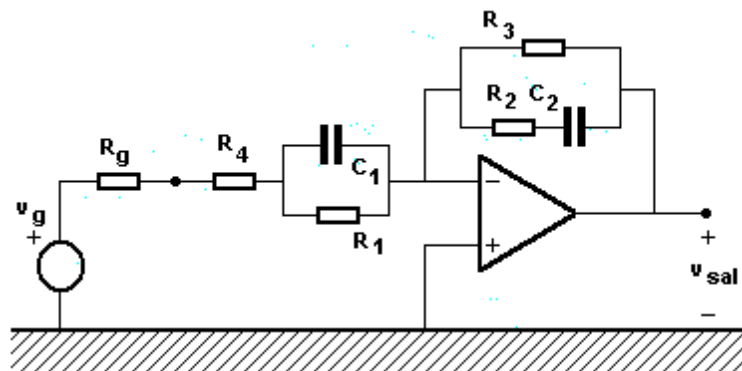
$$R_1 = (R_1 / K) - R_g = \dots$$

$$C_1 = 1 / \omega_{\min} (R_1 + R_g) = \dots$$

### Design band-attenuate

Be the data

$$R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots \quad K = \dots \geq \leq 1$$



Of the I outline of the transfer we obtain

$$v_{sal} / v_g = [- R_2 / (R_2 + R_4 + R_g)] [(s + \omega_1) (s + \omega_2) / (s + \omega_{\min}) (s + \omega_{\max})]$$

$$\omega_{\min} = 1 / (R_2 + R_3)C_2$$

$$\omega_1 = 1 / R_1 C_1$$

$$\omega_2 = 1 / R_2 C_2$$

$$\omega_{\max} = 1 / R_1 // R_3 C_1$$

$$R_3 = R_g + R_4 \text{ (unnecessary simplification)}$$

$$K = R_3 / (R_1 + R_3)$$

we adopt

$$C_2 = \dots$$

$$R_3 = \dots > R_g$$

and we find

$$R_2 = 1 / \omega_{\min} C_2 - R_3 = \dots$$

$$R_1 = R_3 (1 - K) / K = \dots$$

$$C_1 = (R_1 + R_3) / \omega_{\max} R_1 R_3 = \dots$$

$$R_4 = R_3 - R_g = \dots$$

### Slopes of second order with limited plane response (+40 [dB/DEC])

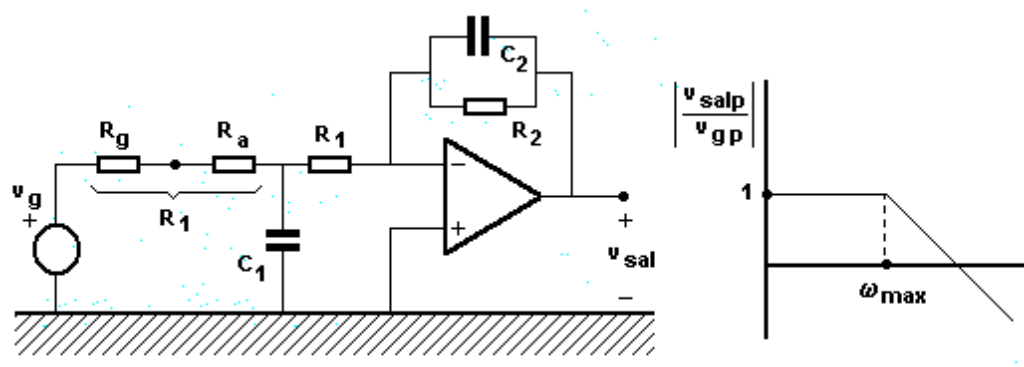
#### Generalities

Here the selectivity is good but it falls to practically 6 [dB] the attenuation in the court frequencies.

#### Design low-pass

Be the data

$$R_g = \dots \quad f_{\max} = \dots$$



With the purpose of simplifying the equations, we make

$$R_1 = R_g + R_a$$

We outline the transfer impedances

$$Z_1 = R_1^2 C_1 (s + 2/R_1 C_1)$$

$$Z_2 = (1/C_2) / (s + 1/R_2 C_2)$$

we express the gain and we obtain the design conditions

$$v_{sal} / v_g = - Z_2 / Z_1 = - \omega_{max}^2 / (s + \omega_{max})$$

$$\omega_{max} = 1 / R_2 C_2$$

$$R_1 = R_2 / 2$$

$$C_1 = 4 C_2$$

and we adopt

$$R_1 = \dots \geq R_g$$

what will allow to be

$$R_a = R_1 - R_g = \dots$$

$$R_2 = 2 R_1 = \dots$$

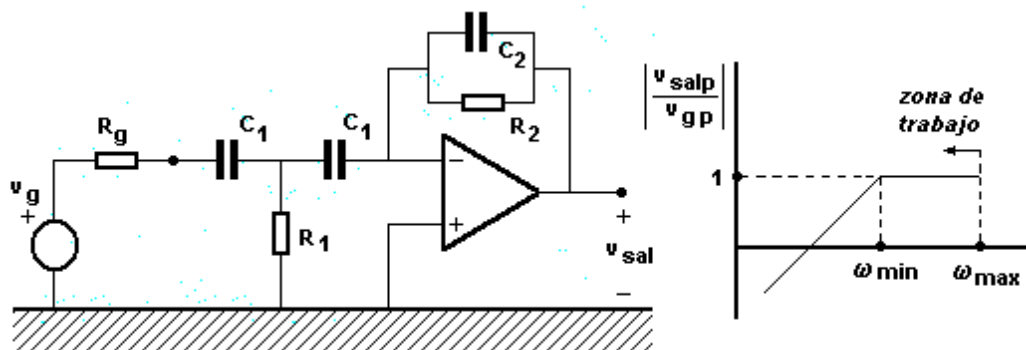
$$C_2 = 1 / \omega_{max} R_2 = \dots$$

$$C_1 = 4 C_2 = \dots$$

## Design high-pass

Be the data

$$R_g = \dots \quad f_{min} = \dots \quad f_{max} = \dots$$



With the purpose of simplifying the equations, we make

$$R_g \ll 1 / \omega_{max} C_1$$

We outline the transfer impedances

$$Z_1 = (2 / C_1) [ (s + 1/R_1 C_1) / s^2 ]$$

$$Z_2 = (1/C_2) / (s + 1/R_2C_2)$$

we express the gain and we obtain the design conditions

$$v_{sal} / v_g = - Z_2 / Z_1 = (- C_1 / 2C_2) s^2 / (s + \omega_{min})$$

$$\omega_{min} = 1 / R_2C_2$$

$$R_1C_1 = R_2C_2 / 2$$

and we adopt

$$C_2 = \dots \geq R_g$$

what will allow to be

$$R_2 = 1 / \omega_{min}C_2 = \dots$$

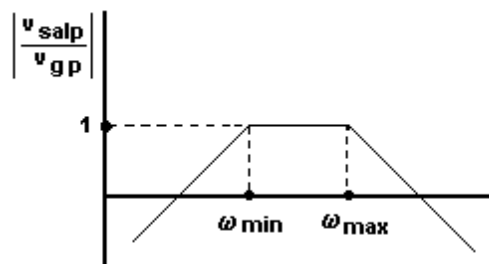
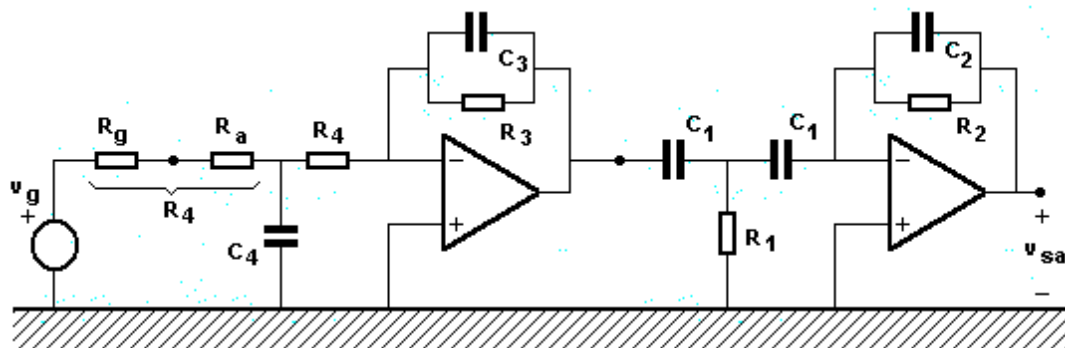
$$C_1 = \dots \ll 1 / \omega_{max}R_g$$

$$R_1 = R_2C_2 / 2C_1 = \dots$$

### Design band-pass (and/or tone-pass)

Be the data

$$R_g = \dots \quad f_{min} = \dots \quad f_{max} = \dots$$



On the other hand, so that the circuit works as *tone-pass* at a  $f_0$  it should simply be made

$$f_0 = f_{min} = f_{max} = \dots$$

As they are same stages that those *low-pass* and *high-pass* that were studied precedently, we adopt

$$R_4 = \dots \geq R_g$$

$$C_1 = \dots$$

$$C_2 = \dots$$

and with it

$$R_a = R_4 - R_g = \dots$$

$$R_2 = 1 / \omega_{\min} C_2 = \dots$$

$$R_3 = 2 R_4 = \dots$$

$$R_1 = R_2 C_2 / 2 C_1 = \dots$$

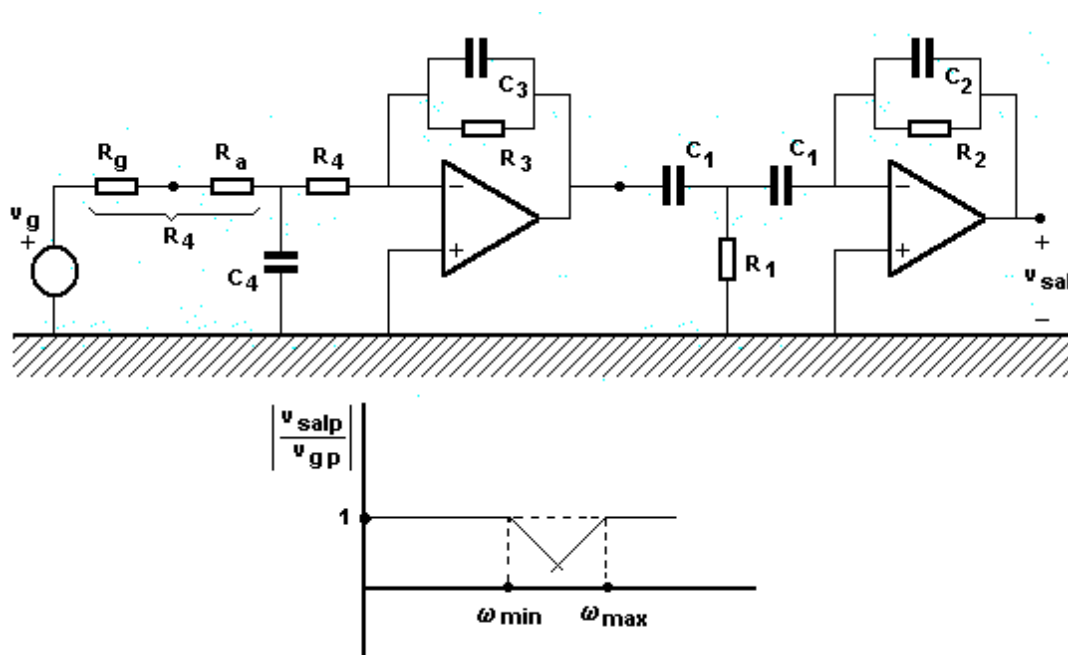
$$C_3 = 1 / \omega_{\max} R_3 = \dots$$

$$C_4 = 4 C_3 = \dots$$

### Design band-attenuate (and/or tone-pass)

Be the data

$$R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$



On the other hand, so that the circuit works as *tone-attenuate* at a  $f_0$  it should simply be made

$$f_0 = f_{\min} = f_{\max} = \dots$$



As they are same stages that those *low-pass* and *high-pass* that were studied precedently, we adopt

$$R_4 = \dots \geq R_g$$

$$C_1 = \dots$$

$$C_2 = \dots$$

and with it

$$R_a = R_4 - R_g = \dots$$

$$R_2 = 1 / \omega_{\max} C_2 = \dots$$

$$R_3 = 2 R_4 = \dots$$

$$R_1 = R_2 C_2 / 2 C_1 = \dots$$

$$C_3 = 1 / \omega_{\min} R_3 = \dots$$

$$C_4 = 4 C_3 = \dots$$

## Chap. 24 Active networks as filters of frequency and displaced of phase (II Part)

Slopes of second order of high plain (+40 [dB/DEC])

Generalities

Design low-pass

Design high-pass

Design band-pass

Design band-attenuate

CIRCUITS OF DISPLACEMENT OF PHASES

Generalities

Design for phase displacements in backwardness (negative)

Design for phase displacements in advance (positive)

FILTERS WITH INVERTER COMPONENTS

Generalities

Filter of simple syntony with girator

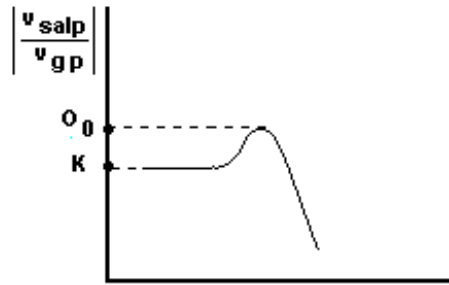
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Slopes of second order of high plain (+40 [dB/DEC])

Generalities

Here the selectivity is good because in the court frequencies a syntony takes place impeding the attenuation, but it deteriorates the plain. This filter responds to the name of Chebyshev. The way to measure this over-magnitude of the gain calls herself *undulation* and we define it in the following way

$$O \text{ [dB]} = 20 \log O \text{ [vec]} = 20 \log ( O_0 \text{ [vec]} / K \text{ [vec]} )$$



in such a way that if it interested us the undulation, it is

$$O [\text{veces}] = K [\text{veces}] \text{ antilog } ( O [\text{dB}] / 20 )$$

For the filters *band-pass* and *band-attenuate*, we will speak of a band width B to power half  $\sim 0,707 K$  and a frequency central  $\omega_0$  dice approximately with the expression (to see the chapter of *radiofrequency amplifiers, § filter impedance*)

$$\omega_0 \sim (\omega_{\max} + \omega_{\min}) / 2$$

$$\xi \sim Q \sim \omega_0 / B$$

The designs will be carried out by means of enclosed table where the resistances will be calculated with the following equation

$$R = 1000 \alpha \beta$$

where the value of «a» it is obtained of this tables, and the other one with the following expression

$$\beta = 0,0001 / f C_0$$

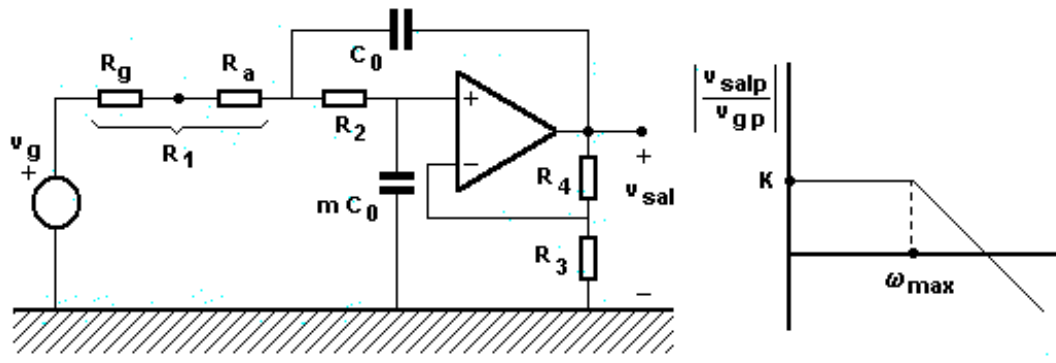
and the other condenser like multiple «m» of  $C_0$ .

## Design low-pass

Be the data

$$R_g = \dots \quad f_{\max} = \dots$$

$$K = \dots (2, 6 \text{ o } 10 [\text{veces}]) \quad O = \dots (1/2, 1, 2 \text{ o } 3 [\text{dB}])$$



We adopt

$$C_0 = \dots$$

and we calculate

$$\beta = 0,0001 / f_{\max} C_0 = \dots$$

so that with the help of the table finally find

$$m = \dots$$

$$mC_0 = \dots$$

$$R_1 = 1000 \alpha_1 \beta = \dots$$

$$R_2 = 1000 \alpha_2 \beta = \dots$$

$$R_3 = 1000 \alpha_3 \beta = \dots$$

$$R_4 = 1000 \alpha_4 \beta = \dots$$

$$R_a = R_1 - R_g = \dots$$

$$m = 1$$

$$K = 2$$

O [dB]	1/2	1	2	3
$\alpha_1$	1,15	1,45	1,95	2,45
$\alpha_2$		1,65	1,6	1,55
$\alpha_3$		5,4	6,2	7,2
$\alpha_4$		5,4	6,2	7,2

$$m = 2$$

$$K = 6$$

O [dB]	1/2	1	2	3
$\alpha_1$	0,54	0,65	0,78	0,88
$\alpha_2$		1,6	1,8	2

$\alpha_3$	2,5	2,9	3,4	3,5
$\alpha_4$	12,8	14,5	16,5	17,5

$$m = 2$$

$$K = 10$$

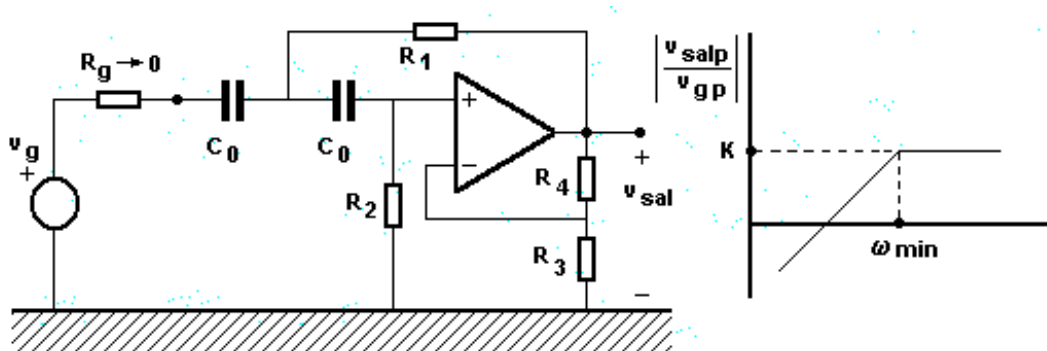
	O [dB]	1/2	1	2	3
$\alpha_1$	0,4	0,48	0,57	0,62	
$\alpha_2$		2,2	2,4	2,7	2,9
$\alpha_3$		2,8	3,2	3,7	3,95
$\alpha_4$		25,5	29	34	35

## Design high-pass

Be the data

$$R_g = \dots \quad f_{\min} = \dots$$

$$K = \dots (2, 6 \text{ o } 10 [\text{veces}]) \quad O = \dots (1/2, 1, 2 \text{ o } 3 [\text{dB}])$$



We adopt

$$C_0 = \dots \ll 1 / \omega_{\min} R_g$$

and we calculate

$$\beta = 0,0001 / f_{\min} C_0 = \dots$$

so that with the help of the table finally find

$$R_1 = 1000 \alpha_1 \beta = \dots$$

$$R_2 = 1000 \alpha_2 \beta = \dots$$

$$R_3 = 1000 \alpha_3 \beta = \dots$$

$$R_4 = 1000 \alpha_4 \beta = \dots$$

$$R_a = R_1 - R_g = \dots$$

<b>K = 2</b>					
	<b>O [dB]</b>	<b>1/2</b>	<b>1</b>	<b>2</b>	<b>3</b>
$\alpha_1$	2,05	1,7	1,38	1,25	
$\alpha_2$		1,35	1,5	1,8	2,05
$\alpha_3$		2,45	3	3,7	4,1
$\alpha_4$		2,45	3	3,7	4,1

<b>K = 6</b>					
	<b>O [dB]</b>	<b>1/2</b>	<b>1</b>	<b>2</b>	<b>3</b>
$\alpha_1$	3,7	3,1	2,65	2,35	
$\alpha_2$		0,7	0,82	0,97	1,05
$\alpha_3$		0,8	1	1,15	1,25
$\alpha_4$		4,15	4,9	5,8	6,3

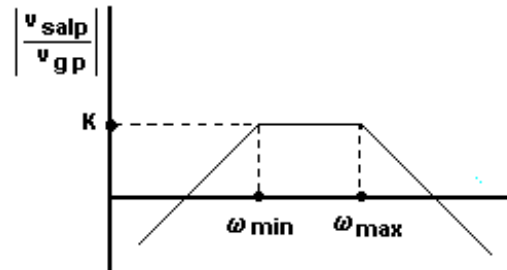
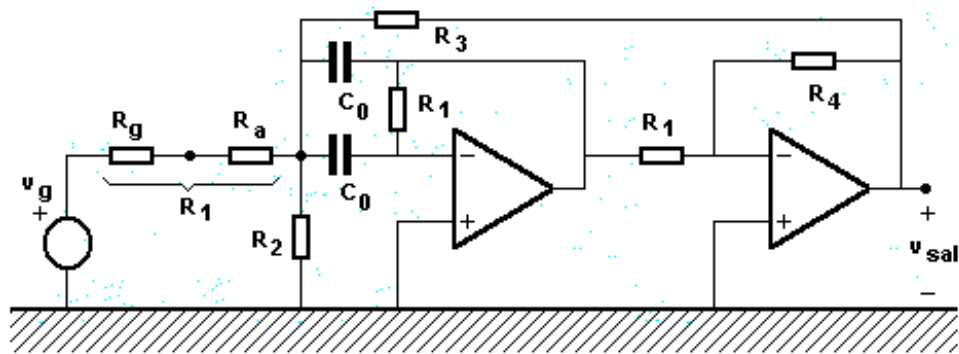
<b>K = 10</b>					
	<b>O [dB]</b>	<b>1/2</b>	<b>1</b>	<b>2</b>	<b>3</b>
$\alpha_1$	4,8	4	3,4	3,1	
$\alpha_2$		0,54	0,64	0,75	0,84
$\alpha_3$		0,6	0,71	0,85	0,92
$\alpha_4$		5,4	6,4	7,4	8,1

## Design band-pass

Be the data

$$R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$

$$K = \dots \text{ (4 o 10 [veces])} \quad Q = \dots \text{ (15, 20, 30 o 40 [veces])}$$



We adopt

$$C_0 = \dots$$

y calculamos and we calculate

$$\beta = 0,0001 / f_0 C_0 = 0,0002 / (f_{\max} + f_{\min}) C_0 = \dots$$

so that with the help of the table finally find

$$R_1 = 1000 \alpha_1 \beta = \dots$$

$$R_2 = 1000 \alpha_2 \beta = \dots$$

$$R_3 = 1000 \alpha_3 \beta = \dots$$

$$R_4 = 1000 \alpha_4 \beta = \dots$$

$$R_a = R_1 - R_g = \dots$$

<b>K = 4</b>					
	<b>Q</b>	<b>15</b>	<b>20</b>	<b>30</b>	<b>40</b>
$\alpha_1$	6,1	7,2	8,6	10,2	
$\alpha_2$		0,5	0,42	0,32	0,28
$\alpha_3$		3,5	3,5	3,5	5,5
$\alpha_4$		6,4	6,4	6,4	6,4

**K = 10**

<b>Q</b>	<b>15</b>	<b>20</b>	<b>30</b>	<b>40</b>
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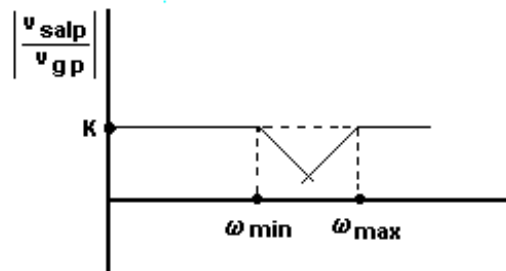
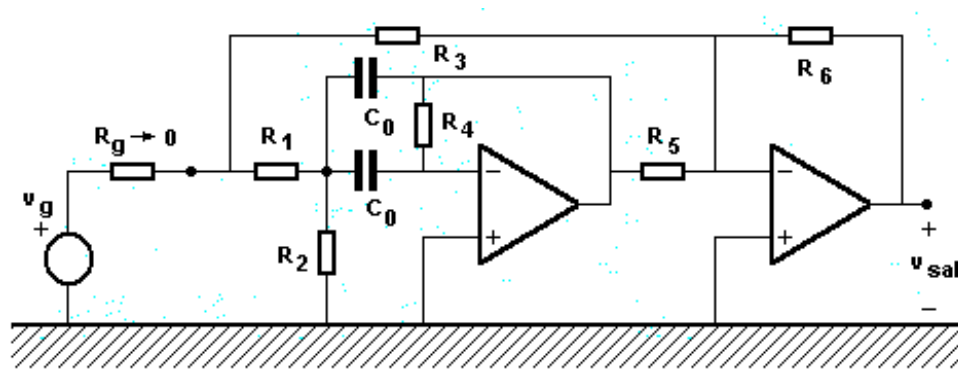
$\alpha_1$	6,1	7,2	8,6	10,2
$\alpha_2$	0,5	0,42	0,32	0,28
$\alpha_3$	9,2	8,9	8,7	8,5
$\alpha_4$	16	16	16	16

## Design band-attenuate

Be the data

$$R_g = \dots \quad f_{\min} = \dots \quad f_{\max} = \dots$$

$$K = \dots \quad (2, 6 \text{ o } 10 [\text{veces}]) \quad Q = \dots \quad (2, 5, 10 \text{ o } 15 [\text{veces}])$$



We adopt

$$C_0 = \dots$$

and we calculate

$$\beta = 0,0001 / f_0 C_0 = 0,0002 / (f_{\max} + f_{\min}) C_0 = \dots$$

so that with the help of the table finally find

$$R_1 = 1000 \alpha_1 \beta = \dots$$

$$R_2 = 1000 \alpha_2 \beta = \dots$$

$$R_3 = 1000 \alpha_3 \beta = \dots$$



$$R_4 = 1000 \alpha_4 \beta = \dots$$

$$R_5 = 1000 \alpha_4 \beta = \dots$$

$$R_6 = 1000 \alpha_4 \beta = \dots$$

$$R_1 // R_3 = \dots \gg R_g$$

<b>K = 2</b>					
	<b>Q</b>	<b>2</b>	<b>5</b>	<b>10</b>	<b>15</b>
$\alpha_1$	1,55	3,9	8	11,8	
$\alpha_2$		0,54	0,16	0,08	0,055
$\alpha_3$		1	1	1	1
$\alpha_4$		6,3	16	36	47
$\alpha_5$	2	2	2	2	
$\alpha_6$	2	2	2	2	

<b>K = 6</b>					
	<b>Q</b>	<b>2</b>	<b>5</b>	<b>10</b>	<b>15</b>
$\alpha_1$	1,55	3,9	8	11,8	
$\alpha_2$		0,54	0,16	0,08	0,055
$\alpha_3$		1	1	1	1
$\alpha_4$		6,3	16	36	47
$\alpha_5$	2	2	2	2	
$\alpha_6$	6	6	6	6	

<b>K = 10</b>					
	<b>Q</b>	<b>2</b>	<b>5</b>	<b>10</b>	<b>15</b>
$\alpha_1$	1,55	3,9	8	11,8	
$\alpha_2$		0,54	0,16	0,08	0,055
$\alpha_3$		1	1	1	1
$\alpha_4$		6,3	16	36	47
$\alpha_5$	2	2	2	2	
$\alpha_6$	10	10	10	10	

## CIRCUITS OF DISPLACEMENT OF PHASES

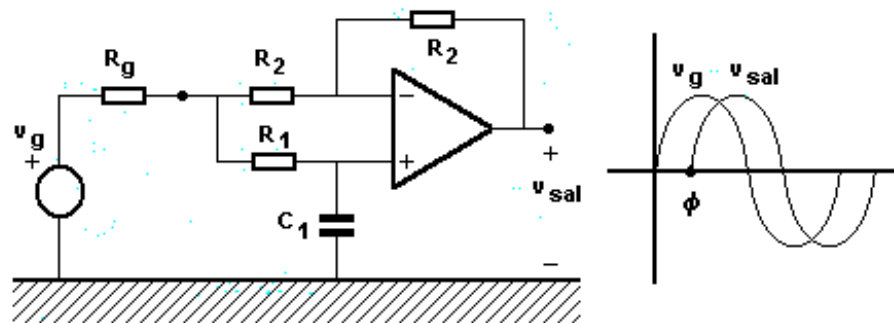
### Generalities

We take advantage of the phase displacement here from a transfer when being used in a frequency  $\omega_0$  different from the plain area. If the useful spectrum is very big (band bases B) the displacement won't be the same one for all the frequencies, and also the widths for each one of them will change.

## Design for phase displacements in backwardness (negative)

Be the data

$$R_g = \dots \quad f_0 = \dots \quad 0 [^\circ] < \phi = \dots < 180 [^\circ]$$



Of the equation of the output ( $R_g = 0$ )

$$v_{sal} = v_g (-R_2/R_2) + v_g [ (1/sC_1) / (R_1 + 1/sC_1) ] (1 + R_2/R_2)$$

it is the transfer

$$v_{sal} / v_g = (1 - sR_1C_1) / (sR_1C_1 + 1) \rightarrow 1 \cdot e^{j(-2 \arctg \omega R_1C_1)}$$

consequently if we adopt

$$C_1 = \dots$$

$$R_2 = \dots \gg R_g$$

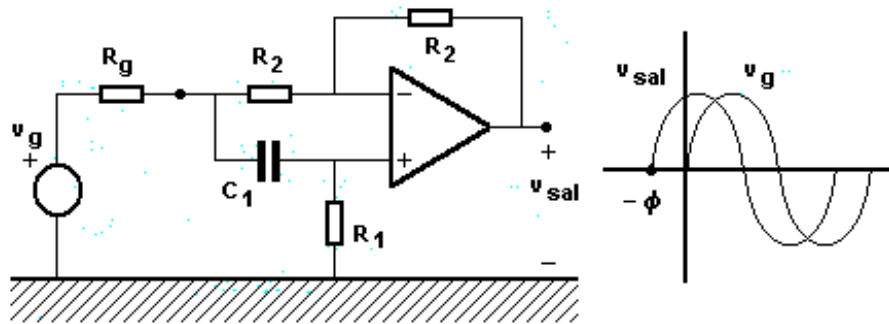
we calculate and we verify

$$R_1 = [ \operatorname{tg} (\phi/2) ] / \omega_0 C_1 = \dots \gg R_g$$

## Design for phase displacements in advance (positive)

Be the data

$$R_g = \dots \quad f_0 = \dots \quad 0 [^\circ] < \phi = \dots < 180 [^\circ]$$



Of the equation of the output ( $R_g = 0$ )

$$v_{sal} = v_g (-R_2/R_2) + v_g [R_1 / (R_1 + 1/sC_1)] (1 + R_2/R_2)$$

it is the transfer

$$v_{sal} / v_g = (sR_1C_1 - 1) / (sR_1C_1 + 1) \rightarrow 1 \cdot e^{j(\pi - 2 \arctg \omega R_1C_1)}$$

consequently if we adopt

$$C_1 = \dots$$

$$R_2 = \dots \gg R_g$$

we calculate and we verify

$$R_1 = [ \operatorname{tg} [(180 - \phi) / 2] ] / \omega_0 C_1 = \dots \gg R_g$$

## FILTERS WITH INVERTER COMPONENTS

### Generalities

With the intention of generalizing, we can classify to these types of networks in the following way

- Convertors of impedance
  - positives (or *escalor*)
    - for voltage (CIPV)
    - for current (CIPI)
  - negatives
    - for voltage (CINV)
    - por corriente (CINI)
- Inverters of impedance
  - positives (or *girator*)
    - for voltage (IIPV)

- for current (IIP)
- negatives
  - for voltage (IINV)
  - for current (IINI)
- Circulators
- Rotators (created by Léon Or-Chua in 1967)
- Mutator (created by Léon Or-Chua in 1968)
- Symmetrizer (or *reflexors*, created by R. Gemin and G. Fravelo in 1968)

### Filter of simple syntony with girator

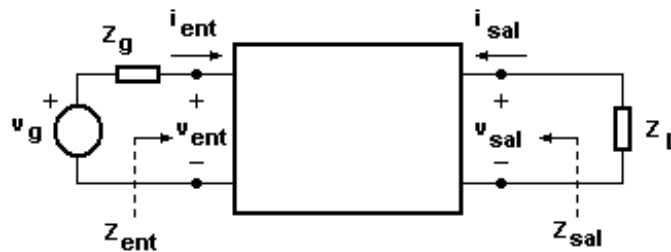
We will use the IIP or girator. It is characterized to possess the following parameters of impedance (to see the chapter *passive networks as adapters of impedance*)

$$v_{\text{ent}} = i_{\text{ent}} Z_{11} + i_{\text{sal}} Z_{12}$$

$$v_{\text{sal}} = i_{\text{ent}} Z_{21} + i_{\text{sal}} Z_{22}$$

$$Z_{11} = Z_{22} = 0$$

$$Z_{21} = -Z_{12} = Z_G \text{ (turn impedance)}$$



what manifests that if we divide member to member the following equations

$$v_{\text{ent}} = -i_{\text{sal}} Z_G$$

$$v_{\text{sal}} = i_{\text{ent}} Z_G$$

we arrive to that

$$S_{\text{ent}} = -S_{\text{sal}}$$

being transferred the power to the load.

Let us study the input subsequently to this network

$$Z_{\text{ent}} = v_{\text{ent}} / i_{\text{ent}} = -(i_{\text{sal}} Z_G) / (v_{\text{sal}} / Z_G) = Z_G^2 / Z_L$$

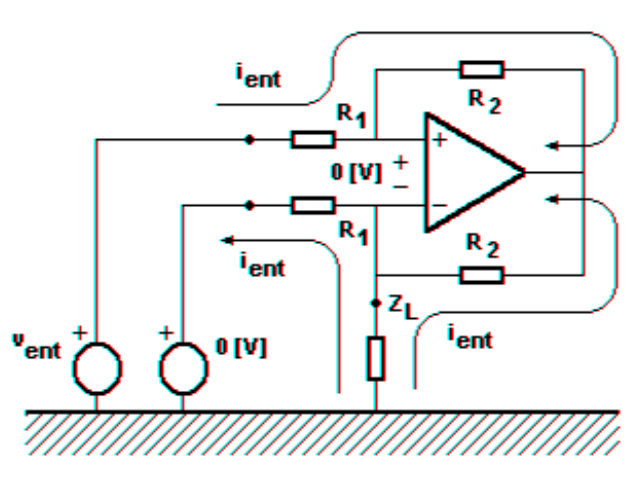
and like it is symmetrical, it will be on the other hand

$$Z_{sal} = Z_G^2 / Z_g$$

what has allowed us to obtain this way the justification of their name as network "inverter of impedances".

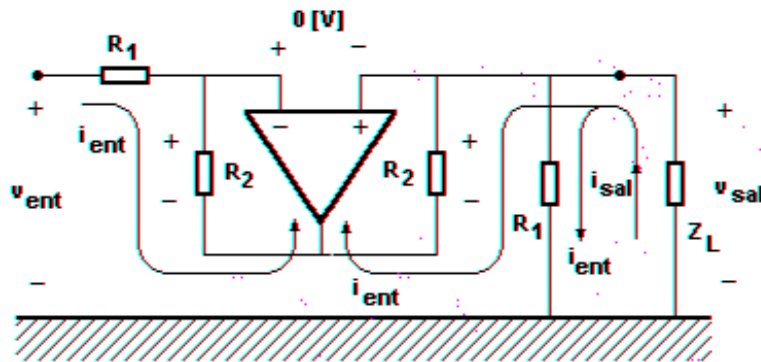
We will implement a possible circuit girator next. For we study it the load of the circuit and let us observe that it behaves as a perfect current generator

$$i_{sal} = 2 i_{ent} = 2 v_{ent} / 2 R_1 = v_{ent} / R_1 \neq i_{sal} (Z_L)$$



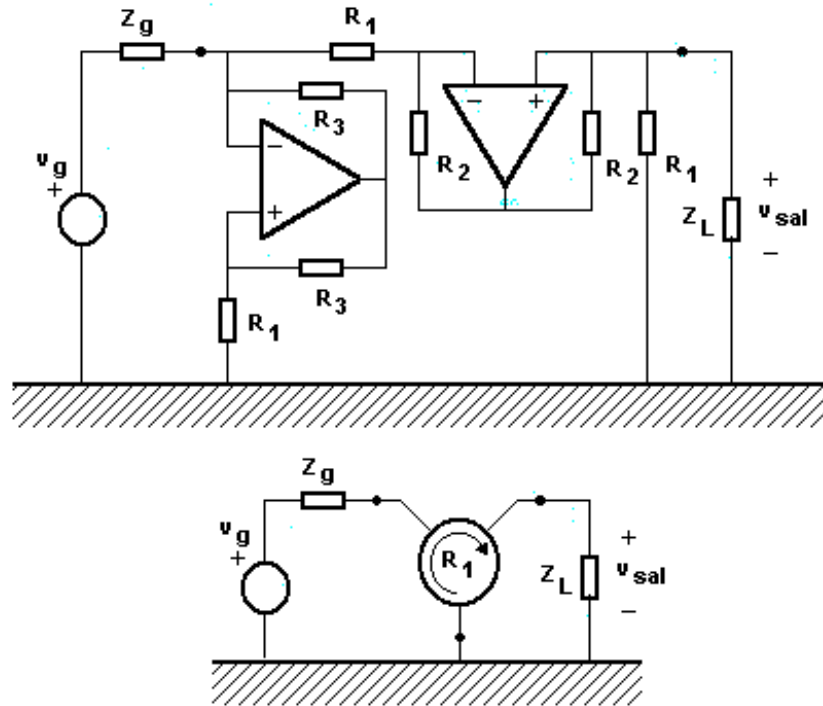
and now the entrance impedance is (to observe that the circuit is the same one but drawn otherwise)

$$Z_{ent} = v_{ent} / i_{ent} = v_{ent} R_1 / (v_{ent} - v_{sal}) = v_{ent} R_1 / [v_{ent} + (v_{ent} / R_1) Z_L] = (1/R_1 + Z_L / R_1^2)^{-1}$$



that it is similar to the previous  $Z_{ent} = Z_G^2 / Z_L$  and, to achieve it perfectly, we will be able to use a CINI (we attach to the implementation the equivalent symbol of the girator)

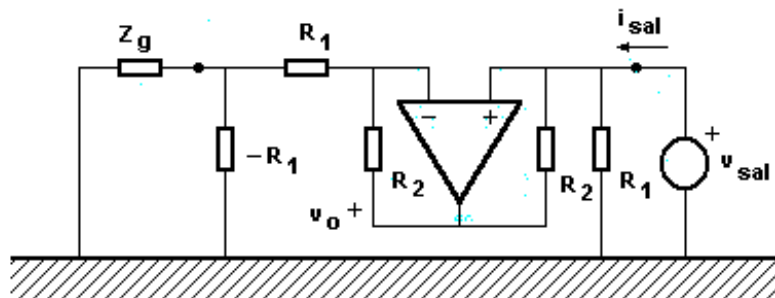
$$Z_G = R_1$$



To make the symmetry of the girator we should verify their output impedance; this is given for

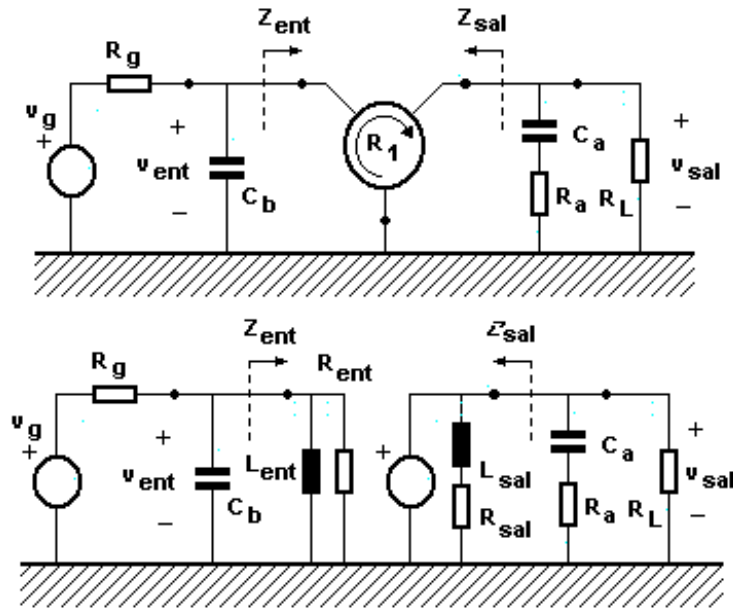
$$v_{sal} = v_o [ R_1 + Z_g / (-R_1) ] / [ R_2 + R_1 + Z_g / (-R_1) ]$$

$$Z_{sal} = v_{sal} / i_{sal} = v_{sal} / [ v_{sal} / R_1 + (v_{sal} - v_o) / R_2 ] = R_1^2 / Z_g$$



If now we connect a RC conforms to shows in the following circuit, we will have a simple syntony to a work frequency if we design

$$|Z_a| = [ R_a^2 + (1/\omega C_a)^2 ]^{1/2} \ll R_L$$



achieving

$$Z_{ent} = R_1^2 / [R_a + (1/sC_a)] = 1 / [(1/R_{ent}) + (1/sL_{ent})]$$

$$Z_{sal} = R_1^2 / R_g // (1/sC_b) = R_{sal} + sL_{sal}$$

with

$$R_{ent} = R_1^2 / R_a$$

$$L_{ent} = R_1^2 C_a$$

$$R_{sal} = R_1^2 / R_g$$

$$L_{sal} = R_1^2 C_b$$

consequently

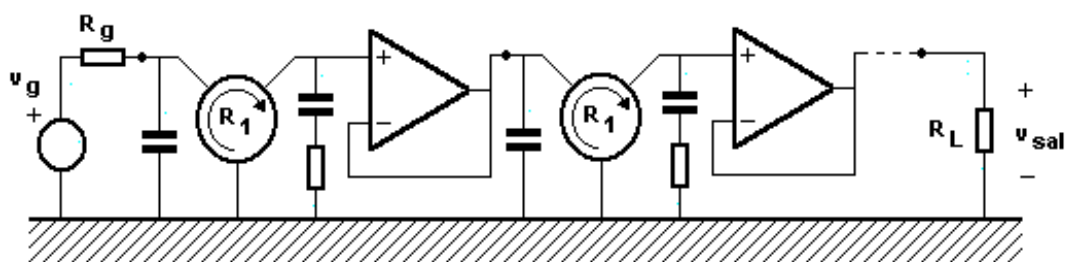
$$v_{sal} = i_{ent} R_1 = R_1 (v_g - v_{ent}) / R_g$$

$$v_{sal} / v_g = R_1 (1 - Z_T / R_g) / R_g$$

$$Z_T = R_g // R_{ent} // sL_{ent} // (1/sC_b)$$

with  $Z_T$  the value of the impedance of the syntonized filter.

If we want to connect several stages of these in cascade to obtain more syntonies in *tip*, of *maximum plain* or of *same undulation*, it is enough with separating them for followers sample the following circuit, and to go to the chapter that treats the topic of *amplifiers of radiofrequency class A*.





## Chap. 25 Amplitude Modulation

### GENERALITIES

Spectral analysis of the signs

Theorem of the sampling

Mensuration of the information

Generalities

The information of a signal

### MODULATION

Generalities

Amplitude Modulation (MA)

Generalities

Double lateral band and carrier (MAC)

Generalities

Generation with quadratic and lineal element

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Generation for product

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Design

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Generalities

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Generalities

Generation for filtrate

Generation for phase displacement

Generation for code of pulses (PCM)

Generation OOK

Generation PAM

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### GENERALITIES

Spectral analysis of the signs

We know that a sign anyone temporary  $v(t)$  it can be expressed in the spectrum, that is to say,

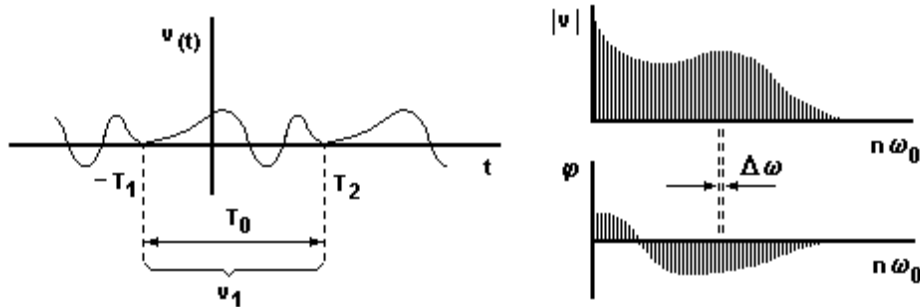
in their content harmonic  $v_{(\omega)}$  and where the module of Laplace  $|v_{(s)}|$  transformation it is their contour.

When it has a period  $T_0$  it can be expressed in the time with the help of the transformation in series of Fourier.

$$v(t) = (1/T_0) \sum_{-\infty}^{\infty} v_{(n\omega_0)} e^{j n \omega_0 t}$$

$$\omega_0 = n 2\pi/T_0 \text{ (con } n = 0, 1, 2, 3, \dots)$$

$$T_0 = T_1 + T_2$$



where  $v_{(n \omega_0)}$  it is the contour in the spectrum

$$v_{(n \omega_0)} = |v_{(n \omega_0)}| e^{j \phi_{(n \omega_0)}} = \int_{-T_1}^{T_2} v_1(t) e^{j n \omega_0 t} dt$$

It is useful many times to interpret this with trigonometry

$$v(t) = (1/T_0) \{ v_0 + 2 \sum_{n=1}^{\infty} |v_{(n \omega_0)}| \cos [n \omega_0 t + \phi_{(n \omega_0)}] \} =$$

$$= \sum_{n=1}^{\infty} V_n \cos [n \omega_0 t + \phi_{(n \omega_0)}]$$

$$v_0 = \int_{-T_1}^{T_2} v_1(t) dt$$

$$|v_{(n \omega_0)}| = (v_{a(n \omega_0)}^2 + v_{b(n \omega_0)}^2)^{1/2}$$

$$\phi_{(n \omega_0)} = - \arctg (v_{b(n \omega_0)} / v_{a(n \omega_0)})$$

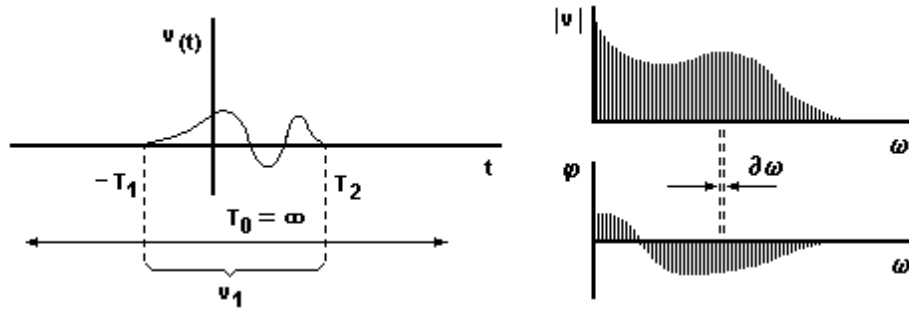
$$v_{a(n \omega_0)} = \int_{-T_1}^{T_2} v_1(t) \cos n \omega_0 t dt$$

$$v_{b(n \omega_0)} = \int_{-T_1}^{T_2} v_1(t) \sin n \omega_0 t dt$$

When the signal is isolated we will have an uncertain content of harmonic

$$v(t) = (1/T_0) \int_{-\infty}^{\infty} v_{(\omega)} e^{j \omega t} d\omega = (1/2\pi) \int_{-\infty}^{\infty} v_{(\omega)} e^{j \omega t} d\omega$$

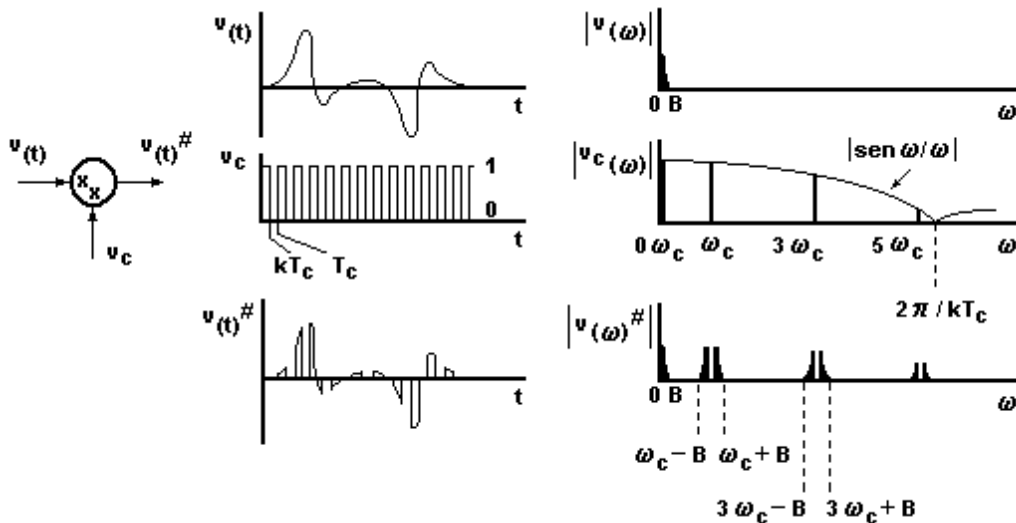
$$v_{(\omega)} = |v_{(\omega)}| e^{j \phi_{(\omega)}} = \int_{-T_1}^{T_2} v_1(t) e^{j \omega t} dt$$



### Theorem of the sampling

When we have a signal  $v(t)$  and it is sampled like  $v(t)^\#$ , it will be obtained of her an information that contains it. In the following graph the effect is shown. That is to say, that will correspond for the useful signal and their harmonics

$$\begin{aligned}
 v(t) &= V_0 + V_1 \cos(\omega_1 t + \phi_1) + V_2 \cos(\omega_2 t + \phi_2) + \dots \\
 v_c(t) &= 1 + k_1 \cos(\omega_c t + \phi_{1c}) + k_3 \cos(3\omega_c t + \phi_{3c}) + \dots \\
 v(t)^\# &= v(t) v_c(t) = \\
 &= V_0 (1 + k_1 + k_3 + \dots) + [V_1 \cos(\omega_1 t + \phi_1) + V_2 \cos(\omega_2 t + \phi_2) + \dots] + \\
 &\quad + (k_1 V_1 / 2) \{ \cos[(\omega_c + \omega_1)t + \phi_{2-1}] + \cos[(\omega_c - \omega_1)t + \phi_{1-1}] \} + \\
 &\quad + (k_3 V_2 / 2) \{ \cos[(3\omega_c + \omega_2)t + \phi_{2-3}] + \cos[(3\omega_c - \omega_2)t + \phi_{1-3}] \} + \dots
 \end{aligned}$$



that is to say that, in  $v(t)^\#$  it is the  $v(t)$  incorporated as

$$V_0 (1 + k_1 + k_3 + \dots) + [V_1 \cos(\omega_1 t + \phi_1) + V_2 \cos(\omega_2 t + \phi_2) + \dots]$$

For applications when the sample is instantaneous  $v(t)^*$  (it is no longer more  $v(t)^\#$ ), the

equations are the same ones but diminishing  $kT_c$  and therefore the spectral contour  $|v_{c(\omega)}|$  it will be plain. In the practical applications these samples are retained by what is denominated a system *Retainer of Order Cero* (R.O.C.) and then coded in a certain digital binary code for recently then to process them in the transcepcions.

Returning to him ours, the *Theorem of the Sampling* indicates the minimum frequency, also well-known as *frequency of Nyquist*, that can be used without losing the useful band B, that is to say to  $v_{(t)}$ . Obviously it will be of empiric perception its value, since to have information of both hemicles of the sine wave more compromising of the useful band B, it will be necessary that we owe sampling to each one at least. Then it says this *Theorem* simply

$$\omega_c \geq 2 B$$

question that can also be observed in the precedent graphs of  $|v_{(\omega)}^\#|$  in those which, for not superimposing the spectra, it should be

$$B \leq \omega_c - B$$

## Mensuration of the information

### Generalities

The signal sources are always contingents, that is to say, possible to give an or another information. For such a reason a way to quantify this is measuring its probability that it exists in a transception channel.

We distinguish something in this: the *message* of the *information*. The first one will take a second, that is to say, it will be the responsible one of transporting the content of a fact that, as such, it will possess «n» objects (symbols) that are presented of «N» available, and they will have each one of them a certain probability « $P_i$ » of appearing, such that:

$$\sum_N P_i = 1$$

and with it, for a source of objects statistically independent (source of null memory) the information «I» it completes a series of requirements; that is

- The information «I» it is a function of the probability  $P_M$  of choosing the message «M»  
 $I = I_{(P_M)}$
- We are speaking of realities of the world
  - The probability  $P_M$  of being transmitted the message «M» it exists  
 $0 \leq P_M \leq 1$
  - The information «I» it exists  
 $0 \leq I$
- The information «I» it is inversely proportional to the probability of the message  $P_M$
- To maximum probability of being given the message «M» it is the minimum information «I»

$$\lim_{(P_M \rightarrow 1)} I = 0$$

— The variations of probabilities in the messages are inversely proportional to their informations

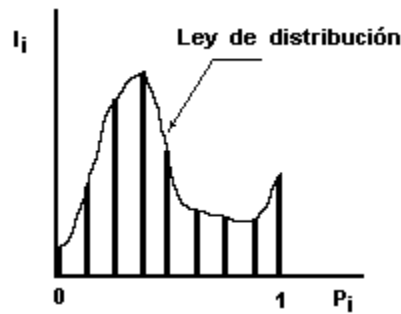
$$P_{M1} < P_{M2} \Rightarrow I_{(P_{M1})} > I_{(P_{M2})}$$

and it has been seen that the mathematical expression that satisfies these conditions is the *logarithm*. Either that we choose the decimal or not, the information then for each symbol it is

$$I_i = \log_{10} P_i^{-1} = \log P_i^{-1} \text{ [Hartley = Ha]}$$

$$I_i = \log_2 P_i^{-1} \text{ [Binary unit = bit]}$$

$$I_i = \log_e P_i^{-1} = \ln P_i^{-1} \text{ [Nats]}$$



For  $n \gg 1$  presented objects will be then the *total information of the message*

$$I = \sum_n I_i$$

the *average information of the source*

$$I_{medf} = N \sum_N P_i I_i \text{ [Ha]}$$

the *entropy of the source* (that is also the *mathematical hope of the information*  $M_{(I)}$ )

$$0 \leq H_f \text{ [H/objeto]} = M_{(I)} = I_{medf} / N = \sum_N P_i I_i \leq \log N$$

the *average information of the message*

$$I_{med} = n \sum_n P_i I_i \text{ [Ha]}$$

and the *entropy of the message*

$$0 \leq H = I_{med} / n = \sum_n P_i I_i = \sum_n P_i \log P_i^{-1} \leq \log N$$

We define a *channel of information* like

"A channel of information comes determined by an input alphabet  $A = \{a_i\}$ ,  $i = 1,$

2, ..., r; an output alphabet  $B = \{b_j\}$ ,  $j = 1, 2, \dots, s$ ; and a group of conditional probabilities  $P(b_j/a_i)$ .  $P(b_j/a_i)$  it is the probability of receiving to the output the symbol  $b_j$  when it sends himself the symbol of input  $a_i$ ."

and this way, indeed, for a channel of information the following concepts are had: the *mutual information* (that is equal to the capacity of the channel)

$$I_{(A; B)} = H_{(A)} - H_{(A/B)}$$

and their *equivocation*

$$E_{(A/B)} = \sum_{A,B} P_{(a,b)} \log P_{(a/b)}^{-1}$$

### The information of a signal

Here we study the signals that are given in the time. They are sampling like it has been seen precedently and the message of them travels along the transception, carrying a quantity of information «I» that we want to evaluate in their form average «  $I_{med}$  ».

If we define then

$kT_c$	period minimum that we obtain of the sign
$T_c$	time in that the information is evaluated (period of sampling)
$P$	probability of being given the sign in a level
$N$	total number of possible levels

we have the following concepts

$C = (1/kT_c) \log_2 N$ [bit/seg = baudio]	quantity of information
$C_{med} = k \sum P_i \log P_i^{-1}$ [bit/seg]	entropy or quantity of average information
$I = T_0 C$ [bit]	information
$I_{med} = T_0 C_{med}$ [bit]	average information

## MODULATION

### Generalities

It is to use the benefits of the high frequencies to transport to the small. In this we have the benefits of the decrease of the sizes of antennas, of the possibility of using little spectrum for a wide range of other useful spectra, of the codes, etc.

We will use the following terminology

$v_m(t)$	modulating signal (to modulate-demodulate)
$v_m(t) = V_m \cos \omega_m t$	harmonic of the band bases useful of $v_m(t)$ ( $\omega_m \ll \omega_c$ )
$m(t) = \alpha \cos \omega_m t$	relative harmonic of the band bases useful of $v_m(t)$

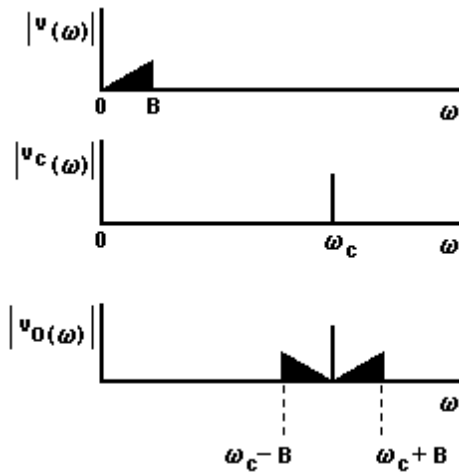
$v_c(t)$	carrier signal and also of sampling
$v_c(t) = V_c \cos \omega_c t$	carrier signal sine wave
$P_m = V_m^2 / 2$	normalized power of the harmonic of the band bases useful
$P_c = V_c^2 / 2$	normalized power of the harmonic of the carrier signal
$P_o$	normalized power of the harmonic of the modulating signal
$P_{BL}$	normalized power of the harmonic of the one lateral band
$v_o(t) = v_o \cos (\omega_c t + \theta)$	modulated signal
$\alpha$	normalized index of amplitude modulation ( $0 \leq \alpha = V_m/1[V] \leq 1$ )
$\beta$	index of frequency modulation (frequency or phase) ( $\beta = \Delta\omega_c/\omega_m$ )
$\Delta\omega_c$	variation of the carrier frequency ( $\Delta\omega_c = K_{OCV}V_m$ )
$K_{OCV}$	constant transfer of the OCV modulator of FM
$B$	B band bases useful of $v_m(t)$ that will contain a harmonic $\omega_m$ ( $B < \omega_c$ )
$\omega_m$	harmonic of the band bases useful of $v_m(t)$ ( $\omega_m < \omega_c$ )
$\omega_c = \partial\phi / \partial t$	fundamental or only harmonica of $v_c(t)$
$\theta$	initial phase of $v_o(t)$
$\phi = \omega_c t + \theta$	angle or phase instantaneous of $v_o(t)$

This way, we know that to the carrier  $v_c(t)$  already modulated as  $v_o(t)$  it will contain, in itself, three possible ways to be modulated

- modulating their amplitude ( $v_o$ ) (MA: *Amplitude Modulation*)
  - carrier and two lateral bands (MAC: *Complete Amplitude Modulation*)
  - two lateral bands (DBL: *Double Lateral Band*)
  - one lateral band (BLU: *Unique Lateral Band*)
  - piece of a lateral band (BLV: *Vestige Lateral Band*)
- modulating their instantaneous phase ( $\phi$ ) (M $\phi$ : *Angle Modulation*)
  - modulating their frequency ( $\omega_c = \partial\phi / \partial t$ ) (MF: *Frequency Modulation*)
  - modulating their initial phase ( $\theta \equiv \phi$ ) (MP: *Phase Modulation*)

Basically it consists on a process of transcription of the band bases  $B$  to the domain of the high frequency of carrier  $\omega_c$  —no exactly it is this way for big modulation indexes in  $M\phi$ . The following drawings explain what is said. When we have these lows index of modulation, then the form of the temporary equation of the modulated sign is practically the same in the MAC that in the  $M\phi$ ; that is

$$\begin{aligned}
 v_o(t) &= V_c \{ \cos (\omega_c t) + (\alpha/2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ] \} && \rightarrow \text{MAC} \\
 v_o(t) &= V_c \{ \cos (\omega_c t) + (\beta/2) [ \cos (\omega_c + \omega_m)t - \cos (\omega_c - \omega_m) ] \} && \rightarrow M\phi
 \end{aligned}$$

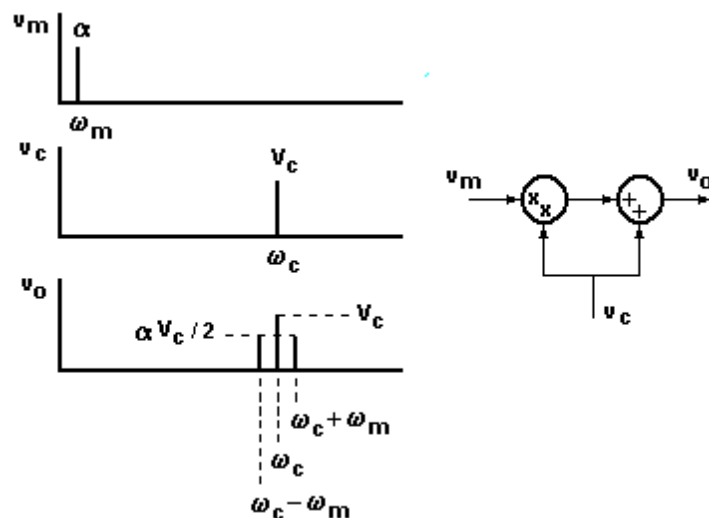


## Amplitude Modulation (MA)

### Generalities

The sign modulated obtained  $v_o(t)$  in the MAC it has the form of the product of the carrier with the modulating, more the carrier

$$\begin{aligned}
 v_o(t) &= m(t) v_c(t) + v_c(t) = (\alpha \cos \omega_m t) (V_c \cos \omega_c t) + V_c \cos \omega_c t = \\
 &= (\alpha V_c / 2) [\cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m)] + V_c \cos \omega_c t = \\
 &= V_c \{ \cos \omega_c t + (\alpha / 2) [\cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m)] \}
 \end{aligned}$$



and a power that depends not only on the modulation, but rather without this same the transmitter loses energy unsuccessfully

$$P_o = 2 P_{BL} + P_c = 2 [0,707(\alpha V_c / 2)]^2 + (0,707 V_c)^2 = V_c^2 (0,5 + 0,25\alpha^2) =$$



$$= P_c (1 + 0,5\alpha^2)$$

The way to transmit suppressing the carrier is denominated DBL, and when it is only made with an alone one we are speaking of BLU. Obviously in both cases there is not energy expense without modulation, but like it will be seen appropriately in the demodulation, the inconvenience is other, that is: it gets lost quality of the sign modulating. This way, respectively for one and another case

$$P_{o(DBL)} = 2 P_{BL} = 2 [0,707(\alpha V_c / 2)]^2 = 0,25 \alpha^2 V_c^2 = 0,5\alpha^2 P_c$$

$$P_{o(BLU)} = P_{BL} = 0,707(\alpha V_c / 2)^2 = 0,125 \alpha^2 V_c^2 = 0,25\alpha^2 P_c$$

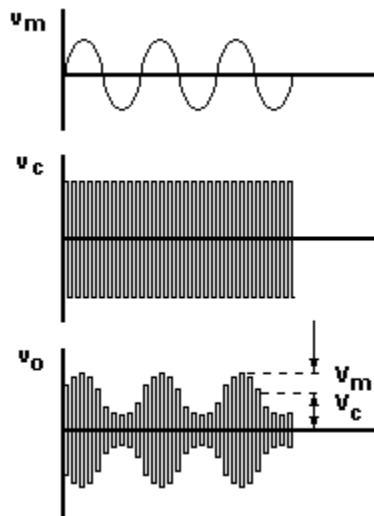
## Double lateral band and carrier (MAC)

### Generalities

It is drawn the form in that observes in an oscilloscope the modulated signal next. Here, in the transmission antenna, it is where finally the true and effective modulation index is measured (without normalizing it at 1 [V])

$$\alpha = V_m / V_c$$

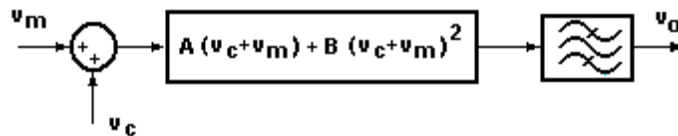
$$\begin{aligned} v_{o(t)} &= m(t) v_{c(t)} + v_{c(t)} = (\alpha \cos \omega_m t) (V_c \cos \omega_c t) + V_c \cos \omega_c t = \\ &= V_c \{ \cos \omega_c t + (\alpha / 2) [ \cos (\omega_c + \omega_m) t + \cos (\omega_c - \omega_m) t ] \} \end{aligned}$$



This modulation type, like it even transports energy without modulation and due to the faulty efficiency of the stages amplifiers in class A, it always uses on high level. That is to say that is only implemented in the output of power of the transmitter; but this doesn't prevent that for certain specific applications that are not surely those of ordinary transception, this is made in low level, that is to say in stages previous to that of output.

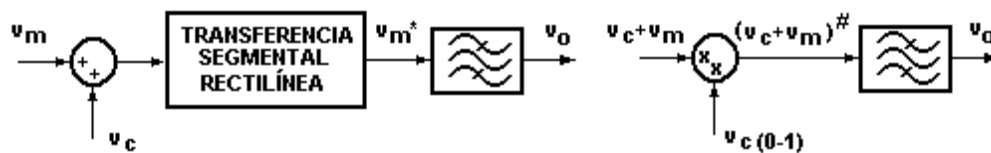
### Generation with quadratic and lineal element

The diagram is presented in the following figure. For example it can be implemented in low level with a JFET, and then with a simple syntony to capture the MA. Truly, it is this case a simplification of any other generality of transfer of more order, since they will always be generated harmonic.

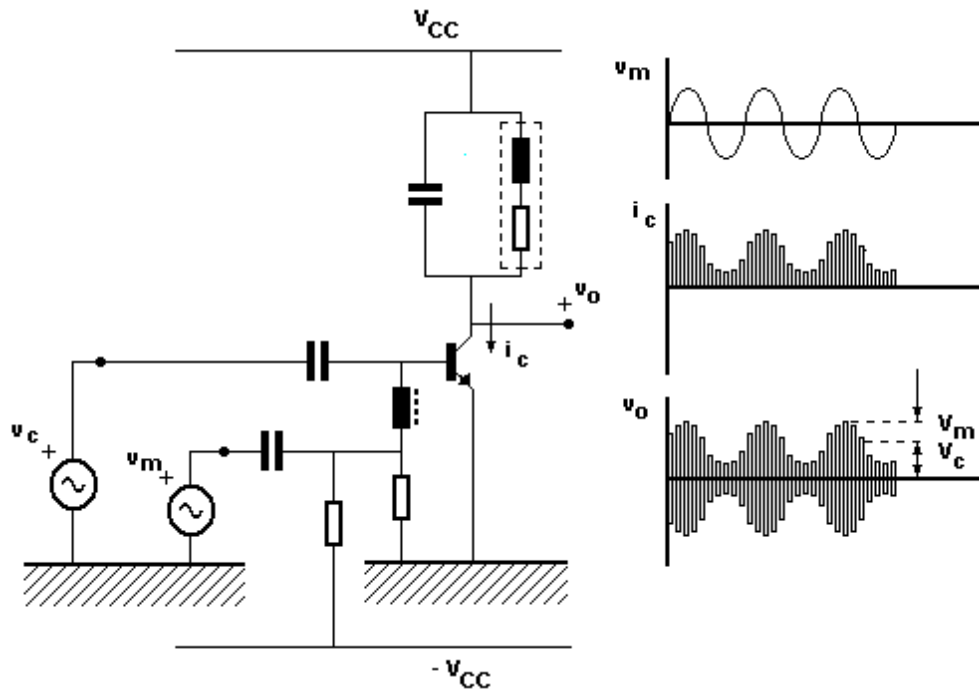


### Generation with element of rectilinear segment

The element of rectilinear segment is a transfer rectificator, that can consist on a simple diode. The diagram following sample the equivalence that has with a samplig, since  $(v_c + v_m)^{\#}$  they are the hemicycles of sine wave  $v_c$  changing their amplitude to the speed of the modulation; then the syntonized filter will capture the 2B necessary and centered to  $\omega_c$  to go out with the MAC.



The following implementation (amplifier in class C analyzed in chapter *amplifiers of RF*) it shows this design that, respecting the philosophy from the old valves to vacuum, they were projected the modulators.



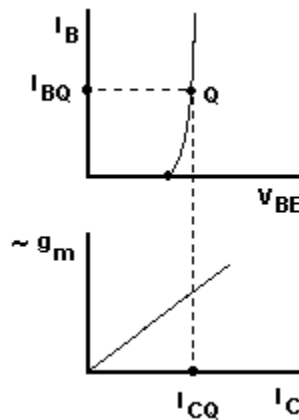
### Generation for product

Taking advantage of the transconductance of a TBJ a modulator of this type can either be implemented in low or on high level. In the trade integrated circuits dedicated to such an end for low level exist. Their behavior equation will be the following one, where a great amplitude of  $v_m$  changes the polarization of the TBJ to go varying its  $g_m$  that will amplify to the small signal of carrier  $v_c$

$$I_B \sim I_{BE0} e^{V_{BE}/V_T}$$

$$I_B = I_{BQ} (1 + \alpha \cos \omega_m t)$$

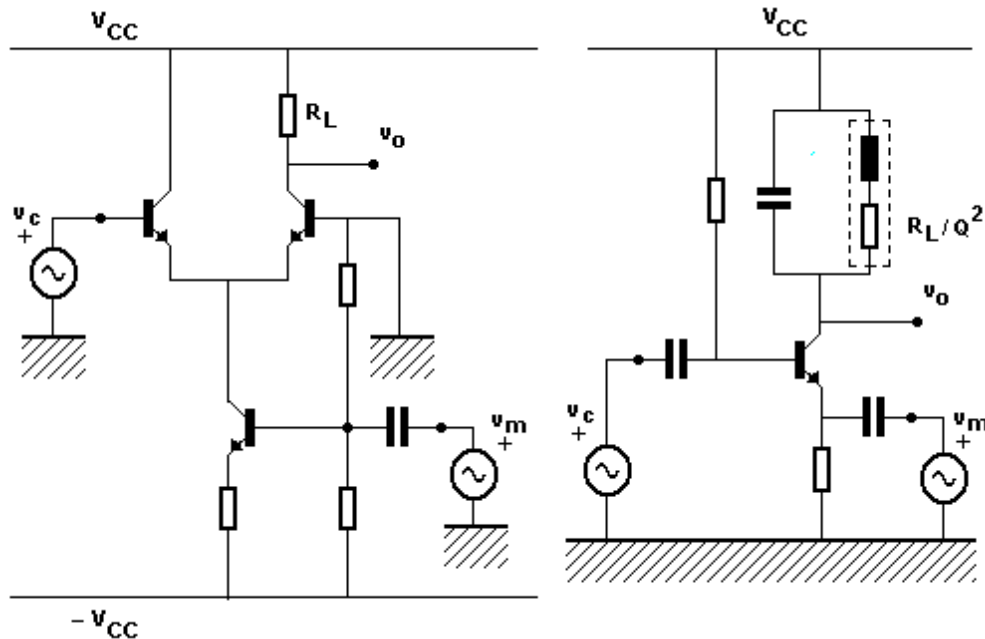
$$g_m = \partial I_C / \partial V_{BE} = I_{BE0} e^{V_{BE}/V_T} / V_T = I_B / V_T = I_C / \beta V_T = I_{CQ} (1 + \alpha \cos \omega_m t) / \beta V_T$$



$$A_v = v_{ce} / v_{be} = g_m R_L = I_{CQ} R_L (1 + \alpha \cos \omega_m t) / \beta V_T$$

$$v_{be} = v_c = V_c \cos \omega_c t$$

$$v_o = v_{ce} = A_v v_{be} = A_v (v_c + v_m) = (I_{CQ} R_L V_c / \beta V_T) (1 + \alpha \cos \omega_m t) \cos \omega_c t = (I_{CQ} R_L V_c / \beta V_T) \{ \cos \omega_c t + (\alpha/2) [\cos (\omega_c + \omega_m) t + \cos (\omega_c - \omega_m) t] \}$$

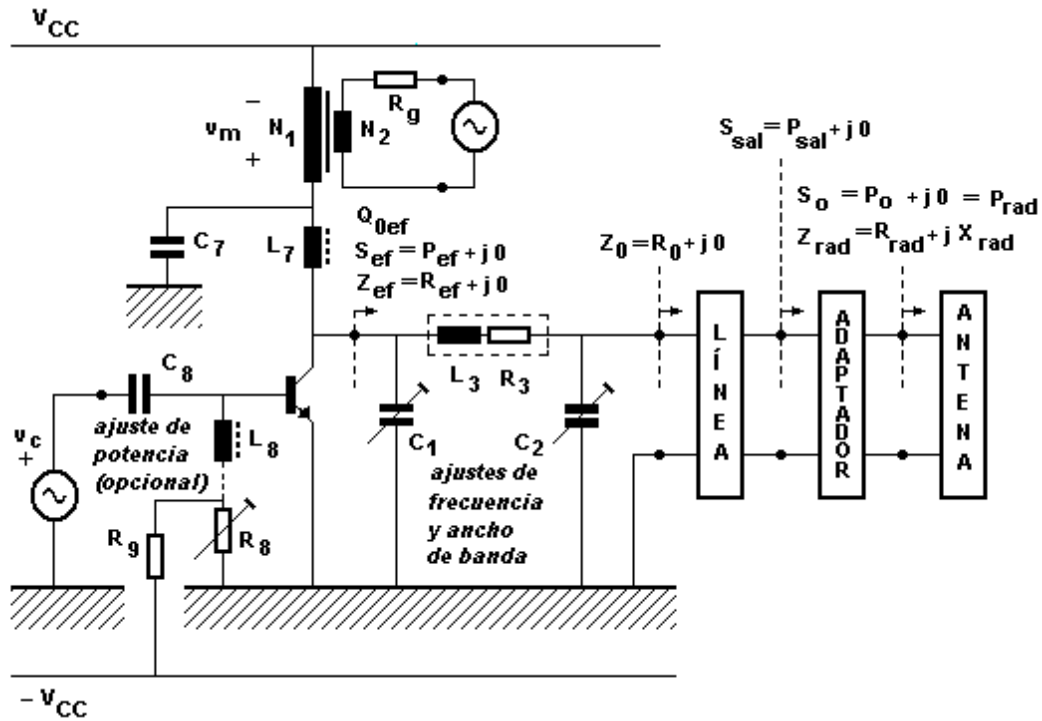


### Generation for saturation of the characteristics of a TBJ

We generate in the transmitter an oscillator that commutes a TBJ in the stage of power output. This frees the amplitude of the carrier oscillator before incorrect polarizations in the base of the class C. This way, we work with carriers that, when being squared, they contain a rich harmonic content and where the fundamental one will have the biggest useful energy and, therefore, it is the convenient one to syntonize as exit.

When we need to increase to the maximum the energy efficiency of this stage (f.ex.: in portable equipment) it will be necessary to adapt the exit stage to the propagation-antenna line (always among these they will be adapted to avoid faulty R.O.E.); it is not this way for powers of bigger magnitude of the common applications.

The circuit following sample a possible implementation in class A of a modulator of MAC disadapted (in a similar way it can be configured in class B type *Push-Pull* or *complementary symmetry* increasing the energy efficiency)



The pulses in class C (to see the analysis of this circuit in the chapter of *amplifiers of RF class C*) in base that are transmitted to the collector and that they will change their intensity with the angle of conduction of the diode base-emitter. To adjust this experimentally we have to the divider  $R_8$ - $R_9$  and the negative source  $V_{CC}$ . Truly this can be omitted if we saturate the TBJ; so this network is unnecessary for practical uses.

We can denominate as *effective voltage* of source  $V_{CCef}$  to which is disacoupled for  $C_7$  in RF

$$V_{CCef} = V_{CC} + v_m$$

$$1 / BC_7 \gg R_g (N_1/N_2)^2$$

$$1 / \omega_c C_7 \ll \omega_c L_7$$

The antenna will have a certain radiation impedance complex  $Z_{rad}$  if it doesn't fulfill the typical requirements, and that it will be able to be measured and adapted according to what is explained in the chapter of *antennas and transmission lines*. Mainly, being portable, their magnitude constantly changes for the effect of the physical environment.

The circuit syntony p that has been chosen presents two important advantages in front of that of simple syntony; that is: it allows us to adjust the adaptation of impedance as well as the band width. On the other hand, in the filter of simple syntony, one of the two things is only possible. To analyze this network we can simplify the things and to divide it in two parts like sample the following figure

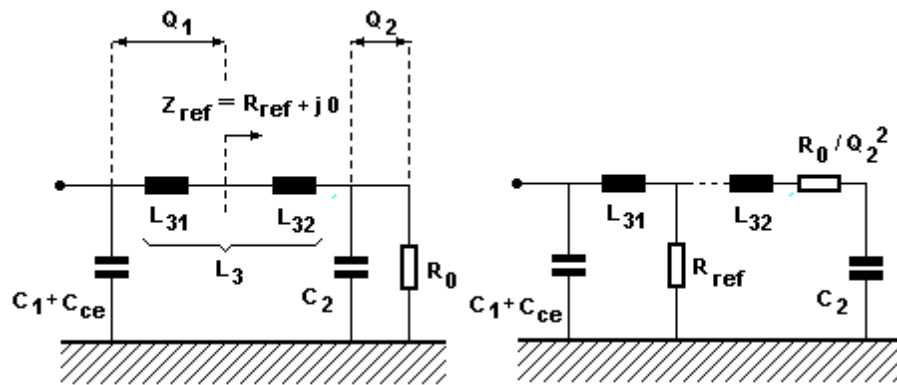
$$\omega_c = 1 / [L_{31}(C_1 + C_{ce})]^{1/2} = 1 / (L_{32}C_2)^{1/2}$$

$$Q_1 = \omega_c L_{31} / R_{ref}$$

$$Q_2 = \omega_c C_2 R_0$$

$$R_{ref} = R_0 / Q_2^2$$

$$P_{\text{salmax}} = V_{CC}^2 (1 + 0,5 \alpha^2) / 2 R_{\text{ref}}$$



where

$$Q_{\text{0ef}} = Q_1 = \omega_c R_0 [C_2^2 / (C_1 + C_{be})]$$

$$R_{\text{ef}} = R_{\text{ref}} Q_1^2 = R_0 [C_2 / (C_1 + C_{be})]^2$$

and the limit of this simple syntony will be given by the band width to transmit  $2B$  and the selectivity that it is needed (although it is not used in the practice, filters of *maximum plain* can be used, of *same undulation*, etc.)

$$Q_{\text{0ef}} \leq \omega_c / 2B$$

## Design

Be the data

$$R_g = \dots \quad N_1/N_2 = \dots \quad f_{\text{mmin}} = \dots \quad f_{\text{mmax}} = \dots \ll f_c = \dots \quad P_{\text{salmax}} = \dots \quad \alpha = \dots \leq 1$$

We adopt a TBJ and of the manual we obtain

$$C_{ce} = \dots$$

$$V_{\text{CEADM}} = \dots$$

what will allow us to choose a source

$$V_{CC} = \dots < V_{\text{CEADM}} / 2$$

We subsequently also adopt an antenna and adapted line (one has examples of this topic in the chapter of *antennas and transmission lines*)

$$Z_0 = R_0 = \dots$$

Now, for the equations seen we obtain

$$R_{ef} = V_{CC}^2 (1 + 0,5 \alpha^2) / 2 P_{salm\max} = \dots$$

$$Q_{0ef} = \dots \leq \omega_c / 2(\omega_{m\max} - \omega_{m\min})$$

$$C_1 = (Q_{0ef} / \omega_c R_{ref}) - C_{ce} = \dots$$

$$C_2 = (C_1 + C_{ce}) (R_{ref} / R_0)^{1/2} = \dots$$

$$L_3 = L_{31} + L_{32} = [ (1 / C_1 + C_{ce}) + (1 / C_2) ] / \omega_c^2 = \dots$$

and for not altering the made calculations we verify

$$R_3 = \dots \ll R_{ref} = 1 / R_0 (\omega_c C_2)^2$$

As for the filter of RF

$$C_7 = \dots \ll 1 / \omega_{m\max} R_g (N_1 / N_2)^2$$

$$L_7 = \dots \gg 1 / \omega_c^2 C_7$$

Subsequently, with the purpose of getting the auto-polarization in the base, we adopt (for a meticulous calculation of  $R_8$ - $R_9$  to appeal to the chapter of *amplifiers of RF class C*)

$$C_8 = \dots$$

and we estimate (the best thing will be to experience their value)

$$R_8 = \dots \gg 2\pi / \omega_c C_7$$

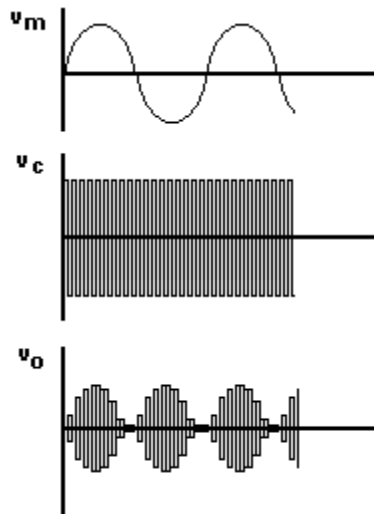
## Double lateral band without carrier (DBL)

### Generalities

The sign modulated obtained  $v_{o(t)}$  in DBL it has only the product of the carrier with the modulating

$$v_{o(t)} = m_{(t)} v_{c(t)} = (\alpha \cos \omega_m t) (V_c \cos \omega_c t) = (\alpha V_c / 2) [ \cos (\omega_c + \omega_m) t + \cos (\omega_c - \omega_m) t ]$$

The form of the signal modulated for an index of modulation of the 100 [%] is drawn subsequently.



### Generation for product

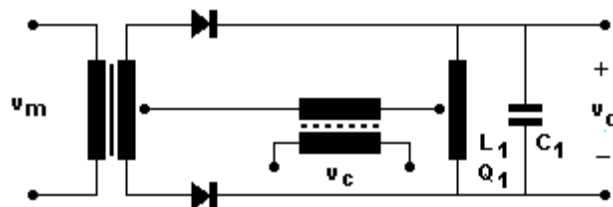
To multiply sine waves in RF is difficult, so a similar artifice is used. Taking advantage of that demonstrated in the precedent equations, it takes a carrier sine wave and it clips it to him transforming it in square wave. They appear this way harmonic odd that, each one of them, will multiply with the sign modulating generating a DBL for the fundamental one and also for each harmonic. Then it is syntonized, in general to the fundamental that is the one that has bigger amplitude.

This way, if we call «n» to the order of the odd harmonic (n = 1 are the fundamental) this harmonic content can be as

$$\begin{aligned}
 v_{CLIM}(\omega) &= \int_{-T_c/4}^{T_c/4} V_c e^{j n \omega_c t} \partial t = (V_c \pi / \omega_c) \text{sen} (n \omega_c T_c / 4) / (n \omega_c T_c / 4) \\
 &= (V_c \pi / \omega_c) \text{sen} (n \pi / 2) / (n \pi / 2) \\
 Q_{(n \omega_c)} &\leq n \omega_c / [(n \omega_c + \omega_m) - (n \omega_c - \omega_m)]
 \end{aligned}$$

Subsequently we show a possible implementation. The carrier becomes present in the secondary polarizing in direct and inverse to the diodes as if they were interruptors. This way then, sampling is achieved to the modulating and then to filter the fundamental in

$$\begin{aligned}
 \omega_c &= 1 / L_1 C_1 \\
 Q_1 &\geq \omega_c / 2B
 \end{aligned}$$



We can also make use of integrated circuits dedicated to such an end. The circuit following



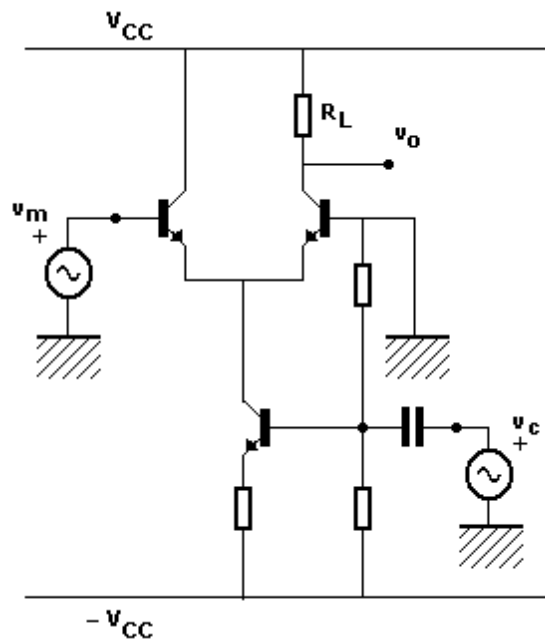
sample the operation approach already explained previously when seeing the topic of *generation of MAC for product*, but changing in the fact to invest the signals; here the great amplitude is due to  $v_c$  that changes the polarization of the TBJ with the purpose of going varying its  $g_m$  that will amplify to the small sign of modulation  $v_m$ .

$$g_m = I_{CQ} (1 + \cos \omega_c t) / \beta V_T$$

$$A_v = v_{ce} / v_{be} = g_m R_L = I_{CQ} R_L (1 + \cos \omega_c t) / \beta V_T$$

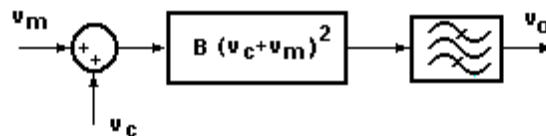
$$v_{be} = v_m = \alpha V_c \cos \omega_m t$$

$$\begin{aligned} v_o = v_{ce} &= A_v v_{be} = A_v v_m = (\alpha I_{CQ} R_L V_c / \beta V_T) (1 + \cos \omega_c t) \cos \omega_m t = \\ &= (I_{CQ} R_L V_c / \beta V_T) \{ \alpha \cos \omega_m t + (\alpha/2) [\cos (\omega_c + \omega_m) t + \cos (\omega_c - \omega_m) t] \} \end{aligned}$$



### Generation for quadratic element

The diagram is presented in the following figure. Only a transfer that distorts without lineal component will guarantee that there is not carrier.



For example it can be implemented in low power with a JFET, and then with a simple syntony to capture the DBL. Their behavior equations are the following

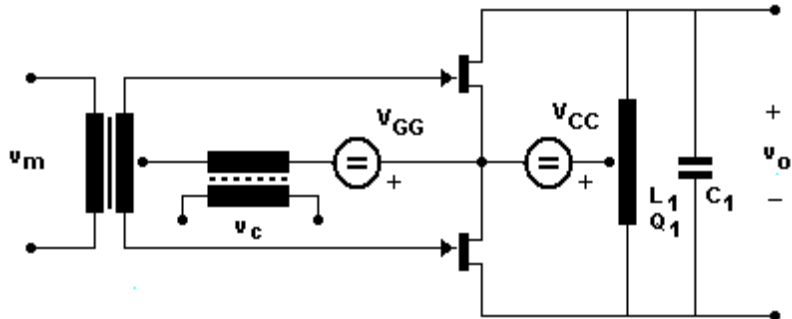
$$\omega_c = 1 / L_1 C_1$$

$$Q_1 \geq \omega_c / 2B$$

$$v_{gs1} = v_m + v_c - V_{GG}$$

$$v_{gs2} = -v_m + v_c - V_{GG}$$

$$\begin{aligned} v_o &= (i_{d1} - i_{d2}) = I_{DSS} \omega_c L_1 Q_1 [ (1 + v_{gs2}/V_P)^2 - (1 - v_{gs1}/V_P)^2 ] = \\ &= (8I_{DSS} \omega_c L_1 Q_1 / V_P^2) [ (V_{GG} - V_P/2) v_m - v_c v_m ] \rightarrow (8I_{DSS} \omega_c L_1 Q_1 / V_P^2) v_c v_m \end{aligned}$$

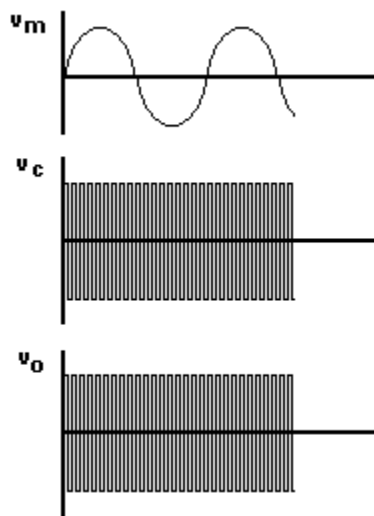


### Unique lateral band (BLU)

#### Generalities

This is a special case of the DBL where one of the bands is filtrate, or a mathematical method is used to obtain it. For simplicity of the equations we will work with the inferior band. The result of the modulation is kind of a MA and combined  $M\phi$ ; subsequently we show their temporary form

$$v_o(t) = (\alpha V_c / 2) \cos(\omega_c - \omega_m) t$$



#### Generation for filtrate

This generation is made firstly as DBL and then with a filter the wanted lateral band is obtained. The inconvenience that has this system is in the selectivity and plain of the filter; for this

reason the approaches are used seen in the filters LC of *maximum plain, same undulation, simple syntonies* producing the selectivity with crystals, or also with *mechanical filters*.

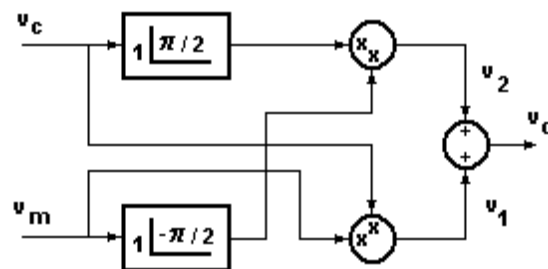
### Generation for phase displacement

The following implementation, among other variants of phase displacements, shows that we can obtain BLU

$$v_1 = v_m v_c = (\alpha \cos \omega_m t) (V_c \cos \omega_c t)$$

$$v_2 = (\alpha \sin \omega_m t) (V_c \sin \omega_c t)$$

$$v_0 = v_1 + v_2 = (\alpha V_c \cos (\omega_c - \omega_m) t)$$

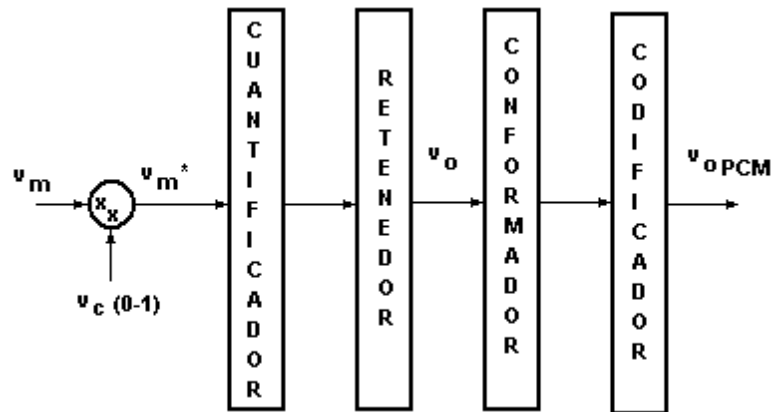


Now then, to displace an angle anyone of the carrier is simple, but to obtain it for an entire band bases B is difficult. Consequently this modulator here finds its limitations, and it is common for this reason to be changed the displacements implementing them otherwise.

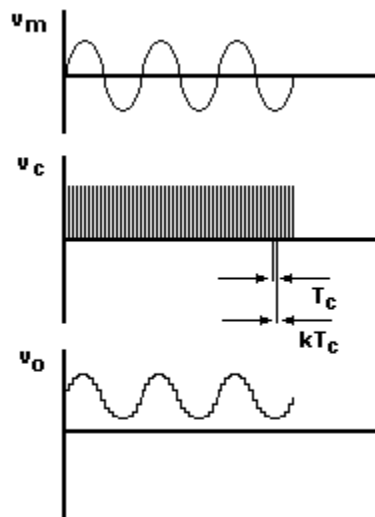
### Generation for code of pulses (PCM)

The code here is like in the following system, where they are

$n$	<i>number of quantification levels</i>
$m$	<i>number of pulses used to take place «n»</i>
$N = n^m$	<i>effective number of levels</i>



being the sign in an oscilloscope the following (in  $v_{oPCM}$  we have only pulses coded without change of amplitude)

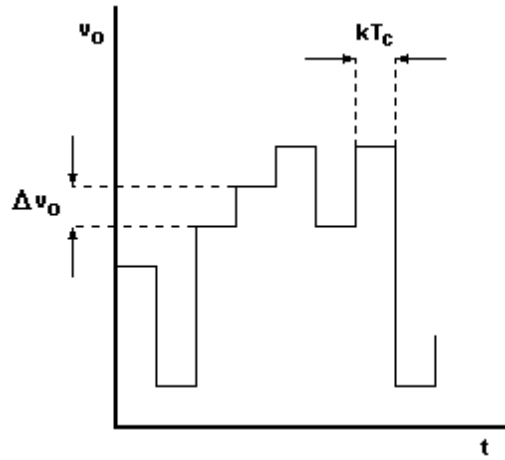


The quantity of information that is used is

$$C = (1/kT_c) \log_2 N = (m/kT_c) \log_2 n$$

and the differences of the system produce a *distortion* that (mistakenly) it is denominated *quantification noise*  $\sigma [V]$ , and being then their magnitude  $\Delta v_o$  will be limited in a proportion  $K$  such that

$$\Delta v_o = K \sigma$$



and knowing that the amplitudes go of  $0 \leq i \leq (n-1)\Delta v_o$ , in steps of magnitude  $\Delta v_o$  like it was said, their value quadratic average is

$$P_{\sigma} = (1/n) \sum_{i=0}^{n-1} (i \Delta v_o)^2 = K^2 \sigma^2 (n-1) (2n-1) / 6$$

and on the other hand as the value average of voltage of the pulses with same probability of to happen is  $K\sigma(n-1)/2$  then the power of the useful signal is

$$S = P_{\sigma} - [K\sigma(n-1)/2]^2 = K^2 \sigma^2 (n^2 - 1) / 12$$

and ordering it as relationship *signal to noise* (S/R) of PCM

$$(S/R)_{PCM} = K^2 (n^2 - 1) / 12$$

that finally replacing it in the quantity of information is obtained

$$C = (m/2kT_c) \log_2 [1 + 12 (S/R)_{PCM} / K^2]$$

We can also be interested in the relationship  $(S/R)_{PCM}$  that has with respect to N. For we call it *sampling error* at the level of decision of the quantification and that it will have a maximum in

$$\varepsilon_{\max} = \Delta v_o / 2$$

and we outline the power of quantification noise normalized at 1 [Ω]

$$R = (1/\Delta v_o) \int_{\varepsilon_{\max}}^{\varepsilon_{\max}} \varepsilon^2 \partial \varepsilon = \Delta v_o^2 / 2$$

as well as the relationship signal to noise voltages

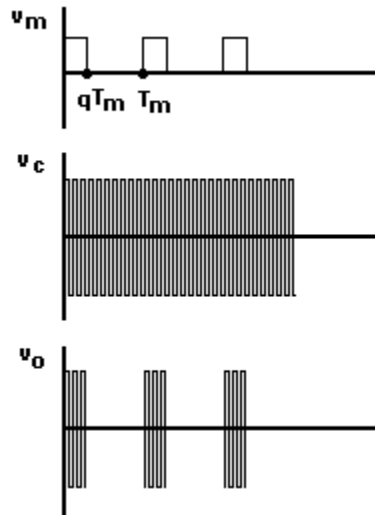
$$(s/r)_{PCM} = (N \Delta v_o) / R^{1/2}$$

so that finally we find

$$(S/R)_{\text{PCM}} = (s/r)_{\text{PCM}}^2 = 12 \text{ N}^2$$

## Generation OOK

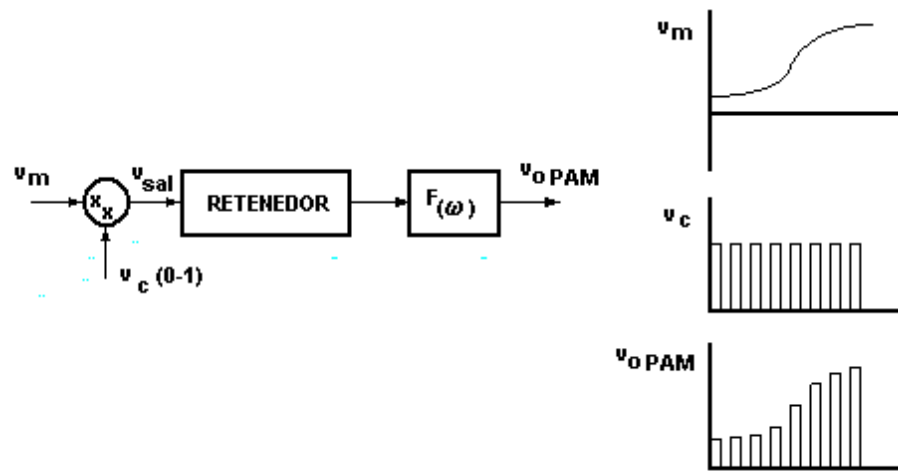
Here it is modulated binarily to the carrier. The effect is shown in the following figure, where we see that the band width for the transmission is double with respect to that of the pulses, that is to say, practically  $2(\omega_c - \pi/qT_m)$ .



## Generation PAM

We call with this name to the modulation for the amplitude of the pulse. The system consists on sampling to the sign useful  $v_m$  to  $v_c$  and to make it go by a filter low-pass  $F(\omega)$  retainer like a period monostable  $kT_c$  and of court in  $\omega_F$ . This filter will allow to pass the sampling pulses to rhythm  $\omega_c$

$$\omega_F > \omega_c$$



## Chap. 26 Demodulación of Amplitude

### Generalities

#### Double lateral band and carrier (MAC)

##### Generalities

Obtaining with quadratic element

Obtaining with element of segment rectilinear

##### Design

#### Double lateral band with suppressed carrier (DBL)

##### Generalities

Obtaining for the incorporation of asynchronous carrier

Obtaining for product

#### Unique lateral band (BLU)

##### Generalities

Obtaining for the incorporation of asynchronous carrier

Obtaining for lineal characteristic

Obtaining for quadratic characteristic

Obtaining for product

Obtaining for the incorporation of synchronous carrier

### Pulses

##### Generalities

Obtaining of coded pulses (PCM)

Obtaining of PAM

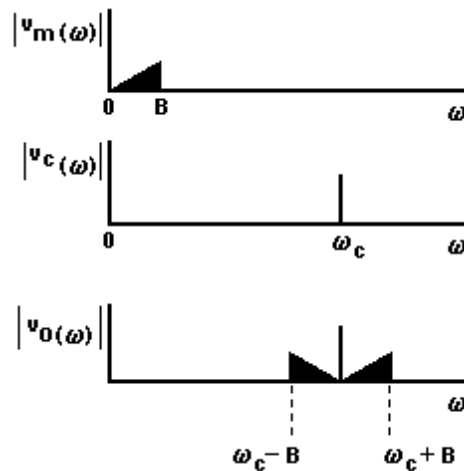
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### Generalities

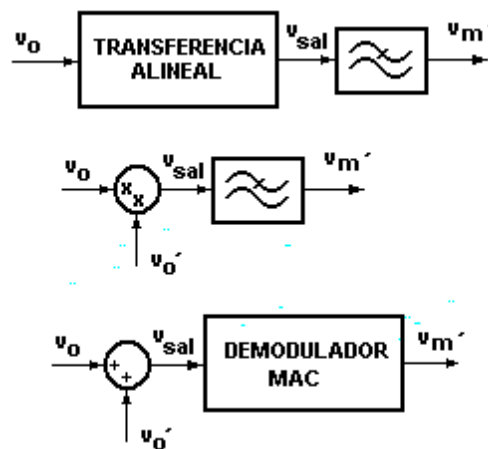
Basically it consists the demodulation a transcription of the band it bases 2B of the domain from the high frequency of carrier  $\omega_c$  to the low B. The following drawings explain what is said. That is, like it was said in the previous chapter

$$\begin{aligned}v_o(t) &= v_o (1 + \alpha \cos \omega_m t) = \\&= V_c (\alpha \cos \omega_m t \cos \omega_c t + \cos \omega_c t) = \\&= V_c \{ \cos (\omega_c t) + (\alpha/2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ] \} \quad \rightarrow \text{MAC} \\v_o(t) &= (\alpha V_c / 2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ] \quad \rightarrow \text{DBL} \\v_o(t) &= (\alpha V_c / 2) \cos (\omega_c - \omega_m) \quad \rightarrow \text{BLU}\end{aligned}$$





The criteria of the demodulation of amplitude are, basically, three: first, to go the modulation by an element non-linear (rectilinear segment, quadratic, etc.). Second, to mix it with a new local carrier, in such a way that in both cases a harmonic content will take place and, surely, a band bases B on low frequencies that then one will be able to obtain a filter low-pass. Third, reinjecting the carrier when it lacks and then to treat her classically.



### Double lateral band and carrier (MAC)

#### Generalities

We repeat their characteristic equation

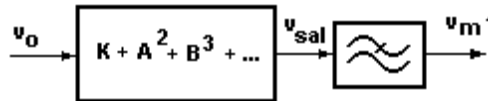
$$\begin{aligned}
 v_o(t) &= v_o (1 + \alpha \cos \omega_m t) = \\
 &= V_c (\alpha \cos \omega_m t \cos \omega_c t + \cos \omega_c t) = \\
 &= V_c \{ \cos (\omega_c t) + (\alpha/2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ] \}
 \end{aligned}$$

## Obtaining with quadratic element

Subsequently this system is drawn. Polarizing a no-linear device, like it can be a diode among 0,6 to 0,7 [V], an area will exist that is practically a quadratic transfer

$$v_{sal} = K v_o + A v_o^2 + \dots = A V_c^2 (\alpha \cos \omega_m t \cos \omega_c t + V_c \cos \omega_c t)^2 + \dots$$

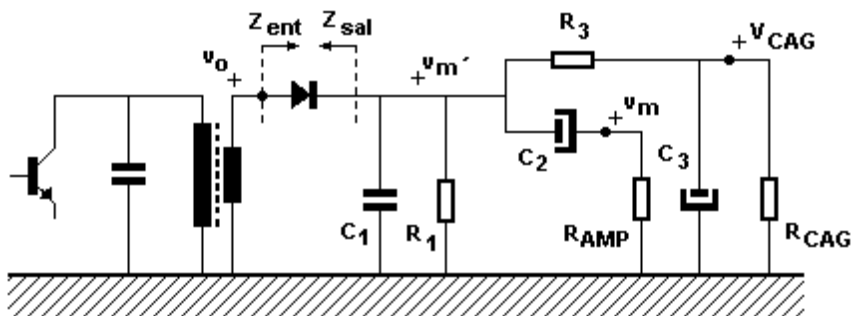
$$v_m' = A V_c^2 \alpha \cos \omega_m t$$



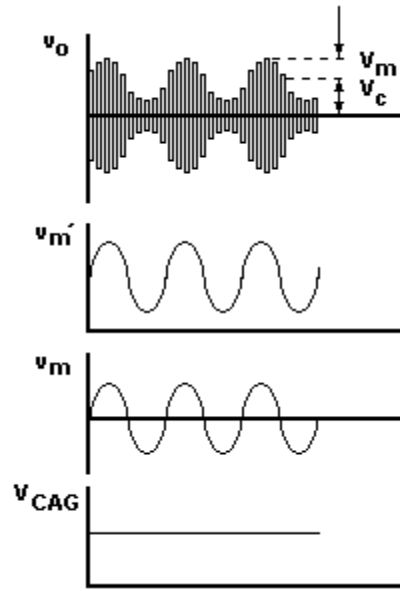
Without being common for applications of low RF, yes on the other hand it is used in microwaves.

## Obtaining with element of segment rectilinear

This obtaining is the most common. The output is rectified and filtered, according to the circuit that we use, so much to get the signal useful modulating  $v_m'$  of AF on an input resistance to the amplifier following  $R_{AMP}$ , like for a continuous in the automatic control of gain CAG of the receiver and that we call  $V_{CAG}$ .



The following graphs express the ideal voltages in each point.



Indeed, for the fundamental (harmonic  $n = 1$ ) it is

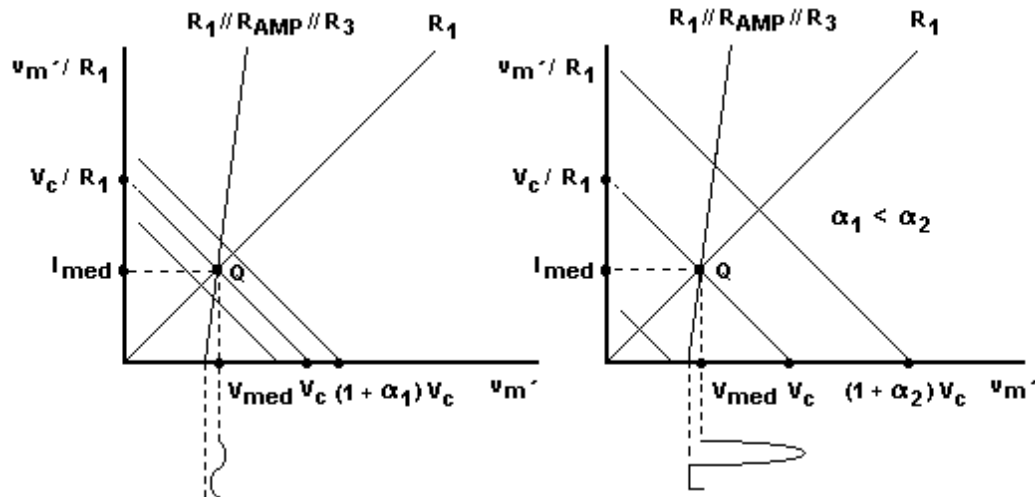
$$\begin{aligned}
 v_{m'} &= v_{sal(n=1)} = \int v_o \partial t = \\
 &= \int V_c \{ \cos(\omega_c t) + (\alpha/2) [ \cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t ] \} \partial t = \\
 &= V_c \{ \sin(\omega_c t) / \omega_c + (\alpha/2) [ \sin(\omega_c + \omega_m)t / (\omega_c + \omega_m) + \sin(\omega_c - \omega_m)t / (\omega_c - \omega_m) ] \} \sim \\
 &\sim V_c / \omega_c \{ \sin(\omega_c t) + (\alpha/2) [ \sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t ] \} = \\
 &= V_c (1 + \alpha \cos \omega_m t) \sin(\omega_c t) / \omega_c \\
 v_{sal(n=0)} &= V_c (1 + \alpha \cos \omega_m t)
 \end{aligned}$$

Truly this demodulator not prevents to have 100 [%] of modulation in the theory —not of this way in the practice for the curve of the rectifier. When the work point Q rotate by the modulation, like they show the graphs (that have been idealized as straight line), a *cutting of the picks* takes place. We obtain the condition

$$I_{med} \sim V_c / R_1 = \alpha V_c / (R_1 // R_{AMP} // R_3)$$

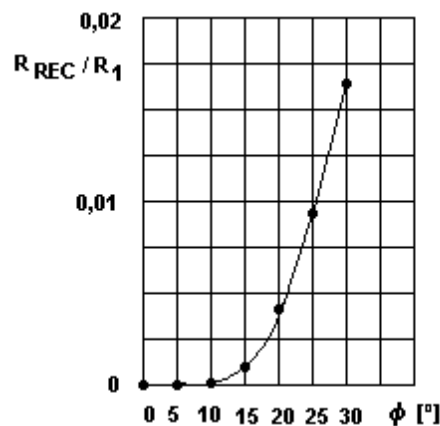
and of here

$$\alpha \leq 1 / [1 + R_1 (1/R_{AMP} + 1/R_3)] = R_{CONTINUA} / R_{ALTERNA}$$



To design the syntony of the filter simple precedent, receiving an *intermediate frequency* reception FI, it is important to know the input impedance to the circuit demodulator  $Z_{ent}$ . With this end we analyze it when the rectifier of half wave possesses a conduction angle  $\phi$  (in the chapter of *sources without stabilized* it called  $\alpha$ ) and a static resistance to the point of dynamic work  $R_{REC}$  (that truly varies with the amplitude of the modulation) and that we can consider average, as well as a resistance reflected by the transformer that, being reducer, it will design it to him preferably of worthless magnitude (this resistance is the simplification of the total series  $R_S$  in the chapter of *sources without stabilizing*). Its magnitude can approach theoretically as

$$R_{REC} \approx R_1 (\text{tg } \phi - \phi) / \pi$$



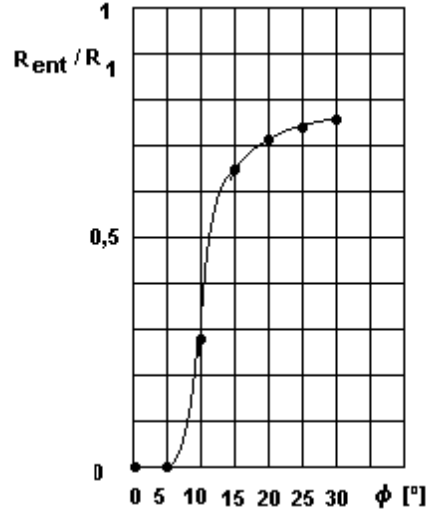
then without modulation (is to say for small  $\alpha$ ) we can find the current average that it enters to the rectifier supposing that in their cathode a continuous voltage it exists practically of magnitude peak  $V_c$

$$I_{med} \sim (2/\pi) \int_0^\phi (v_o - V_c)/R_{REC} \partial \omega_c t = V_c (\text{sen } \phi - \phi \cos \phi) / \pi R_{REC} \sim$$

$$\sim V_c (\phi - \phi \cos \phi) / \pi R_{REC} = V_c \phi (1 - \cos \phi) / \pi R_{REC}$$

approach made for  $\phi < 30^\circ$  that are the practical cases. This allows then to outline

$$Z_{ent} = R_{ent} = V_c / I_{med} = \pi R_{REC} / \phi (1 - \cos \phi) = R_1 (\tan \phi - \phi) / \phi (1 - \cos \phi)$$



expression that is simplified for high detection efficiencies  $\eta$  and relationship  $R_1 / R_{REC}$  bigger than some 10 times, if we plant simply that we don't have energy losses practically in the diode and we equal this power that it enters to the system with the continuous that obtains in null modulation (it lowers)

$$P_{ent} = (0,707 V_c)^2 / R_{ent} \sim (\eta V_c)^2 / R_1$$

then

$$R_{ent} \sim R_1 / 2\eta^2$$

An useful parameter of the demodulator is its *detection efficiency*  $\eta$ . We define it as the voltage continuous average that we obtain to respect the magnitude pick of the carrier without modulating

$$\eta = V_{med} / V_c$$

for that that if we keep in mind the previous expressions

$$V_{med} = I_{med} R_1 = V_c R_1 \phi (1 - \cos \phi) / \pi R_{REC}$$

$$R_1 / R_{REC} \approx \pi / (\tan \phi - \phi)$$

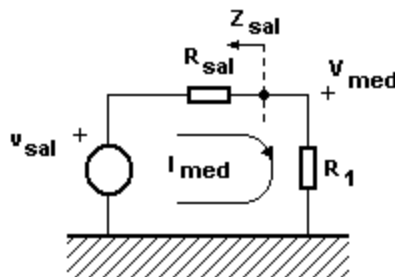
It is

$$\eta = R_1 \phi(1 - \cos\phi) / \pi R_{REC} = \phi(1 - \cos\phi) / (\tan\phi - \phi) = R_{REC} / R_{ent}$$

Truly these equations are very theoretical and distant of the practice. An efficient solution will be to consult the empiric curves of Shade, some of them drawn in the chapter of *sources without stabilizing*.

We can want to know what we see to the output of the rectifier, that is to say the output impedance  $Z_{sal}$  and the voltage available  $v_{sal}$  —for the useful band and not the RF. With this end we outline again

$$\begin{aligned} I_{med} &= V_c (\sin\phi - \sin\phi \cos\phi) / \pi R_{REC} \sim V_c (\sin\phi - \phi) / \pi R_{REC} = \\ &= (V_c \sin\phi / \pi R_{REC}) - (V_c \phi / \pi R_{REC}) \\ I_{med} &= V_{med} / R_1 \sim V_c / R_1 \end{aligned}$$



that we will be able to equal and to obtain

$$V_c \sin\phi / \pi R_{REC} = (V_c / R_1) + (V_c \phi / \pi R_{REC})$$

and now working the equation gets

$$\begin{aligned} v_{sal} &= V_c \sin\phi / \phi = I_{med} (R_1 + R_{sal}) \\ R_{sal} &= \pi R_{REC} / \phi \end{aligned}$$

The condenser of filter  $C_1$  is critical. It should complete three conditions, that is: first, it should be sufficiently big as to filter the RF and that we could simplify with to the following expression

$$1 / \omega_c C_1 \ll R_1$$

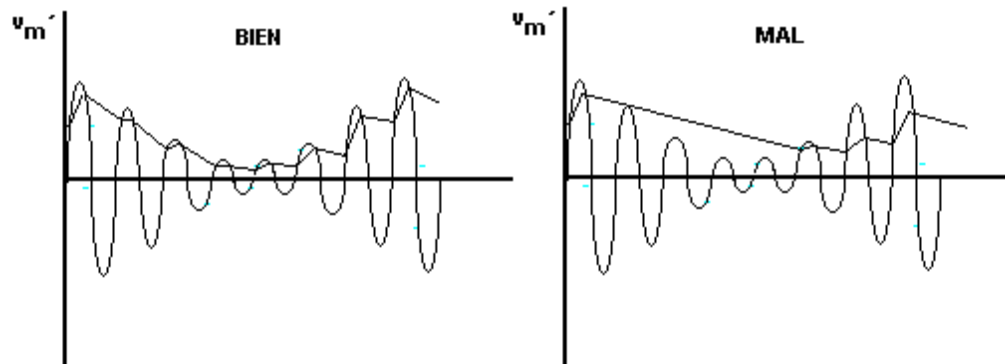
and the sufficiently small as for not filtering the useful band, or to take advantage of it so that it produces the cut frequency in the high frequencies of the band bases having present the resistance of equivalent output  $R_{sal}$  of the rectifier

$$1 / BC_1 = (R_1 // R_{AMP} // R_3) + R_{sal}$$

being the third requirement that, due to the alineality of the system that is discharged without

producing a *diagonal cutting* to the useful signal

$$1 / BC_1 \gg R_1$$



As for the capacitor of it couples  $C_2$ , if we observe that we go out with a voltage in  $R_1$  of worthless resistance, this can be designed for example as so that it cuts in low frequencies

$$1 / \omega_{\min} C_2 = R_{AMP}$$

and at the  $C_3$  as so that it integrates the voltage dedicated to the CAG with the condition

$$1 / \omega_{\min} C_3 \gg R_3 // R_{CAG}$$

keeping in mind that the speed of its tracking in the receiver will be the maximum limit.

## Design

Be the data

$$f_c = \dots \quad f_{\max} = \dots \quad f_{\min} = \dots \quad V_c = \dots \quad z_{\max} = \dots$$

$$R_{AMP} = \dots \quad R_{CAG} = \dots \quad V_{CAG} = \dots$$

We adopt a potentiometer in  $R_1$  to regulate the gain of the receiver in such a way that their magnitude doesn't affect the precedent equations of design (if the modulation is of AF it should be logarithmic)

$$R_{AMP} \gg R_1 = \dots \ll R_{CAG}$$

We can estimate the condenser of filter  $C_1$  keeping in mind the carried out design equations but, truly, it will be better their experimentation. We will only approach to their value with the curves of Shade (to see their abacus in the *sources chapter without stabilizing*)

$$C_1 = \dots$$

Subsequently we find the rest of the components

$$C_2 = 1 / \omega_{\text{mmin}} R_{\text{AMP}} = \dots$$

$$R_3 = R_{\text{CAG}} (V_c / V_{\text{CAG}} - 1) = \dots$$

$$C_3 = \dots \gg (R_3 // R_{\text{CAG}}) / \omega_{\text{mmin}}$$

## Double lateral band with suppressed carrier (DBL)

### Generalities

We repeat their characteristic equation

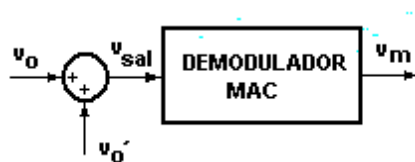
$$v_o(t) = (\alpha V_c / 2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ]$$

Here the demodulation philosophy is in reinjecting the carrier. The problem is in that she is never in true phase with the original of the transmitter, because all the oscillators are never perfect. For example, a displacement of a one part in a million, implies a one digit of cycle of phase displacement in a carrier of 1 [MHz].

We will call to this displacement of phases among carriers  $\psi = \psi(t)$ , and we will have present that changes to a speed that can be the audible.

### Obtaining for the incorporation of asynchronous carrier

Those DBL is excepted with synchronous modulation that, for this case, the modulation contains an exact reference of synchronism .



We can see this way that for this case, calling  $\psi$  to the displacement of phases among carriers and obtaining a previous adjustment in the receiver (to simplify the calculations) to obtain the same carrier amplitude  $V_c$

$$v_o(t) = (\alpha V_c / 2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ]$$

$$v_o(t)' = V_c \cos (\omega_c t + \psi)$$

it is



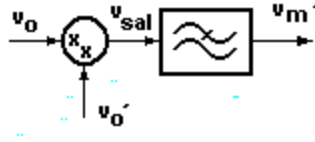
$$\begin{aligned}
v_{sal} &= v_{o(t)} + v_{o(t)}' = V_c [ \alpha \cos \omega_m t \cos \omega_c t + \cos (\omega_c t + \psi) ] = \\
&= V_c' (1 + \alpha' \cos \omega_m t) \\
V_c' &= V_c \cos (\omega_c t + \psi) = V_{c(\psi)} = V_{c(t)} \\
\alpha' &= \cos \omega_c t / \cos (\omega_c t + \psi) = \alpha'_{(\psi)} = \alpha'_{(t)}
\end{aligned}$$

where it is distinguished the deficiency of the system mainly in the amplitude of the carrier like  $V_{c(\psi)}$ , since their phase displacement  $(\omega_c t + \psi)$  it won't affect in a later demodulation of MAC.

### Obtaining for product

The operative is the following one. The same as in all asynchronous demodulation, this system continues suffering of the inconvenience of the quality of the transception

$$\begin{aligned}
v_{o(t)} &= (\alpha V_c / 2) [ \cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m) ] \\
v_{o(t)}' &= \cos (\omega_c t + \psi) \\
\psi &= \psi(t)
\end{aligned}$$



and therefore

$$\begin{aligned}
v_{sal} &= v_{o(t)} v_{o(t)}' = V_c [ \alpha \cos \omega_m t \cos \omega_c t \cdot \cos (\omega_c t + \psi) ] = \\
&= V_c' [ \cos (2\omega_c t + \psi) + \cos \omega_m t ] \\
V_c' &= (\alpha V_c \cos \psi) / 2 = V_{c(\psi)} = V_{c(t)}
\end{aligned}$$

### Unique lateral band (BLU)

#### Generalities

We repeat their characteristic equation

$$v_{o(t)} = (\alpha V_c / 2) \cos (\omega_c - \omega_m)$$

Here the demodulation approach is the same as in DBL for the injection of the carrier. The problem is in that she is never in true phase with the original of the transmitter, because all the oscillators are never perfect. For example, a displacement of a one part in a million, implies a one digit of cycle of phase displacement in a carrier of 1 [MHz].

We will call to this displacement of phases among carriers  $\psi = \psi(t)$ , and we will have present that changes to a speed that can be the audible.

## Obtaining for the incorporation of asynchronous carrier

### Obtaining for lineal characteristic

Here we added to the modulated signal a local carrier. It has as all these demodulations the problem of the phase displacement among carriers.

To simplify our analyses we consider that synchronism exists and then the local carrier is permanently in phase with that of the transmitter. This way, the behavior equations are

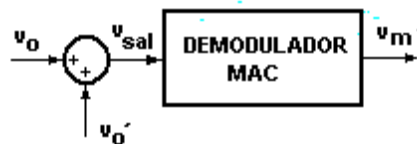
$$v_{o(t)} = (\alpha V_c / 2) \cos (\omega_c - \omega_m)t$$
$$v_{o(t)}' = V_c \cos (\omega_c t + \psi) \rightarrow V_c \cos \omega_c t$$

being

$$v_{sal} = v_{o(t)}' + v_{o(t)} = V_{sal} \cos (\omega_c t + \varphi)$$
$$V_{sal} = [ (\alpha V_c / 2)^2 + V_c^2 + 2(\alpha V_c / 2)V_c \cos \omega_m t ]^{1/2} = V_c [ 1 + (\alpha^2/4) + \alpha \cos \omega_m t ]^{1/2}$$

what tells us that for small modulation indexes we can obtain the sign useful modulating with a simple demodulator of MAC

$$\alpha \ll 1$$
$$v_{sal} = v_{o(t)}' + v_{o(t)} = V_{sal} \cos (\omega_c t + \varphi)$$
$$V_{sal} \sim V_c ( 1 + \alpha \cos \omega_m t )$$
$$v_{m'} = \alpha V_c \cos \omega_m t$$

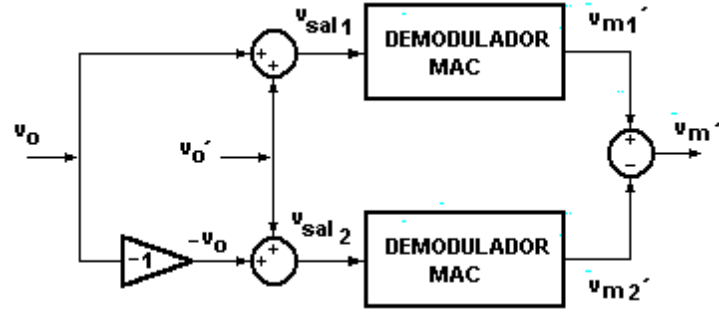


The figure following sample a balanced figure of this demodulator type that eliminates the carrier, and easily it can be implemented taking advantage of the investment of the secondary of a transformer.

$$v_{sal1} = v_{o'} + v_o = V_c ( 1 + \alpha \cos \omega_m t ) \cos (\omega_c t + \varphi)$$
$$v_{sal2} = v_{o'} - v_o = V_c ( 1 - \alpha \cos \omega_m t ) \cos (\omega_c t + \varphi)$$

$$v_{m1'} = V_c ( 1 + \alpha \cos \omega_m t )$$
$$v_{m2'} = V_c ( 1 - \alpha \cos \omega_m t )$$

$$v_{m'} = v_{m1'} - v_{m2'} = 2\alpha V_c \cos \omega_m t$$



### Obtaining for quadratic characteristic

The method here is the following. We added the sign modulated with a local carrier making go their result by an element of quadratic transfer.



Calling  $\psi$  to the phase displacement among carriers obtains

$$v_o(t) = (\alpha V_c / 2) \cos(\omega_c - \omega_m)t$$

$$v_o(t)' = V_c \cos(\omega_c t + \psi)$$

it is

$$v_{sal} = A(v_o(t) + v_o(t)') + B(v_o(t) + v_o(t)')^2 + C(v_o(t) + v_o(t)')^2 + \dots \rightarrow$$

$$\rightarrow B(v_o + v_o')^2 = v_o^2 + 2v_o v_o' + v_o'^2 \rightarrow$$

$$\rightarrow (\alpha V_c B / 2) \{ \cos[(2\omega_c - \omega_m)t + \psi] + \cos(\psi - \omega_m t) \}$$

$$v_m' = \alpha V_c \cos(\psi - \omega_m t)$$

$$\alpha' = B / 2$$

where the deficiency of the system is appreciated in the phase  $(\psi - \omega_m t)$  of the audible frequency.

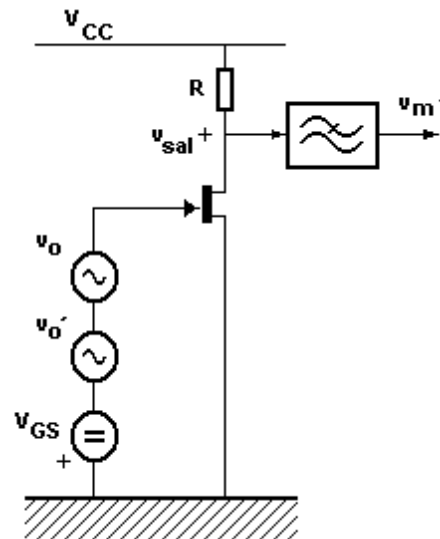
This transfer can be obtained, for example, starting from the implementation with a JFET

$$v_{gs} = v_o + v_o' - V_{GS}$$

$$v_{sal} = i_d R \sim I_{DSS} R (1 + v_{gs}/V_P)^2 = I_{DSS} R [1 + (2v_{gs}/V_P) + (v_{gs}/V_P)^2] \rightarrow$$

$$\rightarrow I_{DSS} R (v_o + v_o' - V_{GS})^2 / V_P^2 \rightarrow B(v_o + v_o')^2$$

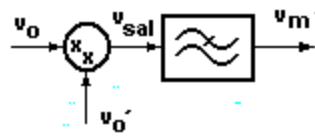
$$B = I_{DSS} R / V_P^2$$



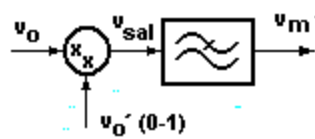
### Obtaining for product

Similar to the previous system, here the BLU multiplies with a local carrier. Their equations are the following

$$\begin{aligned}
 v_{O(t)} &= (\alpha V_c / 2) \cos (\omega_c - \omega_m)t \\
 v_{O(t)}' &= \cos (\omega_c t + \psi) \\
 v_{sal} &= v_{O(t)} v_{O(t)}' = (\alpha V_c / 4) \{ \cos [(2\omega_c - \omega_m)t + \psi] + \cos (\psi + \omega_m t) \} \\
 v_{m'} &= \alpha' V_c \cos (\psi + \omega_m t) \\
 \alpha' &= \alpha / 4
 \end{aligned}$$



This demodulation usually implements with the commutation of an active dispositive that makes the times of switch. In the following figure the effect is shown. The analysis will always be the same one, where now the carrier will have harmonic odd due to the square signal of the commutation, taking place for each one of them (mainly to the fundamental for its great amplitude) the demodulation for product.



### Obtaining for the incorporation of synchronous carrier

When we have a sample of the phase of the original modulating of the transmitter, then the

demodulation calls herself *synchronous*, and it no longer suffers of the problems of quality in the transception. The variation of the phase displacement is it annuls

$$\psi \neq \psi(t)$$

and all the made analyses are equally valid.

## Pulses

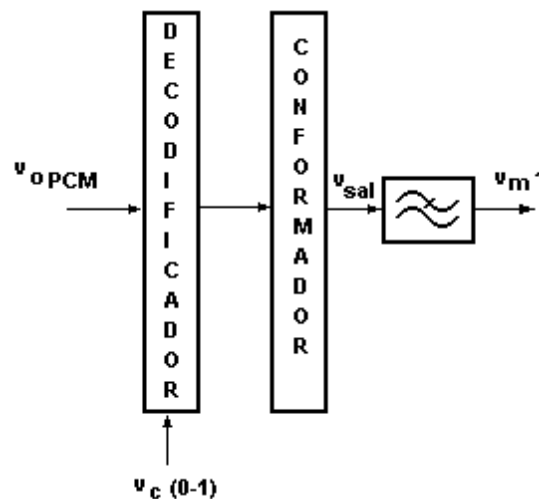
### Generalities

The digital demodulations is practically the same ones that those studied for the analogical. As we work with pulses of frequency  $\omega_m$  with period  $T_m$  and duration  $kT_m$ , then the spectrum of the useful band B will be, practically, of  $3\omega_m$  at  $5\omega_m$  as the efficiency is wanted.

### Obtaining of coded pulses (PCM)

Already demodulated the band base of transmission, receives us PCM that obtains the signal finally useful  $v_m$  again. The following outline, as possible, processes this and where one will have exact reference of the phase of the carrier

$$\psi = 0$$



### Obtaining of PAM

A simple filters low-pass it will be enough to obtain the sign useful  $v_m$ . But the pre-emphasis given by the filter  $F(\omega)$  now it will be reverted as  $1/F(\omega)$ .



## Chap. 27 Modulation of Angle

### Generalities

#### Frequency Modulation (MF)

##### Generalities

##### Generation Armstrong

##### Generation with OCV

##### Modulation in high frequency

##### Design

##### Modulation in low frequency

##### Design

#### Phase Modulation (MP)

##### Generalities

##### Generation for derivation

##### Design

#### Pulses

##### Generation FSK

##### Generation PSK

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### Generalities

The angular modulation  $M\phi$  consists on making that the signal useful  $v_m$  enters in the *instantaneous phase* of the carrier, and that we have denominated  $\phi$ . As this variable it is depending of other two according to

$$v_o(t) = V_c \cos (\omega_c t + \theta)$$

$$\phi = \omega_c t + \theta$$

the  $M\phi$  can be made in two ways

- frequency modulation (MF)
- phase modulation (MP)

that is to say, the first one will imply that the frequency  $\omega_c$  will vary to rhythm of the modulating  $v_m$  (instantly as  $\omega_i$ ), and the second it will be with their *initial phase*  $\theta$ .

We will see that both modulation types are similar, and calling  $\omega_i$  to the frequency of instantaneous carrier, they are related among them for a simple derivation

$$\omega_i = \partial \phi / \partial t$$

and in the transformed field of Laplace

$$\omega_i = s \phi$$

Returning to their generalization like  $M\phi$ , we can deduce that, being the modulating

$$v_m(t) = V_m \cos \omega_m t$$

for both cases the carrier frequency will go varying according to the rhythm of the following expression

$$\omega_i = \omega_c + \Delta \omega_c \cos \omega_m t$$

and of where it is deduced

$$\phi = \int \omega_i \partial t = \omega_c t + \beta \sin \omega_m t$$

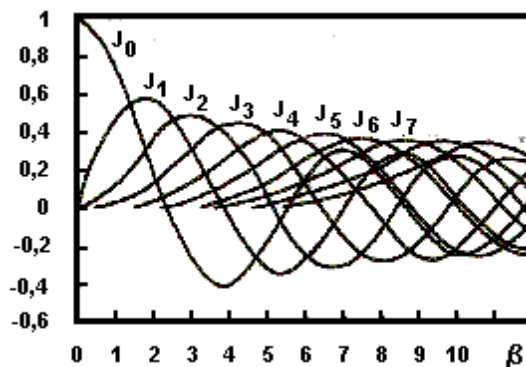
$$\beta = \Delta \omega_c / \omega_m$$

being denominated to  $\beta$  like *index of angular modulation*; and being the modulation finally

$$v_o(t) = V_c \cos \phi = V_c \cos (\omega_c t + \theta) = V_c \cos (\omega_c t + \beta \sin \omega_m t)$$

If we want to know the spectrum of harmonic of this modulation type, we can appeal to the abacus of Bessel according to the following disposition

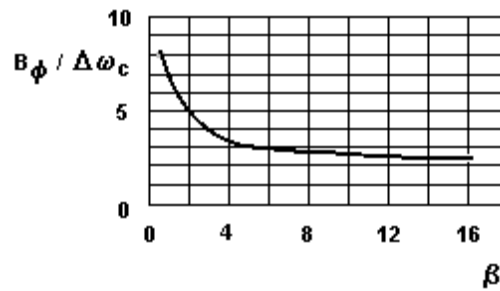
$$v_o(t) = V_c \{ J_0 \cos \omega_c t + J_1 [\cos (\omega_c + \omega_m)t - \cos (\omega_c - \omega_m)t] + J_2 [\cos (\omega_c + 2\omega_m)t - \cos (\omega_c - 2\omega_m)t] + \dots$$



and if what we want to know is the band width *normalized*  $B_\phi$  we use the following one other, where it is defined it in two possible ways

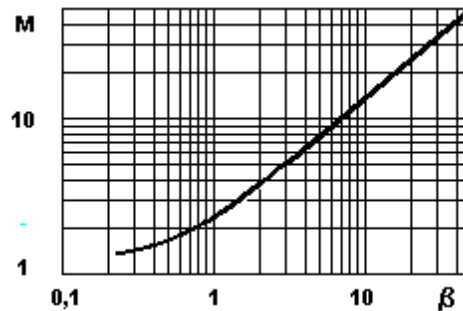


- wide band ( $B_\phi \geq 5$  being  $B_\phi \sim 2\Delta\omega_c$ )
- short band ( $B_\phi \ll \pi/2$  being  $B_\phi \sim 2\omega_m$ )



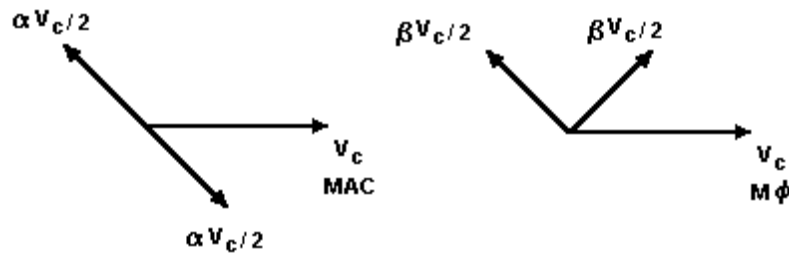
Another way to estimate the band width is with the abacus of Carson. Of this graph a multiplier  $M$  is obtained according to the following form (the original graphs possess as parameter the module of the harmonic  $i\omega_c$  among  $0,01 \leq |J_i| \leq 0,1$  for what we clarify that it is an interpolation average)

$$B_\phi = 2 M \omega_m$$



For short band we can simplify the equation and to arrive to didactic results. That is

$$\begin{aligned}
 v_o(t) &= V_c \cos(\omega_c t + \beta \sin \omega_m t) = \\
 &= V_c [\cos \omega_c t \cos(\beta \sin \omega_m t) - \sin \omega_c t \sin(\beta \sin \omega_m t)] \sim \\
 &\sim V_c [\cos \omega_c t - \sin \omega_c t (\beta \sin \omega_m t)] = \\
 &= V_c \{ \cos(\omega_c t) + (\beta/2) [\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t] \} \\
 B_\phi &= 2 \omega_m
 \end{aligned}$$



On the other hand, when the modulation is made in low frequencies and then it increases it to him, it is necessary to have present that changes the modulation index, and therefore also the band width. Let us see this when it multiplies the frequency N times

$$v_o(t) = V_c \cos N\phi = V_c \cos (N\omega_c t + N\beta \sin \omega_m t)$$

## Frequency Modulation (MF)

### Generalities

In this modulation the apartment of the carrier frequency  $\Delta\omega_c$  is proportional to the amplitude of the modulating  $V_m$ , and the speed of its movement to the frequency of the modulating  $\omega_m$

$$\begin{aligned}\Delta\omega_c &= k_1 V_m \\ \partial\omega_i / \partial t &= k_2 \omega_m = \partial^2\phi / \partial t^2\end{aligned}$$

and we repeat the general expressions

$$\begin{aligned}v_m(t) &= V_m \cos \omega_m t \\ v_o(t) &= v_o \cos (\omega_c t + \theta) \\ \phi &= \omega_c t + \theta = \int \omega_i \partial t = \omega_c t + \beta \sin \omega_m t \\ \omega_i &= \partial\phi / \partial t = \omega_c + \Delta\omega_c \cos \omega_m t \\ \beta &= \Delta\omega_c / \omega_m \text{ (índice de MF)}\end{aligned}$$

The relationship signal to noise in the transception (modulation and demodulation) of this modulation type, as much in microwave as in those of radiofrequency of use commercial of broadcasting, it has been seen that it is necessary to compensate it with a filter called pre-emphasis with the purpose of that this relationship stays the most constant possible along the useful band B. This filter for the applications of microwaves is complex, because it depends on many requirements, so much technical as of effective normativity of the regulation of the telecommunications. As for the broadcasting, this usually makes simpler with a filter in high-pass whose pole is in some approximate 50 microseconds—in truth this is variable.

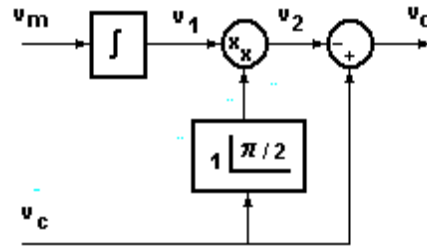
### Generation Armstrong

Belonging to the history, the generation for the method of Armstrong always determines an easy way to take place FM in short band. Their behavior equations are the following ones (the integrative is implemented with a filter low-pass that makes go to the band B by a slope of -20 [dB/DEC])

$$v_1 \equiv \int v_m \partial t = (V_m/\omega_m) \cos \omega_m t \equiv \beta \sin \omega_m t$$

$$v_2 \equiv v_1 v_c e^{-j\pi/2} = \int v_m \partial t \equiv \beta \sin \omega_m t \sin \omega_c t$$

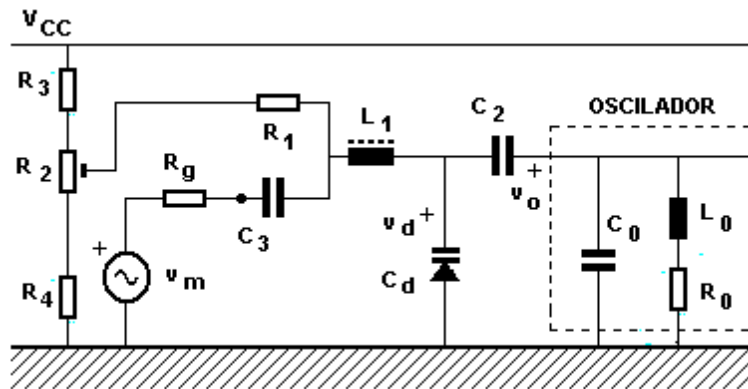
$$v_o = v_c - v_o \equiv \sin \omega_c t - \beta \sin \omega_m t \sin \omega_c t$$



## Generation with OCV

## Modulation in high frequency

Depending logically on the work frequency the circuit will change. Subsequently we observe one possible to be implemented in RF with an oscillator anyone (for their design it can be appealed to the chapter of *harmonic oscillators*).

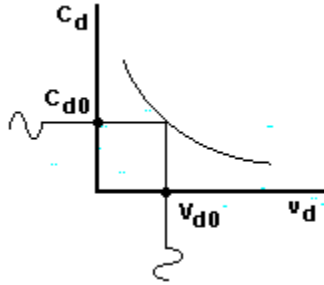


The synton of the oscillator is polarized by the capacitance  $C_{d0}$  of the diode varicap due to the continuous voltage that it provides him the source VCC

$$\omega_c^2 = 1 / L_0(C_0 + C_{d0} + C_p)$$

$$V_{d0} = k V_{CC}$$

$$C_d \sim A / v_d^\gamma$$



being  $C_p$  the distributed capacitance of the connections in derivation with the diode and  $k$  the attenuation of the divider  $R_2$ - $R_3$ - $R_4$ .

For the design we can know the magnitude of  $A$  and of  $\gamma$  if we observe their data by the maker, or for a previous experimentation, since if we obtain the capacitance of the diode for two points of the curve they are

$$C_{dmax} = A / v_{dmin}^{\gamma}$$

$$C_{dmin} = A / v_{dmax}^{\gamma}$$

$$\gamma = \log (C_{dmax} / C_{dmin}) / \log (v_{dmax} / v_{dmin})$$

$$A = C_{dmax} v_{dmin}^{\gamma}$$

As for the distortion that is generated of the graph, studies in this respect that we omit here for simplicity, show that the most important distortion is given by the equation

$$D [\text{veces}] \sim \{ 0,25 (1 + \gamma) - \{ 0,375 \gamma / [ 1 + (C_0 + C_p)/C_{d0} ] \} \} V_m / V_{d0}$$

## Design

Be the data

$$V_{CC} = \dots \quad f_c = \dots \quad C_0 = \dots \quad C_p \approx \dots \text{ (approximately 5 [pF])}$$

$$V_m = \dots \quad f_{mmax} = \dots \quad f_{mmin} = \dots \quad D_{max} = \dots \quad R_g = \dots$$

We choose a diode and of the manual or their experimentation in two points anyone of the curve obtains (f.ex.: BB105-A with  $\gamma \sim 0,46$ ,  $C_{d0} = 11,2$  [pF] and  $V_{d0} = 3,2$  [V])

$$\gamma = \log (C_{dmax} / C_{dmin}) / \log (v_{dmax} / v_{dmin}) = \dots$$

and then we polarize it in the part more straight line possible of the curve

$$V_{d0} = \dots$$

$$C_{d0} = \dots$$

getting

$$L_0 = 1 / (C_0 + C_{d0} + C_p) \omega_c^2 = \dots$$

$$L_1 = \dots \gg L_0$$

$$R_1 = \dots \gg R_g$$

Subsequently we design the capacitances so that they cut in the frequencies of the useful band

$$C_1 = 1 / R_1 \omega_{\min} = \dots$$

$$C_2 = 1 / R_g \omega_{\max} = \dots$$

The resistive dividing, where we suggest one pre-set multi-turn for  $R_2$ , we calculate it as a simple attenuator

$$R_2 = \dots \ll R_1$$

$$I_{R2} = \dots \geq V_m / (R_2 / 2)$$

$$R_3 = [V_{CC} - (V_{d0} + V_m)] / I_{R2} = \dots$$

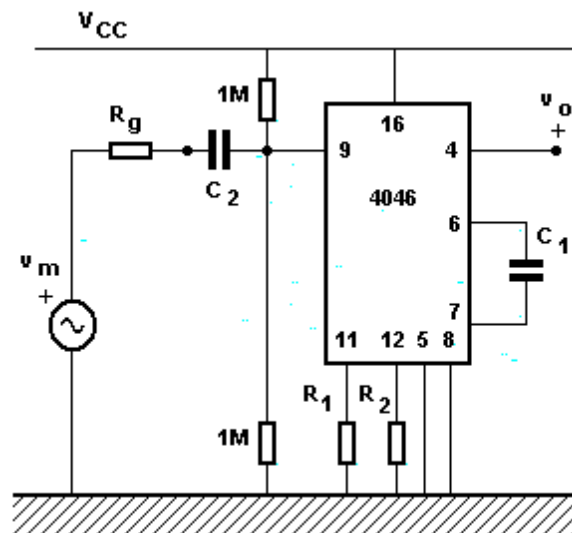
$$R_4 = (V_{d0} - V_m) / I_{R2} = \dots$$

Finally we verify the distortion

$$\{ 0,25 (1 + \gamma) - \{ 0,375 \gamma / [ 1 + (C_0 + C_p) / C_{d0} ] \} \} V_m / V_{d0} = \dots < D_{\max}$$

## Modulation in low frequency

For applications of until some few MegaHertz it is possible the use of the OCV of the integrated circuit 4046 already explained in the *multivibrators* chapter, where the output is a FM of pulses.



## Design

Be the data

$$f_c = \dots \quad V_m = \dots \quad f_{m\max} = \dots \quad f_{m\min} = \dots \quad \beta_{\max} = \dots \ll \pi / 2 \text{ (short band)}$$

We choose a supply

$$15 \text{ [V]} \geq V_{CC} = \dots > 2 V_m$$

and we go to the *multivibrators* chapter adopting, for this integrated circuit 4046, a polarization with the *gain* abacus (here  $f_0$  are  $f_c$ )

$$R_1 = \dots$$

$$C_1 = \dots$$

and being the worst case

$$\beta_{\max} = 2 (f_{c\max} - f_c) / f_{m\min}$$

they are

$$f_{c\max} = f_c + (\beta_{\max} f_{m\min} / 2) = \dots$$

$$f_{c\min} = f_c - (f_{c\max} - f_c) = 2 f_c + f_{c\max} = \dots$$

Then with the third abacus finally find (here  $f_{\max}/f_{\min}$  is  $f_{c\max}/f_{c\min}$ )

$$f_{c\max} / f_{c\min} = \dots$$

$$(R_2 / R_1) = \dots$$

$$R_2 = R_1 (R_2 / R_1) = \dots$$

He couples the we calculate that it produces the cut in low, or simply that it is a short circuit

$$C_2 = \dots \gg 1 / (R_g + 5.10^5) \omega_{m\min}$$

## Phase Modulation (MP)

### Generalities

In this modulation the apartment of the initial phase of the carrier  $q$  is proportional to the amplitude of the modulating  $V_m$ , and the speed of its movement to the frequency of the modulating  $\omega_m$  multiplied by the amplitude of the modulation  $V_m$

$$\theta = \beta \sin \omega_m t$$

$$\beta = k_3 V_m$$

$$\partial \theta / \partial t = \beta \omega_m \sin \omega_m t = k_3 V_m \omega_m \sin \omega_m t$$

and we repeat the general expressions

$$v_m(t) = V_m \cos \omega_m t$$

$$v_o(t) = v_o \cos (\omega_c t + \theta)$$

$$\phi = \omega_c t + \theta = \int \omega_i \partial t = \omega_c t + \beta \sin \omega_m t$$

$$\omega_i = \partial \phi / \partial t = \omega_c + \Delta \omega_c \cos \omega_m t$$

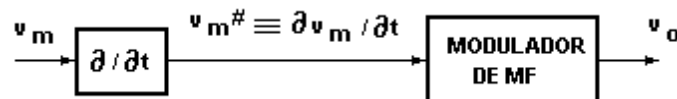
$$\beta = \Delta \omega_c / \omega_m \text{ (índice de MP)}$$

and where we can observe that we will have in consequence a kind of MF

$$\omega_i = \partial \phi / \partial t = \omega_c - \beta \omega_m \sin \omega_m t$$

### Generation for derivation

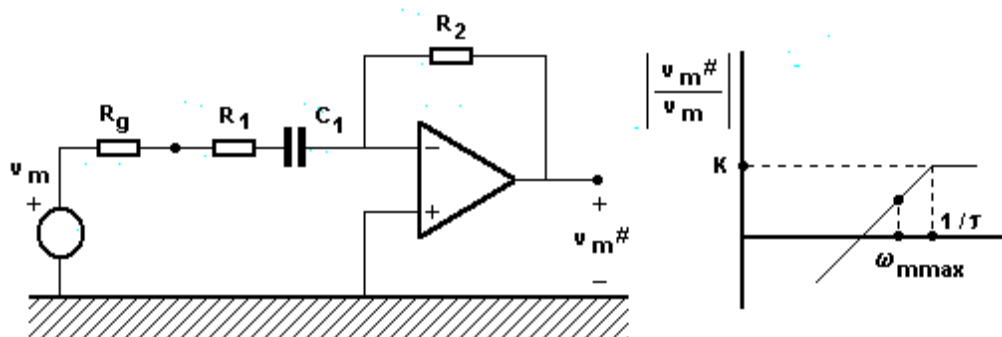
According to the precedent equations, to generate MP we can simply derive the sign modulating and then to modulate it in MF for some of the previous methods



### Design

Be the data (to see their design equations in the chapter of *active networks as filters of frequency and phase displacements*)

$$V_m = \dots \quad f_{m\max} = \dots \quad R_g = \dots \quad K = \dots \geq \leq 1$$



We adopt

$$\tau = \dots \geq 5 \omega_{\text{max}}$$

$$C_1 = \dots$$

and we find

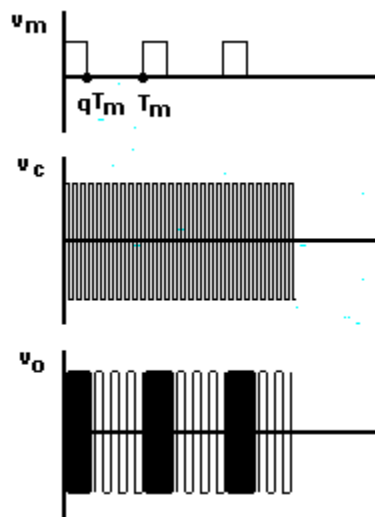
$$R_1 = (\tau / C_1) - R_g = \dots$$

$$R_2 = K (R_1 + R_g) = \dots$$

## Pulses

### Generation FSK

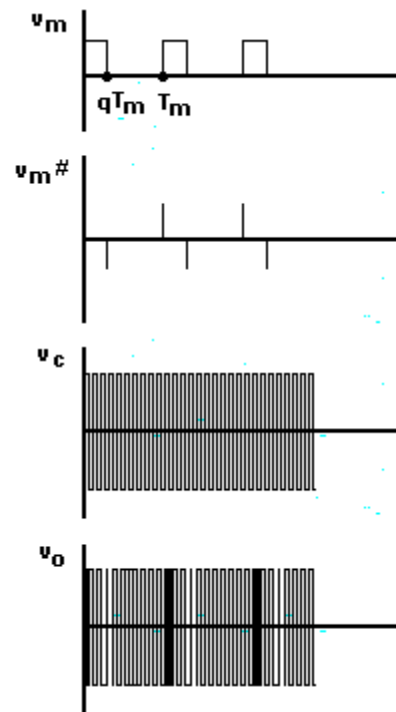
Here it is modulated binarily to the carrier. The effect is shown in the following figure, where we see that for the case of short band the band width for the transmission double that of the pulses; that is to say, practically  $2(\omega_c - \pi/qT_m)$ .



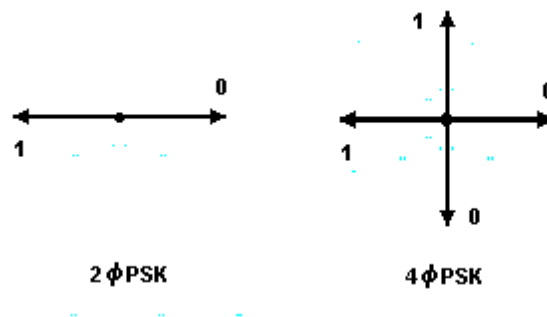
### Generation PSK

The same as the previously seen concepts, here to  $v_m$  we derive it and then we modulate it in frequency.





According to the quantity of binary parity that are had in the modulation, this will be able to be of the type  $2\phi$ PSK,  $4\phi$ PSK, etc.



## Chap. 28 Demodulation of Angle

### Generalities

#### Demodulation of Frequency (MF)

### **Generalities**

#### **Demodulation in high frequency**

#### **Demodulation for conversion to MAC**

#### Obtaining for simple slope

#### Design

#### Obtaining for double slope (Travis)

#### Obtaining with discriminator of relationship

#### Design

#### Obtaining for tracking of phases

#### Design

#### Obtaining for conversion to MP

#### Demodulation of Phase (MP)

### **Generalities**

#### **Obtaining**

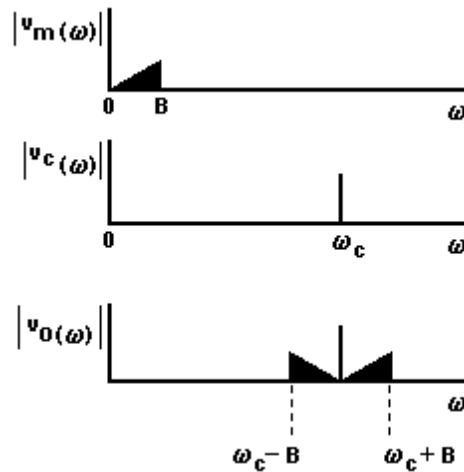
#### **Design**

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### Generalities

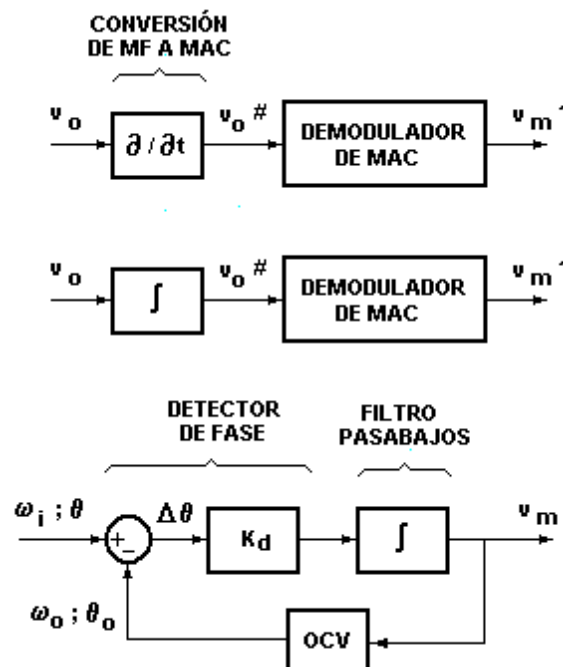
Basically it consists on taking the band bases  $2B$  (short band) of the high frequency of carrier  $\omega_c$  to the  $B$  of the lows. The following drawings they explain what is said. Then, like it was explained

$$v_o(t) = V_c \cos (\omega_c t + \beta \sin \omega_m t) \sim V_c \{ \cos (\omega_c t) + (\beta/2) [\cos (\omega_c + \omega_m)t - \cos (\omega_c - \omega_m)] \}$$



The methods to obtain this demodulation are basically two, that is: one, in transcribing the sign of  $M\phi$  (interpreted as MF) in MAC it stops then to demodulate it classically; the other, with a tracking of phases —*phase lock loop* LFF.

For the first method it is made go the signal of  $M\phi$  by a slope of first order (20 [dB] per decade), either positive or negative, in such a way that the variations of frequency are translated to voltages. It is also accustomed to be used slopes of more order that, although they achieve the discrimination equally, they don't reproduce the modulating correctly.



The second way, something more complex, it detects the phase of the  $M\phi$  and it retro-feeds her through a OCV; the result will be that in permanent state, that is to say when the system is hooked, both frequencies  $\omega_i$  and  $\omega_o$  are same and, therefore, the  $v_m'$  is a reflection of what happened in the modulator with  $v_m$  —the OCV it would reproduce it.

On the other hand, due to the sophistications of the processes of signals that they exist today in day, and also as satisfying to the best intentions in the historical beginning of the broadcasting to transmit acoustic fidelities (forgotten fact since to make screech the hearings as in MAC), this transception type is logically more immune to the atmospheric interferences that those of modulation of amplitude.

## Demodulation of Frequency (MF)

### Generalities

We repeat the characteristic equations of the modulation of frequency

$$\begin{aligned}
 v_m(t) &= V_m \cos \omega_m t \\
 v_o(t) &= v_o \cos (\omega_c t + \theta) \\
 \phi &= \omega_c t + \theta = \int \omega_i \, dt = \omega_c t + \beta \sin \omega_m t \\
 \beta &= \Delta \omega_c / \omega_m \text{ (índice de MF)} \\
 \Delta \omega_c &= k_1 V_m \\
 \omega_i &= \partial \phi / \partial t = \omega_c + \Delta \omega_c \cos \omega_m t \\
 \partial \omega_i / \partial t &= k_2 \omega_m = \partial^2 \phi / \partial t^2
 \end{aligned}$$

and we remember

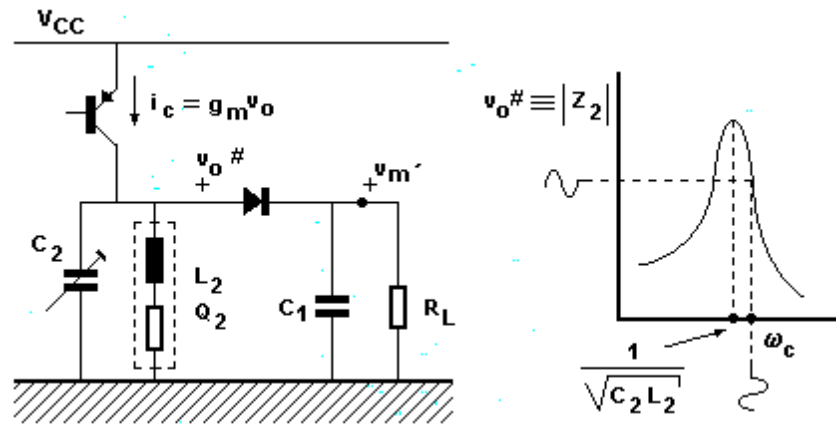
- wide band ( $B_\phi \geq 5$  being  $B_\phi \sim 2\Delta\omega_c$ )
- short band ( $B_\phi \ll \pi/2$  being  $B_\phi \sim 2\omega_m$ )

## Demodulation in high frequency

### Demodulation for conversion to MAC

#### Obtaining for simple slope

This discriminator can be made with a simple filter outside of syntyony to the carrier frequency, either in excess or defect. Although the slope of the filter is not exactly 20 [dB/DEC], the result is, for many cases like for example AF vowel, very efficient.



Another inconvenience of this demodulation consists in that it doesn't avoid the atmospheric interferences, since all interference of amplitude in the carrier will be translated to the output.

### Design

Be the data

$$R_L = \dots \quad f_{\text{mmax}} = \dots \quad f_{\text{mmin}} = \dots \quad f_c = \dots \quad \Delta f_c = \dots \quad (\text{for broadcasting } 75 \text{ [KHz]})$$

We estimate a capacitor of filter of the demodulator of MAC (to see their theoretical conditions in the chapter of *demodulation of amplitude*)

$$C_1 = \dots \sim 1 / \omega_{\text{mmax}} R_L$$

and we choose a syntony that allows the band base of MF that is  $2B$  (we think that the more we come closer to the syntony with the carrier, more will be his amplitude for the detector, but the deformation will also increase)

$$\omega_c \sim < B + 1/(C_2 L_2)^{1/2}$$

$$Q_2 = \dots \sim < \omega_c / 2B$$

for that that if we choose the inductor according to the band width that we need (we remember that the detector reflects a resistance of  $\sim R_L/2$ , and also that the inductor will surely possess a factor of merit much more to the total that we are calling  $Q_2$ )

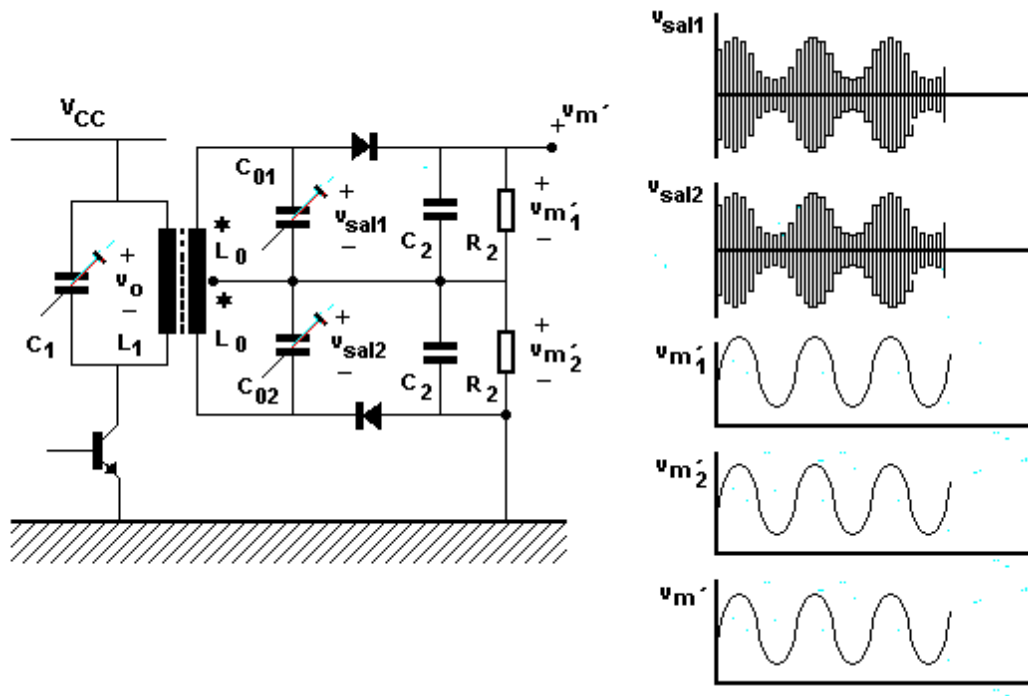
$$L_2 = R_L / 2\omega_c Q_2 = \dots$$

we obtain the estimate (we remember the existence of distributed capacities)

$$C_2 = 1 / L_2 (\omega_c - B) \sim 1 / L_2 \omega_c = \dots$$

### Obtaining for double slope (Travis)

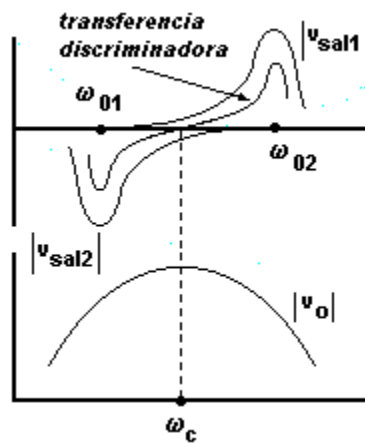
The circuit consists on a double discriminator of simple slope in anti-series.



With a transformer of low coupling ( $k \ll 1$ ), the primary is syntonized in parallel and the secondary in series. It is syntonized to the carrier frequency the primary allowing to pass the width of band of the modulation

$$\omega_c = 1 / (L_1 C_1)^{1/2}$$

$$Q_1 \sim \leq \omega_c / 2B$$



and the secondary ones syntonized for above and below the carrier, they guarantee a lineality in the demodulation that doesn't make it that of simple slope. But it follows the problem of the immunity with

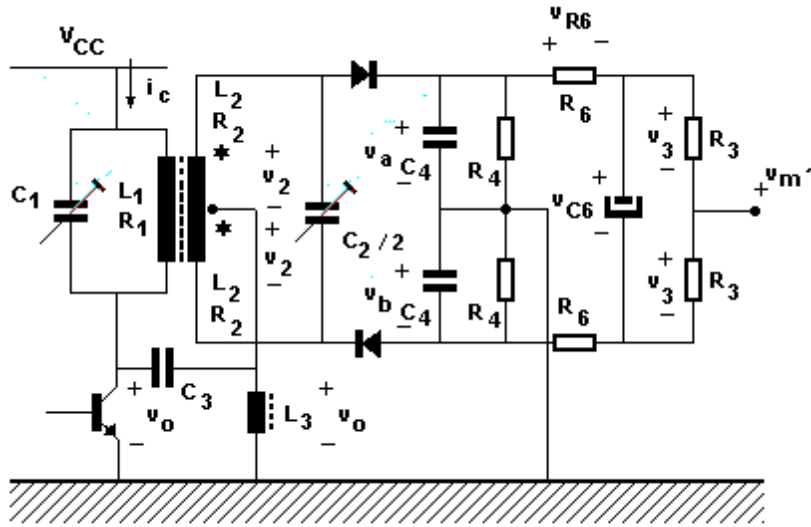
the interferences of amplitude.

$$\omega_{01} = 1 / (L_0 C_{01})^{1/2} < \omega_c - B$$

$$\omega_{02} = 1 / (L_0 C_{02})^{1/2} > \omega_c + B$$

### Obtaining with discriminator of relationship

Being a variant of the discriminator Foster-Seely, we will show that this discriminator attenuates the problems of atmospheric interferences of amplitude. Their configurations and design are in very varied ways, and here we show only the circuit perhaps more classic and more didactic.



The transformer is of low coupling ( $k \ll 1$ ), designed in such a way that allows a double syntony among the coils; for example of maximum plain among  $v_2$  and the current of collector  $i_c$  that it is proportional to the MF.

To simplify we will design an electric separation in the way

$$R_3 \gg R_4$$

then, inside the band pass it will be (to go to the chapter of *radiofrecuencias amplifiers* and that of *demodulación of amplitude*)

$$Z_0 = v_2 / i_c = H_0 \cdot s / (s^4 + s^3 A + s^2 B + s C + D)$$

$$Z_{0(\omega_c)} \sim -j k Q_0 / \omega_c [(C_1 + C_{ce}) C_2]^{1/2}$$

$$\omega_c = 1 / [(L_1(C_1 + C_{22e}))^{1/2}] = 1 / (L_2 C_2)^{1/2}$$

$$Q_0 = [R_1(\omega_c L_1 / R_1)^2 / g_{22e}] / \omega_c L_1 = \omega_c L_2 / [R_2 + (R_4/2) / (\omega_c C_2 R_4/2)^2]$$

expression that it says that the voltage pick of  $v_2$  that we call  $V_2$  will be constant with the frequency like it is shown next ( $I_0$  it is the value pick of the current of MF in the collector)

$$v_2 = Z_0 i_c \sim kQ_0 I_0 / \omega_c [(C_1 + C_{ce}) C_2]^{1/2} = V_2 e^{-j\pi/2}$$

but not so much that of the primary

$$v_o = \{ I_0 R_1 (\omega_c L_1 / R_1)^2 / \{ 1 + [2Q_0(1 - \omega/\omega_c)]^2 \}^{1/2} \} e^{-j \arctan 2Q_0(1 - \omega/\omega_c)}$$

and like we work in short band

$$\omega_c \sim \omega \Rightarrow 2Q_0(1 - \omega/\omega_c) \ll \pi/2$$

it is

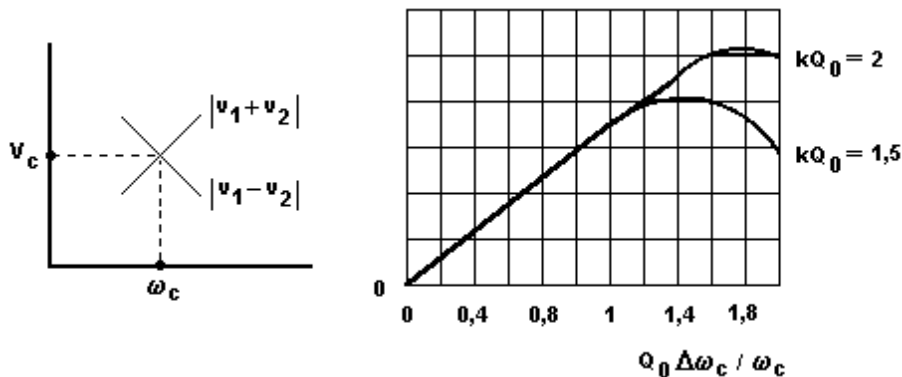
$$v_o \sim [I_0 (\omega_c L_1)^2 / R_1] e^{-j 2Q_0(1 - \omega/\omega_c)} = [I_0 (\omega_c L_1)^2 / R_1] e^{j 2Q_0(\omega/\omega_c - 1)}$$

This way the voltages to rectify are

$$\begin{aligned} |v_1 + v_2| &= \{ \{ [2I_0 Q_0 (\omega_c L_1)^2 / R_1] (\omega/\omega_c - 1) + V_2 \}^2 + [I_0 (\omega_c L_1)^2 / R_1]^2 \}^{1/2} \\ |v_1 - v_2| &= |v_2 - v_1| = \{ \{ [2I_0 Q_0 (\omega_c L_1)^2 / R_1] (1 - \omega/\omega_c) + V_2 \}^2 + [I_0 (\omega_c L_1)^2 / R_1]^2 \}^{1/2} \end{aligned}$$

that they express the transcription of the MF to kind of a MAC, and where the intersection is the amplitude of the carrier

$$V_c = \{ [I_0 (\omega_c L_1)^2 / R_1]^2 + V_2^2 \}^{1/2}$$



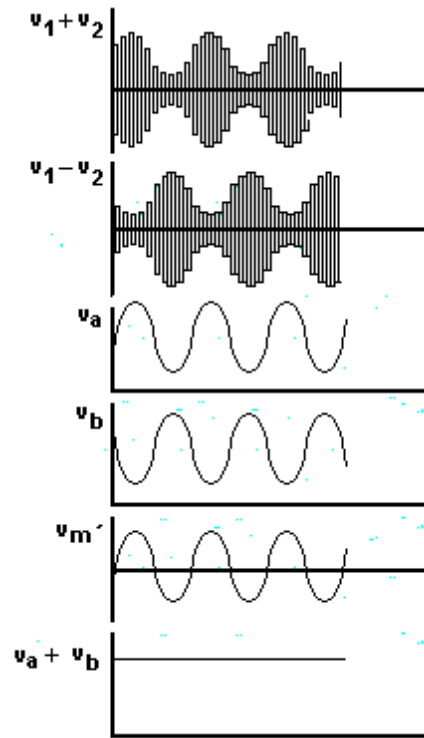
Como  $C_3$  es inevitable en el acople de continua, surgió la necesidad de dar retorno a la continua de las rectificaciones a través del choque de RF por medio de  $L_3$

$$1 / \omega_c C_3 \ll \omega_c L_3$$

consequently, we will say that to the output of each secondary we have

$$\begin{aligned} |v_1 + v_2| &= \pm V_c (1 + \alpha \cos \omega_m t) \\ |v_1 - v_2| &= \pm V_c (1 - \alpha \cos \omega_m t) \end{aligned}$$





that we can deduce in

$$v_a = V_c (1 + \alpha \cos \omega_m t)$$

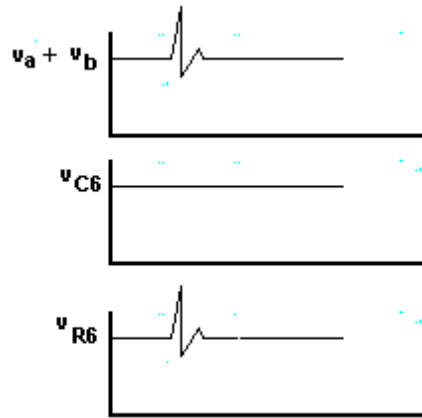
$$v_b = V_c (1 - \alpha \cos \omega_m t)$$

and finally

$$v_m' = v_a - v_b = v_a - (v_a + v_b)/2 = \alpha V_c \cos \omega_m t$$

This discriminator has the advantage in front of the previous ones in that it allows to limit the atmospheric noises of the propagation that arrive modifying the amplitude of the MF. The variant here, denominated by the balanced disposition, it is of *relationship*. The condenser  $C_6$  makes constant a voltages on  $R_3$  proportional to the sum of  $v_a + v_b$ , in such a way that is

$$v_{C6} = (v_a + v_b) 2R_3 / (2R_3 + 2R_6) = (v_a + v_b) / (1 + R_6/R_3)$$



and, although the undesirable interferences appear, these are translated on  $R_3$  and don't on  $C_6$ . Indeed, to limit the annoying noises and to allow to pass the useful band it will be enough to design

$$R_4 \ll R_6 \ll R_3$$

$$2R_3C_6 > 1 / f_{\text{mmin}}$$

Being a broadcasting transmission, we hope the band useful modulating has a pre-emphasis; therefore it is necessary to put as additional to the output of the discriminator an inverse filter or of-emphasis that the equalized in the spectrum. The circuit, simple, consists on a simple low-pass of a resistance and a condenser of constant of time of approximate 50 [ $\mu\text{seg}$ ].

### Design

Be tha data

$$f_{\text{mmax}} = \dots \quad f_{\text{mmin}} = \dots \quad f_c = \dots \quad \pm \Delta f_c = \dots \quad (\text{broadcasting } \pm 75 \text{ [KHz]})$$

We choose a TBJ and we polarize it obtaining of the manual (to see the chapters of *polarization of dispositives* and of *radiofrecuencias amplification in class A*)

$$C_{22e} = \dots$$

$$g_{22e} = \dots$$

Adoptamos (véanse los capítulos de *inductores de pequeño valor* y de *transformadores de pequeño valor*)

$$L_1 = \dots$$

$$R_1 = \dots$$

$$L_2 = \dots$$

$$R_2 = \dots$$

$$L_3 = \dots$$

what will allow us to calculate with the help of the precedent comments (to keep in mind the capacitances distributed in the connections)

$$\begin{aligned}C_1 &= (1 / \omega_c L_1) - C_{22e} = \dots \\C_2 &= 1 / \omega_c L_1 = \dots \\Q_0 &= 1 / \omega_c L_1 [g_{22e} + (R_1 / \omega_c L_1)] = \dots \geq 10 \\R_4 &= 2 / (\omega_c C_2)^2 [(\omega_c L_2 / Q_0) - R_2] = \dots \\C_3 &= \dots \gg 1 / \omega_c L_3 \\R_3 &= \dots \gg R_4\end{aligned}$$

and for the design of the transformer we adopt according to the previous abacus and for not varying the equations

$$k = 1 / Q_0 = \dots$$

The filter anti-noise will be able to be (any electrolytic one bigger than 100 [μF] it will be enough)

$$C_6 = \dots > 1 / 2R_3 f_{\text{mmin}}$$

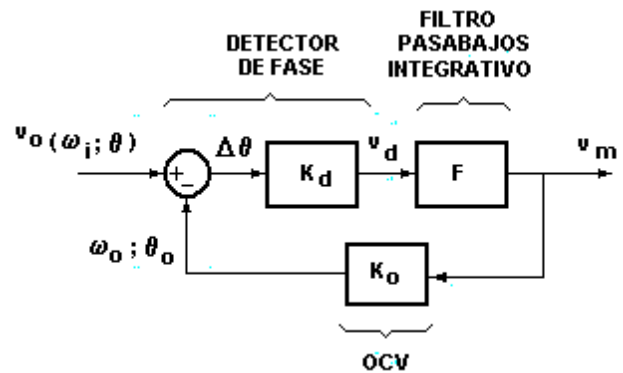
### Obtaining for tracking of phases

We will take advantage of a *Phase Look Loop* LFF to demodulate in frequency. Many ways exist of implementing it, so much in low, high or ultra-high frequencies. We will see only a case of the lows here.

All LFF is based on a detector of phases between the carrier instantaneous  $w_i$  and that of the oscillator local  $w_o$  of constant transfer  $K_d$  [V/rad], that will excite to a filter transfer low-pass  $F$  working as integrative and that it will obtain a continuous average for the control feedback to the transfer OCV as  $K_o$  [rad/Vseg].

This way their basic equations of behavior are (we suppose a filter of a single pole for simplicity and ordinary use, because this can extend to other characteristics)

$$\begin{aligned}K_d &= v_d / \Delta\theta = v_d / (\theta - \theta_o) \\F &= v_m' / v_d = K_f / (1 + s\tau) \\K_o &= \omega_o / v_m'\end{aligned}$$



what will determine a transfer in the way

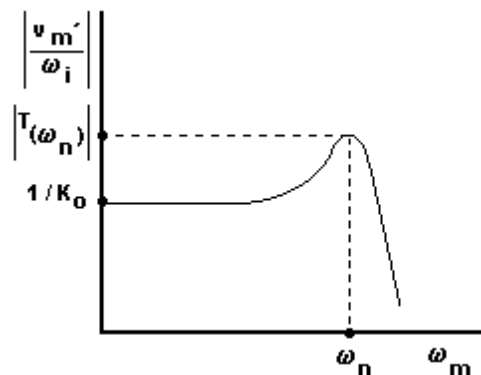
$$T = v_m' / \omega_i = (1 / K_o) / [ s (1 / K_f K_o F) + 1 ] = (1 / K_o) / [ s^2 / \omega_n^2 + s (1 / \tau \omega_n^2) + 1 ]$$

$$\omega_n = (K_f K_o / \tau)^{1/2}$$

$$T(\omega_n) = -j \tau (K_f / K_o)^{1/2}$$

$$T(0) = 1 / K_o$$

$$\xi = 1 / 2 \tau \omega_n \quad (\text{coefficient of damping})$$



where it can be noticed that the complex variable «s» it is the frequency modulating  $\omega_m$  like speed of the frequency of carrier  $\omega_i$ , that is to say, it is the acceleration of the carrier.

For stationary state, that is to say of continuous ( $\omega_m = 0$ ), the total transfer is simplified the inverse of the feedback transfer; this is, at  $1/K_o$ .

So that this system enters in operation it should can "to capture" the frequency of the carrier, for what is denominated *capture range*  $R_c$  to the environment of the central frequency of the local oscillator that will capture the  $\omega_i$  sustaining the phenomenon. Also there will be another *maintenance range*  $R_m$  of which the oscillator won't be been able to leave and that it is he characteristic of its design.

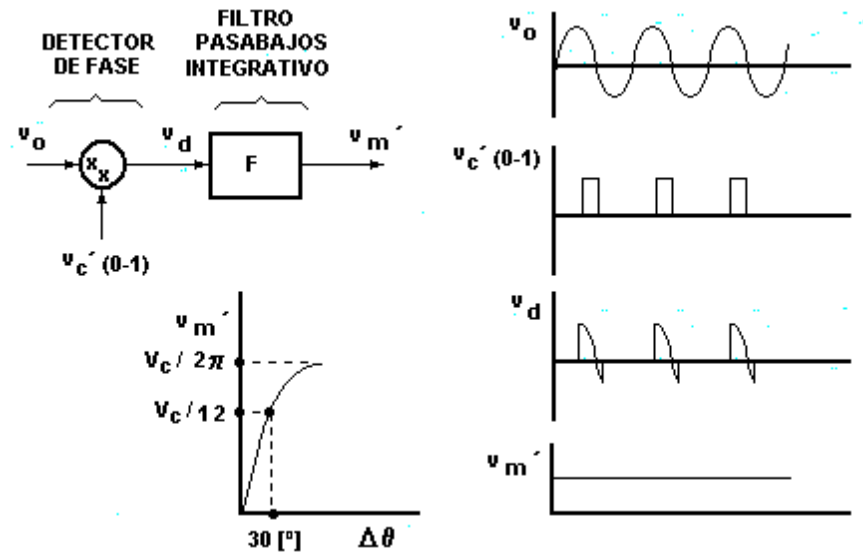
A way to get a detector of phases is with a simple sampling. This is an useful implementation for high frequencies. The output of the circuit that is shown is

$$v_o = V_c \cos \omega_c t$$

$$v_m' = (1/2\pi) \int_0^{\Delta\theta} v_o \partial\omega_c t = (V_c/2\pi) \sin \Delta\theta$$

where it is observed that for low  $\Delta\theta$  the output is lineal

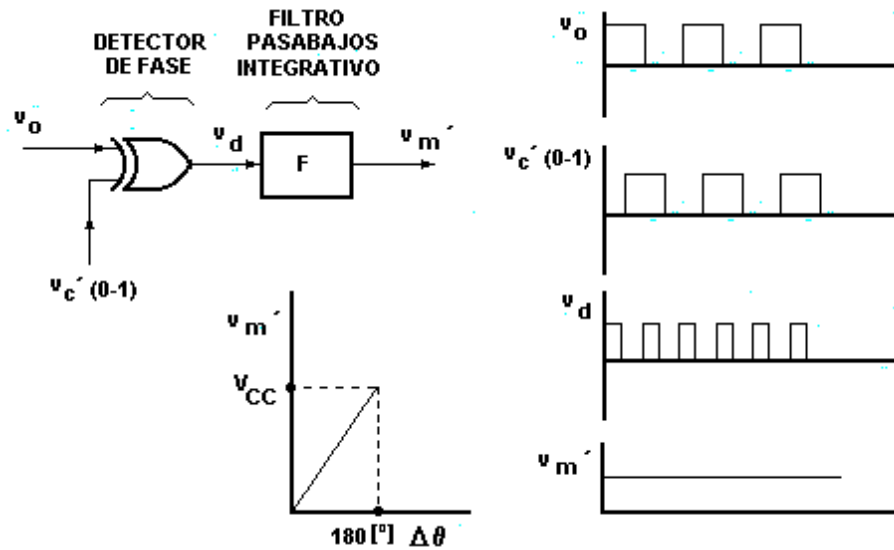
$$K_d = v_m' / \Delta\theta \sim V_c/2\pi$$



Another practical way and where  $\Delta\theta$  can arrive up to  $180^\circ$ , although to smaller frequencies, we can make it with a gate OR-Exclusive as sample the drawing. Their equations are the following

$$v_m' = 2 (1/2\pi) \int_0^{\Delta\theta} v_o \partial\omega_c t = (V_{CC}/\pi) \Delta\theta$$

$$K_d = v_m' / \Delta\theta = V_{CC}/\pi$$



For applications of until some few MegaHertz it is feasible the use of the OCV of the integrated circuit 4046 already explained in the *multivibrators* chapter. As the use technique here is digital, it accompanies to the *chip* a gate OR-Exclusive dedicated to be used as detecting of phases. For this case the behavior equations are the following ones (in the drawing the numbers of the terminals of the integrated circuit are accompanied)

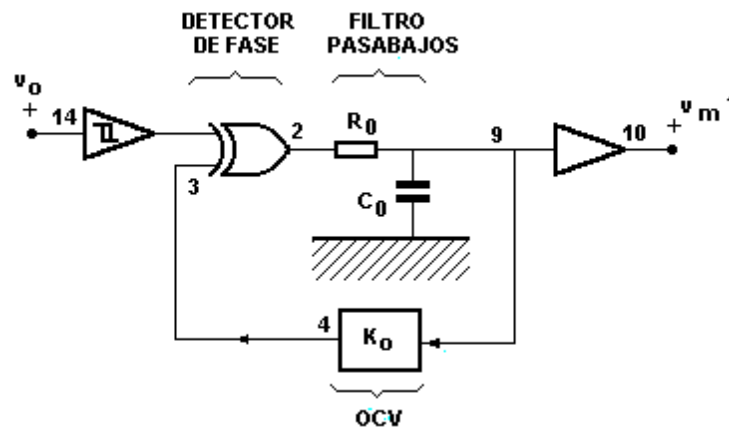
$$K_f = V_{CC} / \pi$$

$$K_o = 2\pi (f_{cmax} - f_{cmin}) / V_{CC}$$

$$\tau = R_0 C_0$$

$$R_m = 2\pi (f_{cmax} - f_{cmin}) = 2 \Delta\omega_c$$

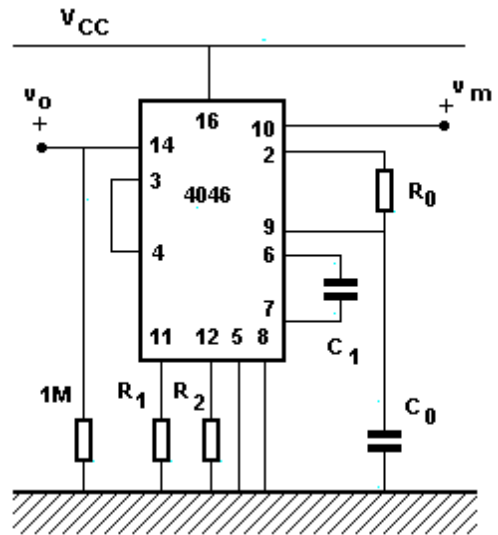
$$R_c = (2 R_m / \tau)^{1/2} = 2 (\Delta\omega_c / \tau)^{1/2}$$



## Design

Be the data

$$f_{\text{mmax}} = \dots \quad f_c = \dots \quad \pm \Delta f_c = \dots \quad V_c = \dots \quad (\text{maximum amplitude or pick of the input } v_o)$$



We choose a polarization with the abacus of rest (here  $f_0$  are our carrier  $f_c$ )

$$V_{CC} = \dots \leq 1,4 V_c \quad (\text{there is guarantee of excitement of gates with 70\% of } V_{CC})$$

$$R_2 = \dots$$

$$C_1 = \dots$$

$$K_f = V_{CC} / \pi = \dots$$

$$K_0 = \Delta\omega_c / V_{CC} = \dots$$

and then with the third (here  $f_{\text{max}}/f_{\text{min}}$  is the  $f_{\text{omax}}/f_{\text{omin}}$  of the OCV )

$$f_{\text{omax}}/f_{\text{omin}} = \dots > (f_c + \Delta f_c) / (f_c - \Delta f_c)$$

$$(R_2 / R_1) = \dots$$

$$R_m = \omega_{\text{omax}} - \omega_{\text{omin}} = \dots$$

$$R_1 = R_2 / (R_2 / R_1) = \dots$$

If we adopt a damping and a filter capacitor for example

$$\xi = \dots < 1 \quad (\text{típico } 0,7)$$

$$C_0 = \dots$$

it will determine

$$\tau = 1 / 4 \xi^2 K_f K_0 = \dots \leq 1 / \omega_{\text{mmax}}$$

$$R_0 = 1 / \tau C_0 = \dots$$

Verificamos finalmente que el LFF logre capturar la MF

$$R_c = (2 R_m / \tau)^{1/2} = \dots > 2 (\Delta \omega_c / \tau)^{1/2}$$

### Obtaining for conversion to MP

The following outline shows the operation. The filter is syntonized outside of the carrier frequency and then the detector of phases, of the type for product for example, obtains a voltage proportional average to this difference. Truly any other filter can be used provided that it produces the displacement of phases, but what happens with the syntony series is that it presents the advantages of the amplification of the voltage and that of a good slope to have conjugated poles.

Their behavior equations are the following ones

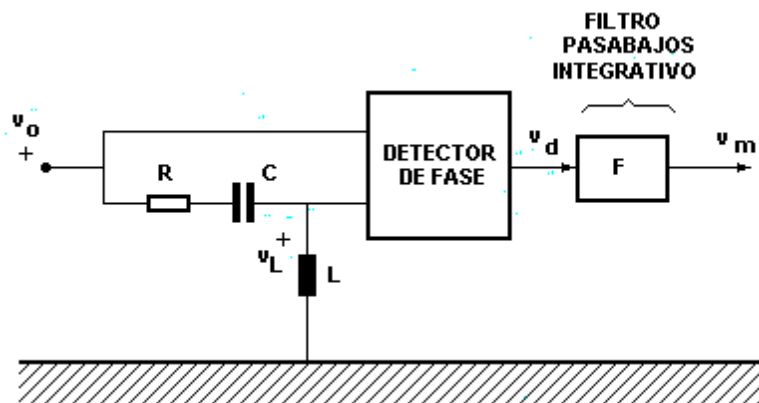
$$T = v_m / v_o = M [ 1 + j / Q(x - 1/x) ] = |T| e^{j\phi}$$

$$M = x^2 / [ 1 + 1/Q(1 + x^2) ]$$

$$x = \omega / \omega_o$$

$$\omega_o = (LC)^{1/2} = RQ/L = 1/QRC > \sim \omega_c$$

$$\phi = \text{arc tg } [1/Q(1 + x^2)]$$



where is for  $\omega_i \sim \omega_o$

$$\phi \sim \text{arc tg } x/Q \sim x/Q$$

for what the detection of phases is

$$\partial \phi / \partial x = 1/Q$$

$$\Delta \phi = (1/Q \omega_o) \Delta \omega_c$$

$$v_m' = K_d F \Delta \phi = (K_d F / Q \omega_o) \Delta \omega_c$$

### Demodulation of Phase (MP)

#### Generalities

We repeat the characteristic equations of the phase modulation



$$\begin{aligned}
v_m(t) &= V_m \cos \omega_m t \\
v_o(t) &= v_o \cos (\omega_c t + \theta) \\
\phi &= \omega_c t + \theta = \int \omega_i \, dt = \omega_c t + \beta \sin \omega_m t \\
\theta &= \beta \sin \omega_m t \\
\partial \theta / \partial t &= \beta \omega_m \sin \omega_m t = k_3 V_m \omega_m \sin \omega_m t \\
\beta &= k_3 V_m = \Delta \omega_c / \omega_m \text{ (index of MP)} \\
\omega_i = \partial \phi / \partial t &= \omega_c + \Delta \omega_c \cos \omega_m t
\end{aligned}$$

and where we can appreciate that we will have in consequence a MF

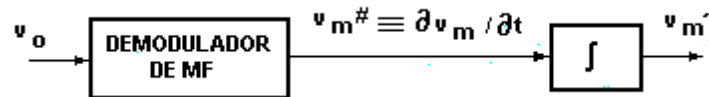
$$\omega_i = \partial \phi / \partial t = \omega_c - \beta \omega_m \sin \omega_m t$$

and we also remember

- wide band ( $B_\phi \geq 5$  being  $B_\phi \sim 2\Delta\omega_c$ )
- short band ( $B_\phi \ll \pi/2$  being  $B_\phi \sim 2\omega_m$ )

## Obtaining

We have explained that when being modulated it derives to the sign useful  $v_m$  and then it transmits it to him as MF. Now, to demodulate it, we use anyone of the demodulators of MF explained previously and we put an integrative one to their output

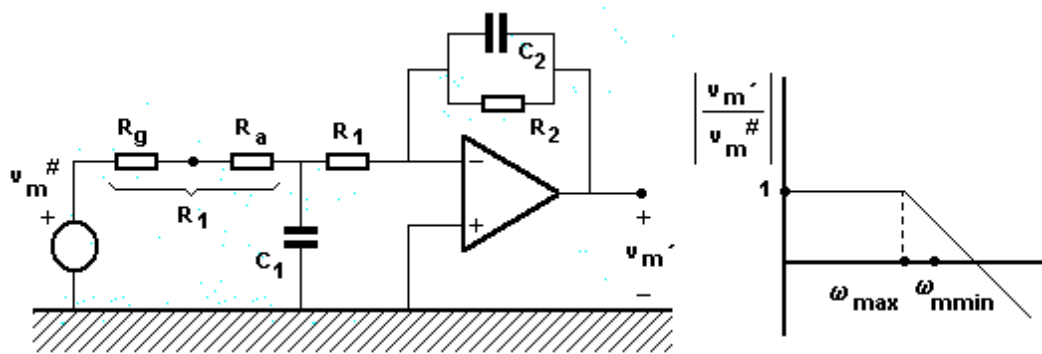


## Design

Be the data (to see their design equations in the chapter of *active networks as filters of frequency and displacements of phases*)

Be the data

$$R_g = \dots \quad f_{mmin} = \dots$$



With the purpose of simplifying the equations we make

$$R_1 = R_g + R_a$$

Of the transfer impedances

$$Z_1 = R_1^2 C_1 (s + 2/R_1 C_1)$$

$$Z_2 = (1/C_2) / (s + 1/R_2 C_2)$$

we express the gain and we obtain the design conditions

$$v_{sal} / v_g = -Z_2 / Z_1 = -\omega_{max}^2 / (s + \omega_{max})$$

$$\omega_{max} = 1 / R_2 C_2$$

$$R_1 = R_2 / 2$$

$$C_1 = 4 C_2$$

and we adopt

$$R_1 = \dots \geq R_g$$

$$\omega_{max} = \dots \leq \omega_{min} / 5$$

what will allow us to calculate

$$R_a = R_1 - R_g = \dots$$

$$R_2 = 2 R_1 = \dots$$

$$C_2 = 1 / \omega_{max} R_2 = \dots$$

$$C_1 = 4 C_2 = \dots$$

## Chap. 29 Heterodyne receivers

### GENERALITIES

#### STAGES OF THE RECEIVER

##### Stage heterodyne

##### Stage intermediate frequency amplifier

##### Stage of demodulation

##### Stage audioamplifier

##### Stage of automatic control

#### REJECTIONS OF UNDESIRABLE FREQUENCIES

#### WHITE NOISE

##### Generalities

##### Figure of noise and their equivalent Temperature

##### In components

##### In a diode

##### In a TBJ

##### In transfers

##### In a filter low-pass

##### In a filter hig-pass derivator

##### In demodulations

##### In demodulations of MAC

##### In demodulations of Mf

##### In demodulations of BLU

##### In demodulations of DBL

##### In demodulations of PCM

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### GENERALITIES

We will study to the basic receiver, guiding our interest to the topics of the commercial broadcasting of MA and MF.

#### STAGES OF THE RECEIVER

##### Stage heterodyne

The first stage of a receiver is the heterodyne. With this name, *converter*, *mixer*, etc., those systems are known that, maintaining the band bases of the sign modulating (in our case of short band 2B), they change the frequency of their carrier «N» times. Their basic principle of operation consists on the product of this sign received  $v_o(t) = v_o \cos (\omega_c t + \theta)$  for another of an oscillator local  $v_x(t) = V_x \cos \omega_x t$  whose phase displacement won't consider because it won't affect to our studies. The equations that define the behavior are based on the product of cosines like it has been presented in previous chapters. The result will be, putting a filter to the band of the spectrum that we want (usually that of smaller order, that is to say  $N = 1$ , to be that of more amplitude and to avoid to change in the index of angular modulation  $\beta$ )

$$v_o(t) = v_o \cos (\omega_c t + \theta) = v_o \cos \omega_c t$$

$$v_x(t) = V_x \cos \omega_x t$$

$$v_y(t) = v_o(t) v_x(t) = v_y \cos (\omega_y t + \theta_y) = v_y \cos \omega_y t$$

$$\omega_y = \omega_c \pm n \omega_x \rightarrow \omega_c - \omega_x$$

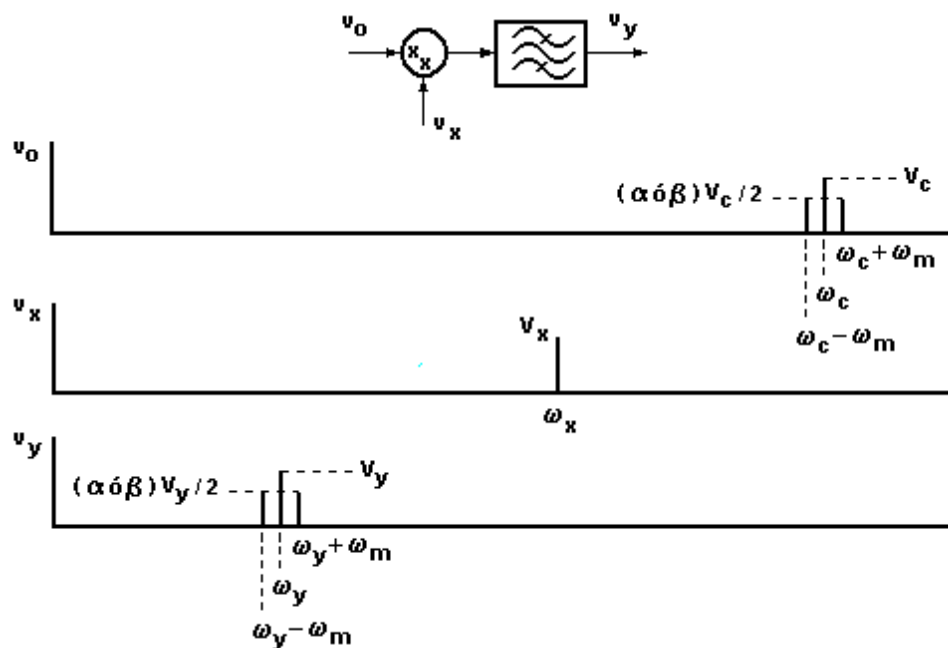
$$\beta_y = N \beta = \beta \omega_c / \omega_y$$

modulated signal

signal of the local oscillator

signal of output of the converter

fundamental inferior (elect)



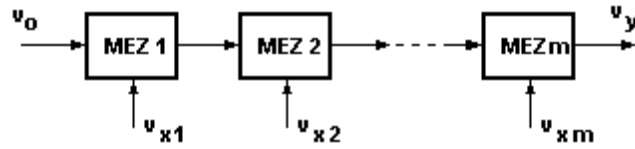
When the frequency  $\omega_c$  is very high, and the difference that we obtain  $\omega_c - \omega_x$  is not the sufficiently small as to work her comfortably, a *multiple conversion* is used. This is, a mixer followed by another

$$v_y(t) = v_o(t) v_{x1}(t) v_{x2}(t) \dots v_{xm}(t)$$

$$[(\omega_c - \omega_{x1}) - \omega_{x2}] - \dots = \omega_c - \omega_{x1} - \omega_{x2} - \dots \omega_{xm}$$

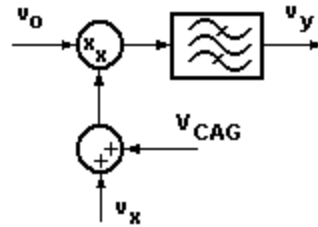
signal of output of the converter  $\omega_y =$

fundamental inferior (elect)



but their inconvenience won't only be in the necessary stability of the local oscillators (that multiply its unstability to each other) but in that change the index of angular modulation a lot ( $N \gg 1$ ).

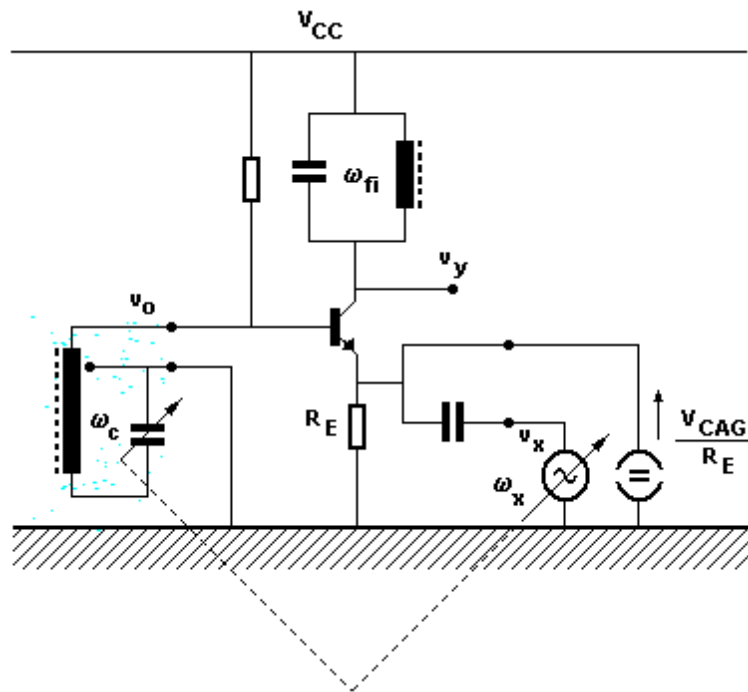
To implement these systems, the idea is to make "walk" the small antenna signal for the great dynamic sign that provides the local oscillator and it changes the polarization of the dispositive; this is, to multiply both signals. Added this, for the receivers of MA, the signal coming from the CAG will modify the point of polarization of this multiplication.



This circuit mixer can contain in itself to the local oscillator or it can have it independently. The approach for the auto-oscillation it is explained in the chapter of *harmonic oscillators*, adding to the topic that an independence will exist among the antenna syntonies and of the local oscillation to be to very distant frequencies ( $\omega_{fi} = \omega_c - \omega_x \gg \omega_m$ ).

The circuit of the figure shows the effect for an independent excitement of sine wave, where the equations that determine it are the following

$$\begin{aligned}
 g_m &= \partial I_C / \partial V_{BE} \approx \beta \partial [I_{BE0} (1 - e^{V_{BE}/V_T})] / \partial V_{BE} = \beta I_{BE0} e^{V_{BE}/V_T} / V_T = I_C / V_T \sim \\
 &\sim 20 I_C = 20 (v_x + V_{CAG}) / R_E = g_m(v_x + V_{CAG}) \\
 v_{ce(\omega_{fi})} &= v_{y(\omega_{fi})} \sim g_m R_L v_o(\omega_c) = 20 (v_x + V_{CAG}) R_L v_o(\omega_c) / R_E = \\
 &= V_y \{ \cos \omega_{fi} t + (\alpha/2) [\cos (\omega_{fi} + \omega_m) t + \cos (\omega_{fi} - \omega_m) t] \} \\
 V_y &= 20 V_x V_{CAG} R_L / R_E
 \end{aligned}$$

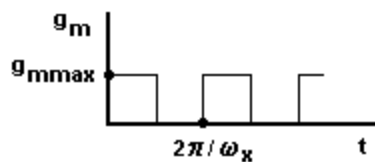


The same effect can also be achieved with transistors of field effect MOS, where the design is simplified if it has two gates, because then each input is completely independent of the other

A comfortable variant and practice of this conversion, are simply achieved commuting to the dispositive. This avoids difficult designs of the stability of the local oscillator. Their *conversion transconductance* then taking the form of a square wave and it is equal to consider it with a harmonic content

$$g_m = (g_{mmax}/2) + (2g_{mmax}/\pi) \cos \omega_x t + (2g_{mmax}/3\pi) \cos 3\omega_x t + \dots$$

$$g_{mmax} = g_{m(ICmax)} = V_{CC}/R_E V_T \sim 20 V_{CC}/R_E$$



where each harmonic will mix with  $v_o$  and the collector filter will obtain the  $\omega_{fi}$ .

### Stage intermediate frequency amplifier

After the conversion stage we meet with a low comfortable frequency of amplifying. Usually of two syntonized amplifiers, they make jointly with the third synton of the conveter a group that it will be designed appropriately. In the chapter of *amplifiers of RF of low level class A* different possibilities were analyzed already in this respect.

What we will add like useful fact to the designer and/or man-caliper of these syntonies, is that it always suits to make it of behind forward, so that the loads of final stages leave giving to the first ones.

## Stage of demodulation

This stage has already been seen in the *demodulation* chapters.

## Stage audioamplifier

This stage has already been seen in the chapters of *amplifiers of AF in low level class A* and in the one of *high level classes A and B*.

## Stage of automatic control

Either for MA like MF it is convenient to feedback the receiving system so that the average volume of reception doesn't fluctuate for reasons like the physiquess of the land and ambient, the pedestrian's mobility, changes of the local oscillator, etc.

Subsequently we present their general outline, where it is that, being the sign useful  $x_o$  (either  $V_c$  in MA or  $\omega_c$  MF) it will be transferred as  $x_m'$  (it either corresponds to  $v_m'$  in MA or MF) or demodulated output, and that we seek to diminish their changes  $\Delta x_o$ . It is this way, if we put a filter low-pass to the output of the demodulator that integrates the spectrum of the band bases B, that is to say with a pole in  $1/\tau$ , we have the following equations

$$G = G_c G_{fi} \eta$$

$$H = H_0 / (1 + s\tau)$$

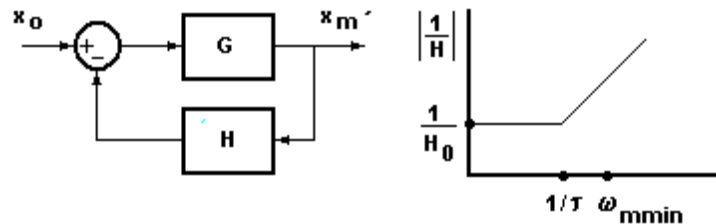
$$\Delta x_m' \sim \Delta x_o / H = \Delta x_o (1 + s\tau) / H_0$$

where

$G_c$  gain of the converter

$G_{fi}$  gain of the stages of intermediate frequency

$\eta$  gain or demodulation efficiency (or detection)



that it allows to observe that, only outside of the band bases B, that is to say for the carrier, the magnitude  $H_0$  should be made the biggest thing possible.

This way, when we speak of MA it is

$$\Delta v_m' \sim \Delta V_c / H = \Delta V_c (1 + s\tau) / H_0$$

and when we make it of MF

$$\Delta v_m' \sim \Delta \omega_c / H = \Delta \omega_c (1 + s\tau) / H_0$$

## REJECTIONS OF UNDESIRABLE FREQUENCIES

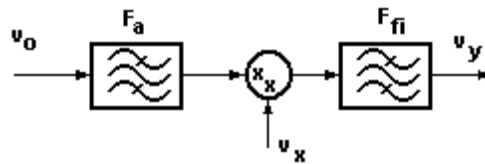
We can say that we have three frequencies that a receiver should reject

- all *frequency of magnitude of the intermediate* one that it receives to their antenna input
- all *frequency image* of the local oscillator that it receives to their antenna entrance
- all *frequency of adjacent channel* to the one syntonized that it penetrates for contiguity

With this end we should keep in mind the syntonized filters, usually of simple syntony that they are used so much in the antenna  $F_a$  as in the converter like first intermediate frequency  $F_{fi}$ . As we have seen in the chapter *amplifiers of RF of low level class A*, the transfer for the simple syntonized circuit will have the following form

$$F_a = 1 / [ 1 + j 2 ( \omega - \omega_c ) / B ) ] \sim 1 / [ 1 + j 2Q_a ( \omega - \omega_c ) / \omega_c ) ]$$

$$F_{fi} = 1 / [ 1 + j 2 ( \omega - \omega_{fi} ) / B ) ] \sim 1 / [ 1 + j 2Q_a ( \omega - \omega_{fi} ) / \omega_{fi} ) ]$$



and their modules

$$|F_a| = 1 / \{ 1 + [ 2Q_a(\omega - \omega_c) / \omega_c ]^2 \}^{1/2}$$

$$|F_{fi}| = 1 / \{ 1 + [ 2Q_{fi}(\omega - \omega_{fi}) / \omega_{fi} ]^2 \}^{1/2}$$

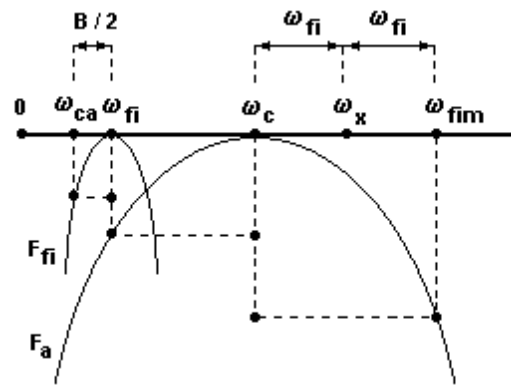
what will allow to be defined, respectively, the *rejections to the intermediate frequency*  $R_{fi}$ , *frequency image*  $R_{fim}$  and *frequency of the adjacent channel*  $R_{ca}$

$$R_{fi} = 1 / \{ 1 + [ 2Q_a(\omega_{fi} - \omega_c) / \omega_c ]^2 \}^{1/2} \rightarrow \text{reject in the antenna syntony}$$

$$R_{fim} = 1 / \{ 1 + [ 2Q_a(\omega_{fim} - \omega_c) / \omega_c ]^2 \}^{1/2} \rightarrow \text{reject in the antenna syntony}$$

$$R_{ca} = 1 / \{ 1 + [ 2Q_{fi}(\omega_{ca} - \omega_{fi}) / \omega_{fi} ]^2 \}^{1/2} \rightarrow \text{reject in the FI syntony}$$



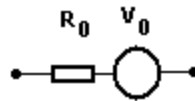


## WHITE NOISE

### Generalities

It is known that the *white noise* consists on a stochastic molecular action of constant spectral energy density that follows the following law of effective voltage  $V_0$  on a resistance  $R_0$ , physics or distributed as it can be that of an antenna, to a temperature  $T_0$  and in a width of band  $B_0$

$$V_0^2 = 4KT_0R_0B_0$$



with

$$K \sim 1,38 \cdot 10^{-23} \text{ [J/seg}^0\text{K]}$$

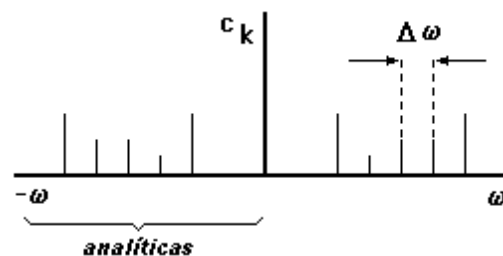
$$R_0 \text{ [}\Omega\text{]}$$

$$T_0 \text{ [}^0\text{K]}$$

$$B_0 \text{ [Hz]}$$

On the other hand and generalizing, keeping in mind that to the noise like it is aleatory and it possesses average null value, it can express it in function of their harmonics as

$$n = \sum_{-\infty}^{\infty} c_k \cos (k \Delta\omega t + \theta_k)$$



it is defined therefore of the same one their *spectral density of power normalized* in the considered band width

$$G = \sum_0^\infty c_k^2 / B_0$$

and what will allow to be defined the *normalized power of noise* on 1 [ $\Omega$ ]

$$N = \int G \partial f$$

and then with this to find the total of the whole spectrum

$$N_T = \int_{-\infty}^0 G_{(\omega)} \partial(-f) + \int_0^\infty G_{(\omega)} \partial f = 2 \int_0^\infty G_{(\omega)} \partial f$$

and that it will determine the concept in turn of *wide of equivalent or effective band of noise*  $B_{eq}$

$$N_T = \int_0^\infty G_{(\omega)} \partial f = G_{max} B_{eq}$$

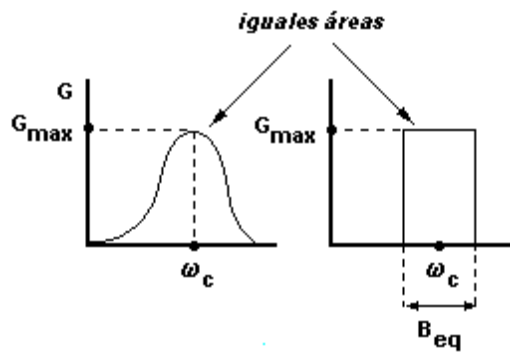


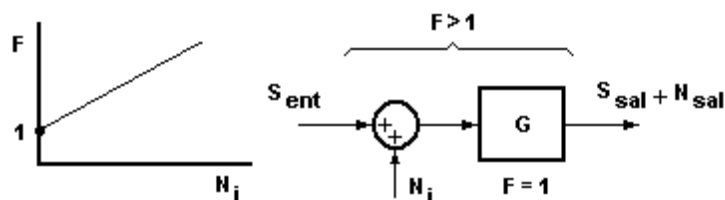
Figure of noise and their equivalent Temperature

Be an amplifier of gain of power  $G$

$$G = S_{sal} / S_{ent}$$

that it possesses internal white noise of power  $N_i$ . We can find him their factor of inefficiency like the one denominated *figure of noise*  $F$

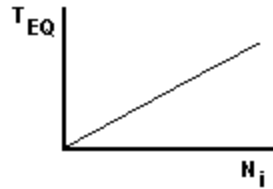
$$F = (S_{ent} / N_i) / (S_{sal} / N_{sal}) = (S_{ent} / S_{sal}) (N_{sal} / N_i) = (N_{sal} / N_i) / G = \\ = [G (N_{ent} + N_i) / N_i] / G = 1 + N_i / N_{ent}$$



and if it is to an ambient temperature  $T_A$  we can also say that this internal noise is produced by an *equivalent temperature*  $T_{EQ}$

$$F = 1 + N_i/N_{ent} = 1 + (4KT_{EQ}R_0B_0 / 4KT_AR_0B_0) = 1 + (T_{EQ} / T_A)$$

$$T_{EQ} = T_A (F - 1)$$



When we have two stages in cascade it is

$$G_T = N_{sal} / N_{ent} = G_1 G_2$$

$$N_{sal} = N_{en} G_T F_T = (N_{sal1} + N_{i2}) G_2 = N_{ent} G_2 [G_1 F_1 + (F_2 - 1)]$$

$$F_T = F_1 + (F_2 - 1)/G_1$$

$$T_{EQT} = T_{EQ1} + T_{EQ2}/G_1$$

and for more stages

$$F_T = F_1 + (F_2 - 1)/G_1 + (F_3 - 1)/G_1 G_2 + \dots$$

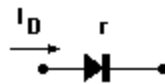
$$T_{EQT} = T_{EQ1} + T_{EQ2}/G_1 + T_{EQ3}/G_1 G_2 + \dots$$

## In components

### In a diode

We have the following expression when it polarizes it in direct

$$V_0^2 \sim (4KT_0 - 2eI_F r) r B_0$$



where

$r [\Omega]$	<i>dynamic resistance</i>
$e = 1,6 \cdot 10^{-19} [\text{Cb}]$	<i>charge of the electron</i>
$I_D [\text{A}]$	<i>direct current of polarization</i>

### In a TBJ

We have the following expression when it polarizes it in direct

$$I_{0E}^2 \sim 2eI_E B_0$$

$$I_{0C}^2 \sim 2eI_C B_0 (1 - \alpha) \{ 1 + [\omega / \omega_\alpha (1 + \alpha)^{1/2}]^2 \} / [1 + (\omega / \omega_\alpha)^2]$$

$$V_{0B}^2 \sim 4KT_0 B_0 r_{bb'}$$



and the maker of dispositives of RF the expressed thing in tables and abacus according to the polarization and work frequency, preferably offering the *figure of noise*.

## In transfers

### In a filter low-pass

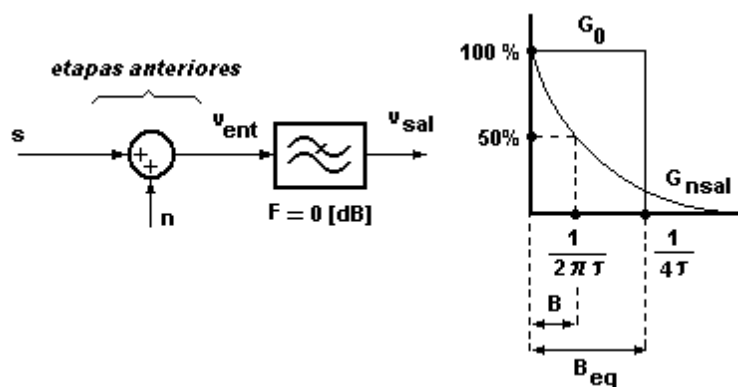
According to the following drawing we have for a signal «s» and a noise «n»

$$v_{sal} / v_{ent} = 1 / (1 + s\tau)$$

$$v_{ent} = s + n$$

$$G_{nent} = G_0 \text{ (constant)}$$

$$G_{sal} / G_{ent} = |v_{sal} / v_{ent}|^2 = 1 / [1 + (\omega\tau)^2]$$



and applying overlapping

$$G_{nsal} = G_{nent} |v_{sal} / v_{ent}|^2 = G_0 / [1 + (\omega\tau)^2]$$

$$N_{salmax} = \int_0^\infty G_{nsal} \partial f = G_0 / 4\tau$$

$$B_{eq} = N_{salmax} / G_{nsalmax} = 1 / 4\tau$$

## In a filter hig-pass derivator

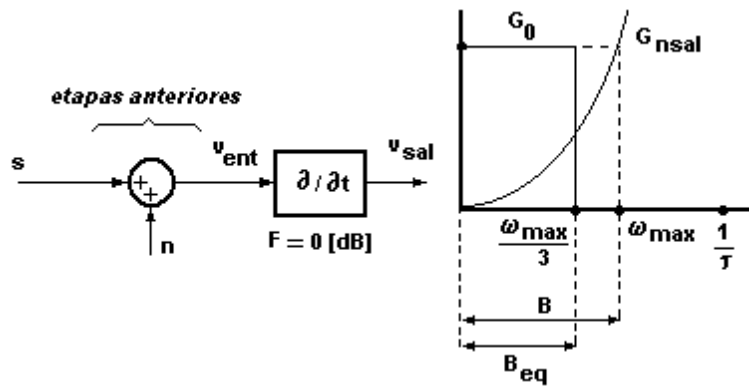
According to the following drawing we have for a signal «s» and a noise «n» in the band in passing derivative

$$v_{sal} / v_{ent} = s\tau / (1 + s\tau) \sim s\tau \quad (\text{derivation condition } \omega_{max} \ll 1/\tau)$$

$$v_{ent} = s + n$$

$$G_{nent} = G_0 \text{ (constante)}$$

$$G_{sal} / G_{ent} = |v_{sal} / v_{ent}|^2 = (\omega\tau)^2$$



and applying overlapping

$$G_{nsal} = G_{nent} |v_{sal} / v_{ent}|^2 = G_0 (\omega\tau)^2$$

$$N_{salmax} = \int_0^{\omega_{max}} G_{nsal} df = G_0 \tau^2 \omega_{max}^2 / 3$$

$$B_{eq} = N_{salmax} / G_{nsalmax} = \omega_{max}^2 / 3$$

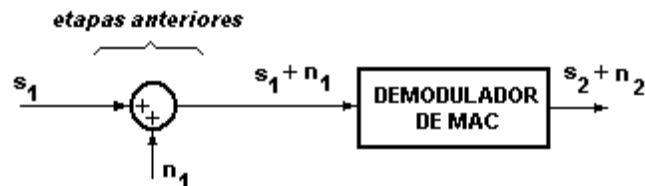
## In demodulations

### In demodulation of MAC

For the following reception we have (to go to the chapter of *modulation of amplitude*)

$$s_1 = v_o(t) = V_c (\alpha \cos \omega_m t \cos \omega_c t + \cos \omega_c t)$$

$$G_{n1} = G_{n10} \text{ (constant)}$$



then

$$S_1 = (V_c^2/2) + S_{1(2BL)}$$

$$S_{1(2BL)} = 2 S_{1(BLU)} = 2 [ (\alpha V_c/2) / \sqrt{2} ]^2 = \alpha^2 V_c^2/4$$

of where it is deduced

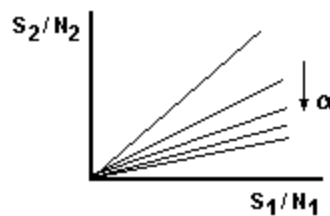
$$S_1 = S_{1(2BL)} (1 + 2/\alpha^2)$$

If now we suppose that the module of the transfer demodulation (detection efficiency) it is unitary

$$S_2 / N_2 = S_{1(2BL)} / N_{1(2BL)} \quad \text{(the carrier } V_c^2/2 \text{ one doesn't keep in mind for not being modulation, and the noise doesn't have carrier)}$$

we can obtain finally

$$F = (S_1 / N_1) / (S_2 / N_2) = (S_1 / N_{1(2BL)}) / (S_{1(2BL)} / N_{1(2BL)}) = (1 + 2/\alpha^2)$$

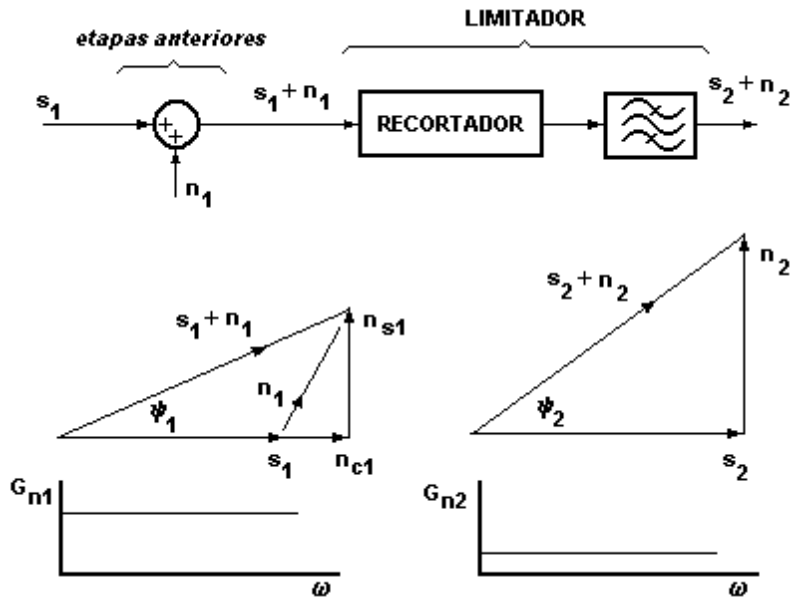


### In demodulation of $M\phi$

For the following reception with a limiter of amplitude see that although it improves the relationship sign to noise of amplitude ( $G_{n2} < G_{n1}$ ), it is not this way in the angular ( $\psi_2 > \psi_1$ )

$$G_{n1} \text{ (constant)}$$

$$G_{n2} \text{ (constant)}$$

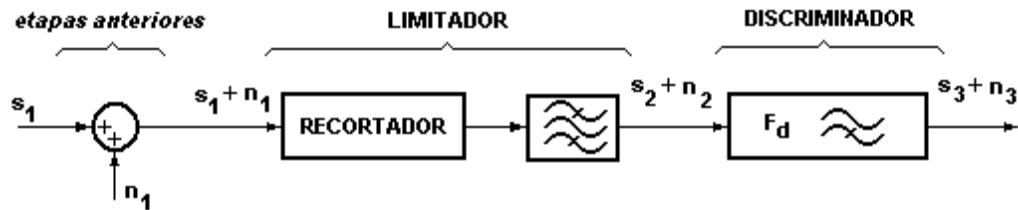


Now proceed to detect (to go to the chapter of *angle modulation*)

$$s_1 = v_o(t) = v_o \cos \phi = V_c \cos (\omega_c t + \beta \sin \omega_m t) = V_c \cos (\omega_c t + k \int V_m \cos \omega_m t \, dt)$$

$$s_3 = k_c (\partial \phi / \partial t) = k k_c V_m \cos \omega_m t$$

$$S_3 = [k k_c (V_m / \sqrt{2})]^2 / 2$$



it is

$$s_1 + n_1 = [(V_c + n_{c1})^2 + n_{s1}^2]^{1/2} \cos \{ \phi + \arctan [n_{s1} / (V_c + n_{c1})] \}$$

$$s_2 + n_2 = (V_c^2 + n_{c1}^2)^{1/2} \cos [ \phi + \arctan (n_{s1} / V_c) ] \approx V_c \cos [ \phi + (n_{s1} / V_c) ]$$

$$s_3 + n_3 = k_c \{ \partial [ \phi + (n_{s1} / V_c) ] / \partial t \} = k_c (\partial \phi / \partial t) + k_c (\partial n_{s1} / \partial t)$$

where the output noise is observed

$$n_3 = k_c (\partial n_{s1} / \partial t)$$

If the transfer of the discriminator is supposed like derivative

$$F_d \sim s \tau$$

we will be able to find

$$G_{n3} = G_{n2} |F_d|^2 \rightarrow G_{n2} (\omega\tau)^2$$

$$N_3 = (1/2\pi) \int_0^B G_{n3} d\omega = G_{n2} B^3 \tau^2 / 6\pi$$

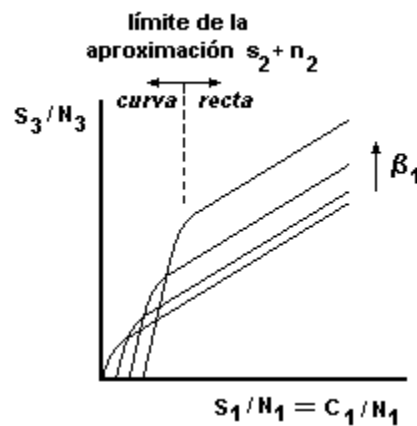
$$S_3 / N_3 = \{ [k k_c (V_m / \sqrt{2})]^2 / 2 \} / (G_{n2} B^3 \tau^2 / 6\pi) = k_0 [S_{1(2BL)} / N_{1(2BL)}] [(V_m^2 / 2) / B]$$

or

$$S_3 / N_3 = \beta_1^2 [3\pi G_{n1} k^2 k_c^2 / V_c^2] (C_1 / N_1)$$

$$\beta_1 = V_m / B$$

$$C_1 / N_1 = S_1 / N_1 = (V_c^2 / 2) / G_{n1} B$$



## In demodulation of BLU

Be the following reception (to go the chapter of *demodulation of MA*)

$$s_1 = v_o(t) = (\alpha V_c / 2) \cos (\omega_c + \omega_m)t$$

$$v_o(t)' = V_c \cos (\omega_c t + \psi) \rightarrow V_c \cos \omega_c t \quad (\text{it doesn't interest } \psi)$$

$$S_1 = (\alpha V_c / 2)^2 / 2$$

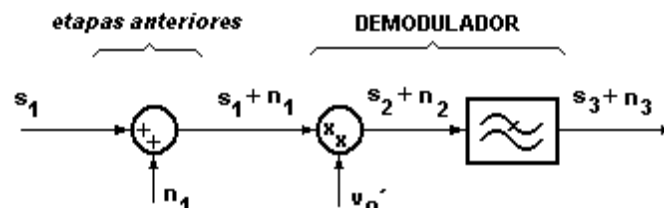
$$G_{n1} \text{ (constant)}$$

$$s_2 = s_1 v_o(t)' = [(\alpha V_c / 2) / 2] [\cos (2\omega_c + \omega_m)t + \cos \omega_m t]$$

$$s_3 = [(\alpha V_c / 2) / 2] \cos \omega_m t$$

$$S_3 = [(\alpha V_c / 2) / 2]^2 / 2 = (\alpha V_c / 2)^2 / 8 = G_T S_1$$

$$G_T = S_3 / S_1 = 1 / 4 \quad (\text{gain of power of the total system demodulator})$$

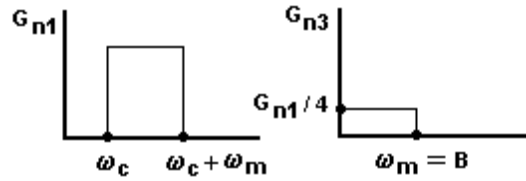




of where it is deduced by overlapping

$$N_1 = G_{n1} B_{eq}$$

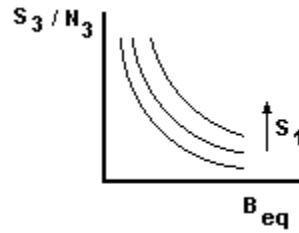
$$N_3 = G_T N_1 = N_1 / 4$$



being finally

$$S_3 / N_3 = [(\alpha V_c / 2)^2 / 8] / (G_{n1} B_{eq} / 4) = (\alpha V_c / 2)^2 / 2 G_{n1} \omega_m = S_1 / G_{n1} \omega_m$$

$$F = (S_1 / N_1) / (S_2 / N_2) = 1$$



### In demodulation of DBL

Be the following reception (to go the chapter of *demodulation of MA*)

$$s_1 = v_o(t) = (\alpha V_c / 2) \cos [\cos (\omega_c + \omega_m)t + \cos (\omega_c - \omega_m)t]$$

$$v_o(t)' = V_c \cos (\omega_c t + \psi) \rightarrow V_c \cos \omega_c t \quad (\text{it doesn't interest } \psi)$$

$$S_1 = 2 \cdot (\alpha V_c / 2)^2 / 2 = (\alpha V_c / 2)^2$$

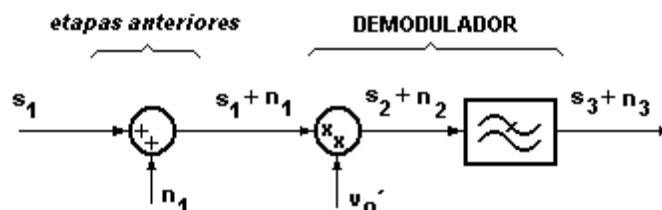
$$G_{n1} \text{ (constant)}$$

$$s_2 = s_1 v_o(t)' = [(\alpha V_c / 2) / 2] (1 + \cos 2\omega_c t)$$

$$s_3 = (\alpha V_c / 2) \cos \omega_m t$$

$$S_3 = (\alpha V_c / 2)^2 / 2 = G_T S_1$$

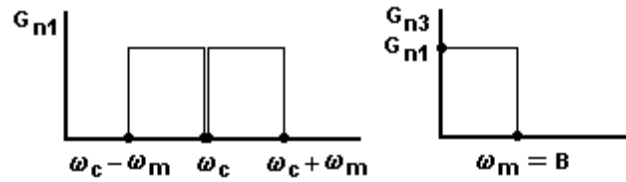
$$G_T = S_3 / S_1 = 1 / 2 \quad (\text{gain of power of the total system demodulator})$$



of where it is deduced by overlapping

$$N_1 = G_{n1} B_{eq}$$

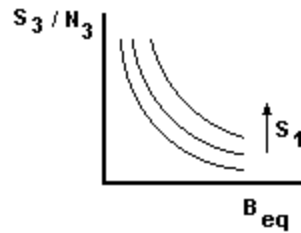
$$N_3 = G_T N_1 = N_1 / 2$$



being finally

$$S_3 / N_3 = [(\alpha V_c / 2)^2 / 2] / (G_{n1} B_{eq} / 2) = (\alpha V_c / 2)^2 / 2 G_{n1} \omega_m = S_1 / G_{n1} \omega_m$$

$$F = (S_1 / N_1) / (S_2 / N_2) = 1$$



and if we compare BLU with DBL we see the equality

$$(S_3 / N_3)_{DBL} = (S_3 / N_3)_{BLU} = S_{1BLU} / G_{n1} B_{eq}$$

## In demodulation of PCM

See you the chapter of *modulation of amplitude*.

## Chap. 30 Lines of Transmission

### GENERALITIES

### STRUCTURES PHYSICS

### ADAPTATION OF IMPEDANCES

#### Generalities

#### Transformation of $\lambda/4$

#### Design

#### Adapting stubs

#### Generalities

#### Design of an admittance

#### Design of adaptation with a known load

#### Design of adaptation with an unknown load

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### GENERALITIES

We summarize the introductory aspects of the equations that we will use subsequently

$$Z_a \sim Z_0 \sim 377 [\Omega] + j 0$$

impedance of the air or vacuum

$$\varepsilon = \varepsilon_r \varepsilon_0 [\text{F/m}]$$

electric impermeability

$$\varepsilon_0 \sim 88,5 \cdot 10^{-12} [\text{F/m}]$$

electric impermeabilidad of the vacuum

$$\varepsilon_r [\text{veces}]$$

relative electric impermeability to the vacuum

$$\mu = \mu_r \mu_0$$

magnetic permeability

$$\mu_0 \sim 12,6 \cdot 10^{-7} [\text{A/m}]$$

magnetic permeability of the vacuum

$$\mu_r [\text{veces}]$$

relative magnetic permeability to the vacuum

$$\Gamma = A + j B$$

space function of propagation

$$A = \omega / v [\text{Neper/m}]$$

space function of attenuation

$$B = 2\pi / \lambda [\text{rad/m}]$$

space function of phase

$$Z_{\text{ent}} = Z_0 [Z_L + Z_0 \tanh \Gamma x] / [Z_0 + Z_L \tanh \Gamma x]$$

Input impedance to a transmission line at a distance «x» of their load  $Z_L$

$$Z_0$$

characteristic impedance

$$Z_L$$

load impedance

$$v = 1 / (\mu \varepsilon)^{1/2}$$

propagation speed

$$c = 1 / (\mu_0 \varepsilon_0)^{1/2} \sim 3 \cdot 10^8 [\text{m/seg}^2]$$

speed of propagation of the light in the vacuum

$$\text{ROE} = V_{\max} / V_{\min} = \\ = (1 + |\rho_v|) / (1 - |\rho_v|) [\text{veces}]$$

relationship of stationary wave of voltage

$$\rho_v = |\rho_v| e^{j\phi}$$

coefficient of reflection of the electric field (or also call of voltage)

$$E_Q = Z_a H_Q$$

electric field in a point «Q» of the space of air or vacuum

$$\vec{P} = \vec{E} \times \vec{H} \text{ [W/m}^2\text{]}$$

vector of power of Pointing

and their characteristic magnitudes

	VACÍO	AIRE	AGUA	GOMA	PARAFINA	MICA	PLÁSTICO	BAQUIELITA
$\epsilon_r$ [veces]	1	~ 1	≈ 80	≈ 3	≈ 2,1	≈ 6	≈ 2,5	≈ 5
$\mu_r$ [veces]	1	~ 1	≈ 1					

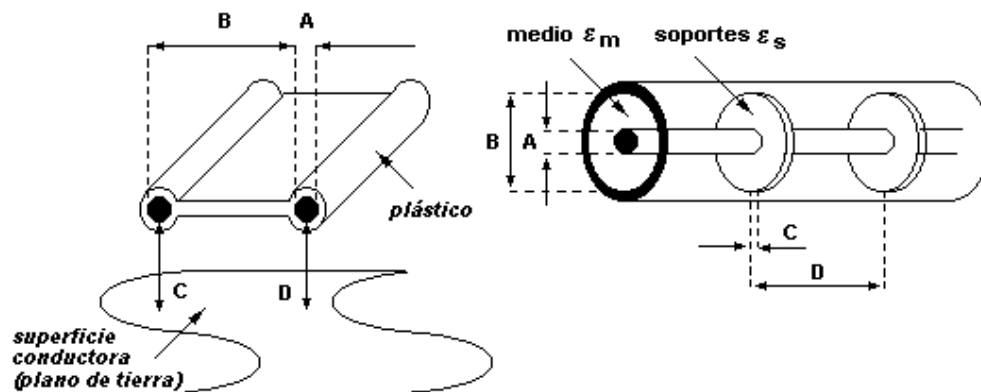
## STRUCTURES PHYSICS

A transmission line is a symmetrical network and, therefore, they are valid all our studies made in the chapter of *passive networks as adapters of impedance*.

We show two classic types, the *coaxial* and the *parallel*. For each case their impedances characteristic is

$$Z_{0(\text{COAXIAL})} = R_0 \sim (138 \log B/A) / \{ \epsilon_{rm} + [ (\epsilon_{rs} - \epsilon_{rm})(C/D) ] \}^{1/2} [\Omega]$$

$$Z_{0(\text{PARALELO})} = R_0 \sim 276 \log \{ (2B/A) / [ 1 + (B^2/4CD) ] \}$$



where

$$\epsilon_{rm} = \epsilon_m / \epsilon_o [\text{veces}]$$

## ADAPTATION OF IMPEDANCES

### Generalities

It is possible to adapt impedances  $Z_0$  among lines, loads and generators, with the help of properly cut pieces of other lines of transmission of characteristic impedance  $Z_{00}$ . We will work here with lines of worthless losses, this is

$$\begin{aligned} B &\geq 5 A \\ \Gamma &\sim 0 + j B \\ Z_{ent} &\sim Z_0 [Z_L + j Z_0 \tanh Bx] / [Z_0 + j Z_L \tanh Bx] \end{aligned}$$

and estimating a speed inside them of the order of that of the light

$$v \sim c$$

### Transformation of $\lambda/4$

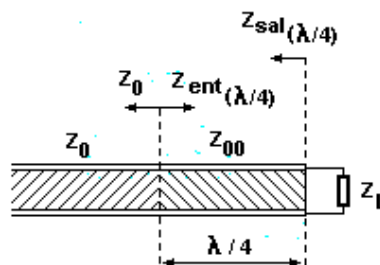
#### Design

Be the data

$$Z_L = \dots \quad Z_0 = \dots$$

The technique consists on adding him a piece of  $\lambda/2$

$$\begin{aligned} Z_{ent(\lambda/4)} &= Z_{00} [Z_L + j Z_{00} \tanh B\lambda/4] / [Z_{00} + j Z_L \tanh B\lambda/4] = Z_{00}^2 / Z_L \\ Z_{sal(\lambda/4)} &= Z_{00}^2 / Z_0 \end{aligned}$$



consequently, if we design

$$Z_{00} = (Z_0 Z_L)^{1/2} = \dots$$

It is

$$\begin{aligned} Z_{ent(\lambda/4)} &= Z_0 \\ Z_{sal(\lambda/4)} &= Z_L \end{aligned}$$

### Adapting stubs

#### Generalities

As we will always work with oneself line of transmission  $Z_0$  ( $Z_{00} = Z_0$ ), then we will be able to normalize the magnitudes of the impedances and admittances as much for the generator as for the load

$$z_g = Z_g / Z_0 = r_g + j x_g$$

$$y_g = Z_0 / Z_g = g_g + j b_g$$

$$z_L = Z_L / Z_0 = r_L + j x_L$$

$$y_L = Z_0 / Z_L = L_L + j b_L$$

We will use for this topic Smith's abacus that we reproduce subsequently. We will try to interpret it; for we express it the normalized reflection coefficient

$$\rho_v = (Z_L - Z_0) / (Z_L + Z_0) = (z_L - 1) / (z_L + 1) = u + j w$$

and if we work

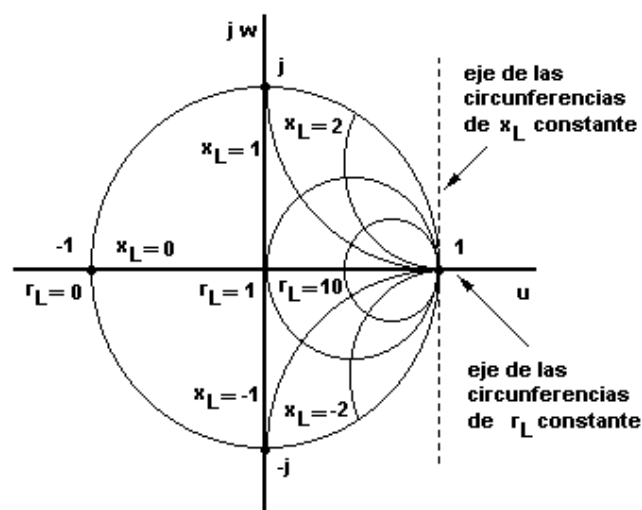
$$z_L = r_L + j x_L = (1 - u^2 - w^2 + j 2w) / (1 - 2u + u^2 + w^2)$$

we will be under conditions of drawing the circles of constant  $r_L$  and of constant  $x_L$

$$r^2 = (u - m)^2 + (r - n)^2$$

$$r_L = \text{cte} \Rightarrow \begin{aligned} r &= 1 / (1 + r_L) \\ m &= r_L / (1 + r_L) \\ n &= 0 \end{aligned}$$

$$x_L = \text{cte} \Rightarrow \begin{aligned} r &= 1 / x_L \\ m &= 1 \\ n &= 1 / x_L \end{aligned}$$

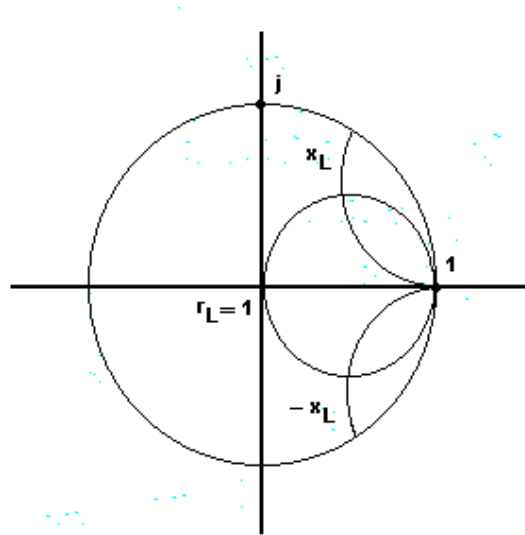


Now look for that is to say in the graph those points that mean perfect adaptation, as

$$\rho_v = 0 + j 0$$

$$z_L = 1 + j 0$$

being determined with it the curve  $r_L = 1$  when being connected the adapting stubs

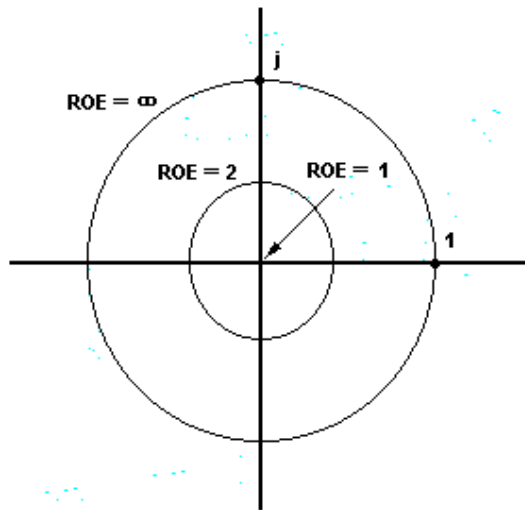


Now we can find the points of this graph of it ROE it constant

$$ROE = (1 + |\rho_v|) / (1 - |\rho_v|) = [1 + (u^2 + v^2)^{1/2}] / [1 - (u^2 + v^2)^{1/2}] = \text{cte}$$

originating

$$[(ROE - 1) / (ROE + 1)]^2 = u^2 + v^2$$



On the other hand, like on the load we have

$$\rho_{v(0)} = (Z_L - Z_0) / (Z_L + Z_0) = |\rho_{v(0)}| e^{j\phi(0)} = u_{(0)} + j w_{(0)}$$

and at a generic distance «x»

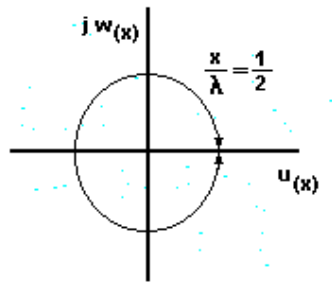
$$Z_{ent(x)} = Z_0 [Z_L + j Z_0 \tanh \Gamma x] / [Z_0 + j Z_L \tanh \Gamma x]$$

$$\rho_{v(x)} = (Z_{ent(x)} - Z_0) / (Z_{ent(x)} + Z_0) = \rho_{v(0)} e^{-j 2\Gamma(x)} = u_{(x)} + j w_{(x)}$$

what will determine rejecting the losses

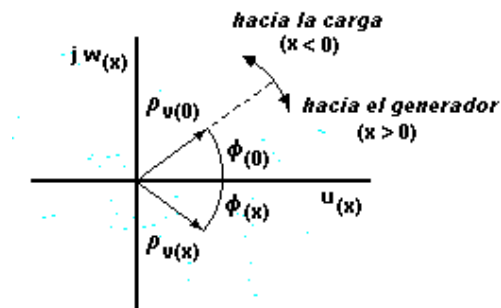
$$\rho_{v(x)} \sim \rho_{v(0)} e^{-j 2B(x)} = \rho_{v(0)} e^{-j 2(2\pi/\lambda x)}$$

that is to say that can have represented this space phase on the abacus if we divide their perimeter in fractions of  $x/\lambda$ .



Also, if we consider the new angle here  $\phi$ , it is

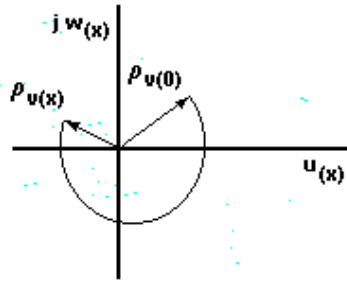
$$\rho_{v(x)} = |\rho_{v(x)}| e^{j \phi(x)} = |\rho_{v(0)}| e^{j \phi(x)} = |\rho_{v(0)}| e^{j [\phi(0) - 2(2\pi/\lambda x)]}$$



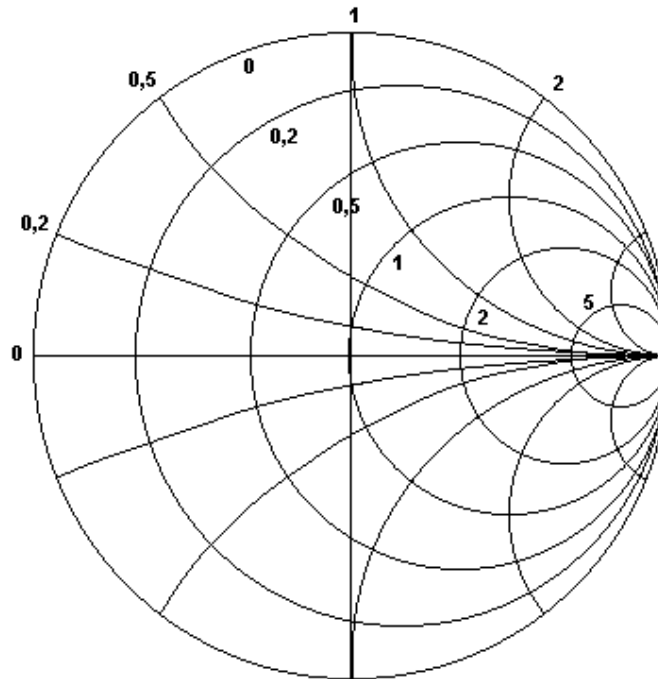
When the previous equation is not completed  $\rho_{v(x)} \sim \rho_{v(0)} e^{-j 2(2\pi/\lambda x)}$  it is of understanding that

$$|\rho_{v(x)}| < |\rho_{v(0)}|$$





Subsequently we reproduce Smith's abacus



### Design of an admittance

With the purpose of to adapt or to syntonize loads, we can appeal to this method to connect in derivation.

Be the data

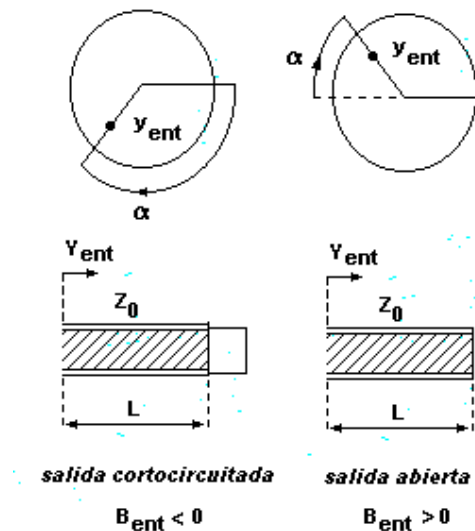
$$Z_0 = R_0 = \dots \quad f = \dots \quad Y_{ent} = G_{ent} + j B_{ent} = \dots \quad B_{ent} \geq \leq 0$$

We find the normalization

$$y_{ent} = g_{ent} + j b_{ent} = G_{ent} R_0 + j B_{ent} R_0 = \dots$$

and we work on Smith's abacus like it is indicated next with the purpose of diminishing the longitude of the stub, and we obtain

$$\alpha = \dots$$



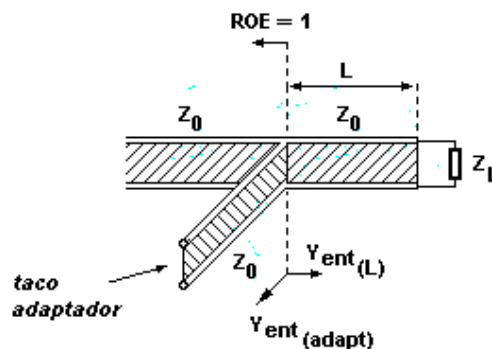
what will allow us to calculate their dimension finally

$$L = \alpha \lambda = \alpha f / c \sim 3,33 \alpha f 10^{-9} = \dots$$

### Design of adaptation with a known load

Be the data

$$Z_0 = R_0 = \dots \quad f = \dots \quad Y_L = G_L + j B_L = \dots \quad B_L \geq \leq 0$$



We find the normalization

$$y_L = g_L + j b_L = G_L R_0 + j B_L R_0 = \dots$$

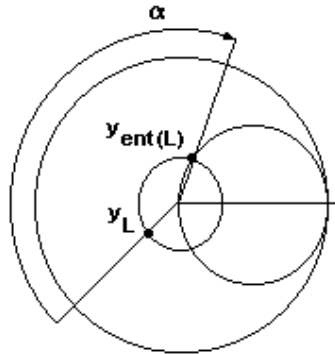
and we work on Smith's abacus like it is indicated next with the purpose of diminishing the longitude of the stub, and we obtain

$$\alpha = \dots$$

$$y_{ent(L)} = 1 + j b_{ent(L)} = \dots$$

$$L = \alpha \lambda = \alpha f / c \sim 3,33 \alpha f 10^{-9} = \dots$$

$$y_{ent(adap)} = -j b_{ent(L)} = \dots$$



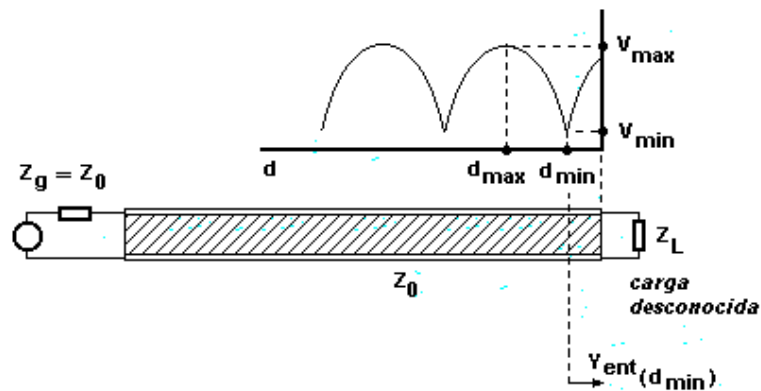
and finally

$$Y_{ent(adap)} = y_{ent(adap)} / R_0 = \dots$$

### Design of adaptation with an unknown load

Be the data

$$Z_0 = R_0 = \dots \quad d_{max} = \dots \quad d_{min} = \dots \quad ROE_{medida} = V_{max} / V_{min} = \dots$$



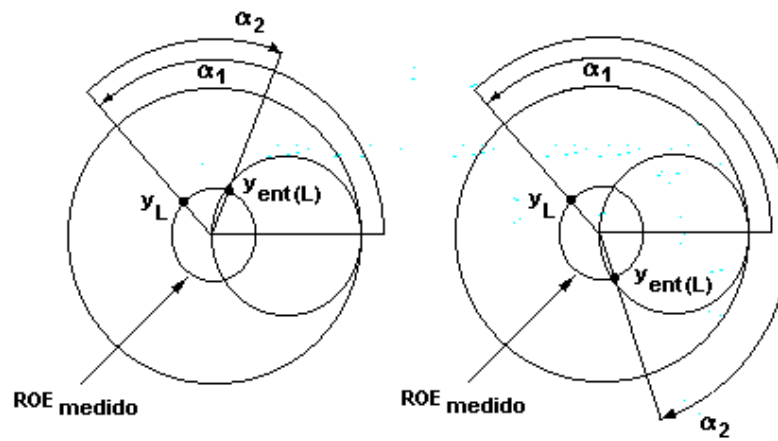
We obtain the wave longitude

$$\lambda \sim 4 (d_{max} - d_{min}) = \dots$$

what will allow us to obtain according to the case for the smallest longitude in the adapting stub

$$\alpha_1 = d_{min} / \lambda = \dots$$

$$\alpha_2 = \dots$$

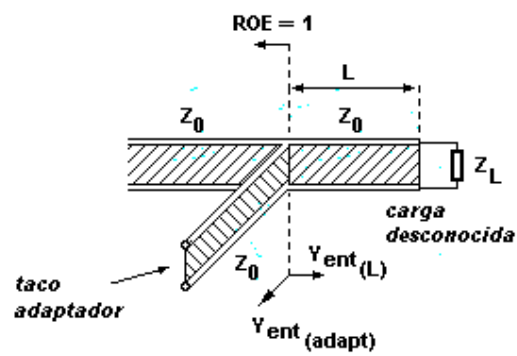


for what is finally

$$L = \alpha_2 \lambda = \dots$$

$$y_{\text{ent}(L)} = 1 + j b_{\text{ent}(L)} = \dots$$

$$Y_{\text{ent(adap)}} = -j b_{\text{ent}(L)} / R_0 = \dots$$



## Chap. 31 Antennas and Propagation

### GENERALITIES

#### RADIATION

##### Generalities

##### Small conductor

#### DIPOLE ANTENNA

##### Short dipole

##### Dipole of half wave

##### Dipoles of half wave folded

##### Dipole of half wave with earth plane

##### Dipole of half wave with elements parasites

##### Dipole of half wave of wide band

##### Dipole of half wave of short band

#### SQUARE ANTENNA

##### Antenna with ferrite

#### PARABOLIC ANTENNA

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### GENERALITIES

The antennas have reciprocity in their impedances, so much is of transmission as of reception; their magnitudes are the same ones and we will call them

$$Z_{\text{rad}} = Z_{\text{rec}}$$

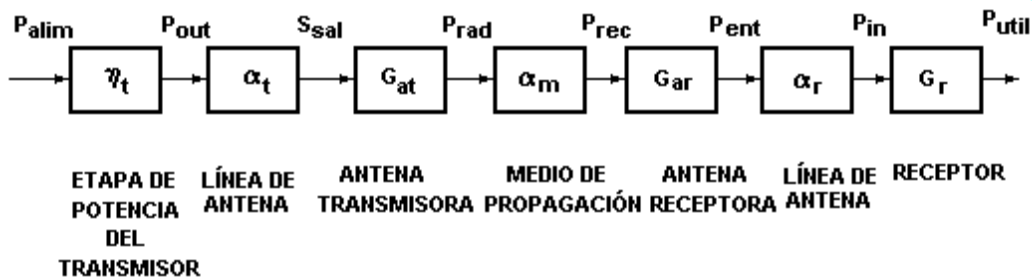
In the propagation of the electromagnetic wave in the vacuum or atmosphere, she finds a ambient practically pure resistive and consequently the antennas that absorb an apparent power will make effective only their active part

$$S_{\text{rad}} = P_{\text{rad}} + j 0$$

$$S_{\text{rec}} = P_{\text{rec}} + j 0$$

The diagram of energy flow of the figure following sample how the useful band goes being transferred along the transception. We observe that, as it gets used, the transmissions of power are called as efficiency  $\eta$ , gain  $G$  or attenuation to according to the situation. Clearing will be that there will be a commitment in all this with respect to the noise, that is to say to the *white noise* (constant

density of spectral power), because as it improves the total gain, we fight against this factor that increases also, being the efficiency of the transception dedicated to the technological ability with which both variables don't increase in the same proportion.

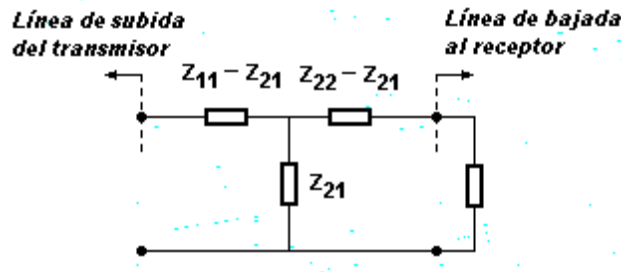


We can interpret the system between antennas like a symmetrical network ( $Z_{12} = Z_{21}$ ), where

$$Z_{\text{rad}} = Z_{11} - Z_{21} \quad (\text{transmission antenna})$$

$$Z_{\text{rec}} = Z_{22} - Z_{21} \quad (\text{reception antenna})$$

$$Z_{21} \rightarrow 0 \quad (\text{mutual impedance})$$



An useful way to specify the utility of an antenna is by means of its *effectiveness*; that is to say that in antennas of a single dimension it is spoken of *effective longitude* (it is always proportional to the long physique of the antenna), and for those of two of *effective area* (it is always proportional to the physical area of the antenna). This is given such that their product for the electric field or power that it receives (or it transmits) it determines their reception in vacuum (or transmission in vacuum). This way we have

$$L_{\text{ef}} = k_L L_{\text{FÍSICA}}$$

$$A_{\text{ef}} = k_A A_{\text{FÍSICA}}$$

$$V_{\text{ef}} = L_{\text{ef}} E_{\text{RECIBIDO}} = L_{\text{ef}} E_{\text{TRANSMITIDO}}$$

$$V_{\text{ef}} = A_{\text{ef}} P_{\text{RECIBIDA}} = A_{\text{ef}} P_{\text{TRANSMITIDA}}$$

being

$$P_{\text{RECIBIDA}} = E_{\text{RECIBIDO}}^2 / 377[\Omega]$$

$$P_{\text{TRANSMITIDA}} = E_{\text{TRANSMITIDO}}^2 / 377[\Omega]$$

## RADIATION

### Generalities

When a current of sine wave (harmonic of an entire band bases useful) it circulates for a conductor an electric interference of field it settles down in the atmosphere that generates in turn other magnetic and so successively, instantly one another, and they make it in space quadrature; and above this phenomenon spreads to the propagation. Nobody has been able to explain their reason.

Physical, mathematical, etc., they have offered their lives to the study but without being able to understand their foundation —if it is that it has it. They have progressed, that is certain, but always with "arrangements" like they are it the *potential*, the *optic bubbles*, the *origin of the universe*, etc. Truly, seemed not to have this phenomenon a physical formation, but rather of being metaphysical and therefore to belong to the *nomenon*.

Returning to him ours, it calls himself *isotropic radiator* to that antenna that radiates (or it receives) omni-directionally; that is to say that their *directivity lobe* is a sphere. The way to measure this lobe in a real application consists on moving from the transmission antenna to constant radio and with a meter of electric field to obtain the effective intensity that one receives; this will give an angular diagram that represents the significance of the space selectivity.

### Small conductor

For a small conductor in the free space and of differential magnitude (small with respect to the wave longitude) it is completed that the electric field in a point distant Q to the longitude of the wave is

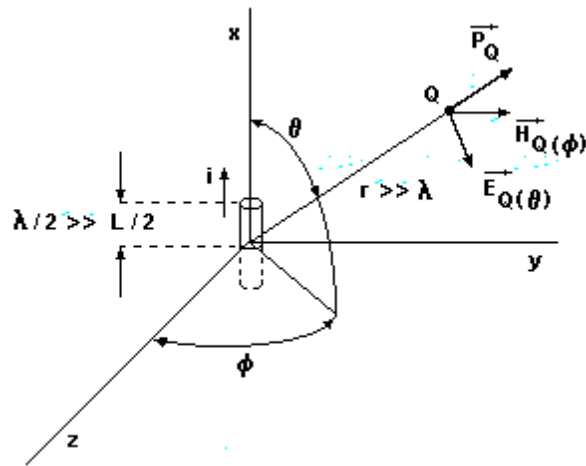
$$E_Q = Z_0 H_Q \sim [ ( I_p Z_0 L \sin \theta ) / 2r\lambda ] e^{j(\omega t - \beta r)} = E_{Qp} e^{j(\omega t - \beta r)}$$

where

$$i = I_p \sin \omega t$$

$$P \rightarrow = E \rightarrow \times H \rightarrow [W/m^2]$$

L *longitude of the conductor*

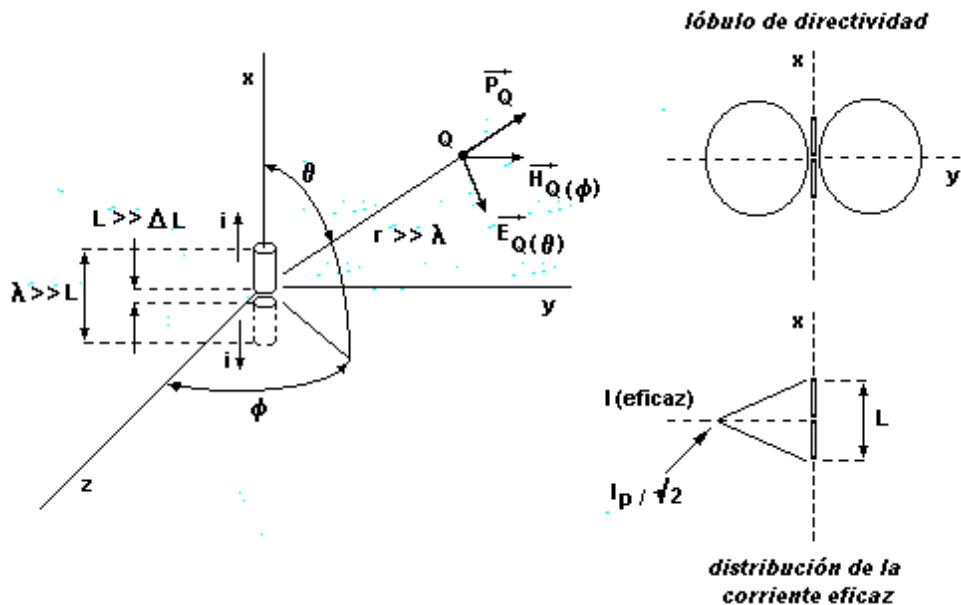


## DIPOLE ANTENNA

### Short dipole

The power radiated instantaneous total of a *short dipole* is the integration of all the differential points of *small conductors* that it form and they affect to the infinite points Q in its around

$$\begin{aligned}
 P_{\text{rad}} &= \int_s P_{(Q)} \rightarrow \partial s \rightarrow = \iiint [E_{Qp}^2 / Z_0 e^{j(\omega t - \beta r)}] \partial x \partial y \partial z = \\
 &= \iiint [E_{Qp}^2 / Z_0 e^{j(\omega t - \beta r)}] r^2 \cos(\pi/2 - \theta) \partial(\pi/2 - \theta) \partial y \partial z = \\
 &= 2 [(Z_0 I_p^2 L^2 / 4\lambda^2) e^{j(\omega t - \beta r)}] \int_{-\pi/2}^{\pi/2} [\int_{-\pi/2}^{\pi/2} \sin^2 \pi \cos(\pi/2 - \theta) \partial(\pi/2 - \theta)] \partial \phi = \\
 &= (Z_0 I_p^2 L^2 \pi / 3\lambda^2) e^{j(\omega t - \beta r)} = P_{\text{radp}} e^{j(\omega t - \beta r)}
 \end{aligned}$$





and consequently

$$P_{\text{rad}} = \int_0^{2\pi} P_{\text{radmed}} e^{j(\omega t - \beta r)} \partial(\omega t - \beta r) \rightarrow \int_0^{2\pi} (P_{\text{radp}}/2) \sin(\omega t - \beta r) \partial(\omega t - \beta r) =$$

$$= Z_0 I_p^2 L^2 \pi / 3 \lambda^2$$

$$R_{\text{rad}} = P_{\text{rad}} / (I_p / \sqrt{2})^2 = (2Z_0 \pi / 3) (L / \lambda)^2 \sim 790 (L / \lambda)^2$$

### Dipole of half wave

Observing the representative drawing sees that for a point generic and differential P of the conductor has

$$\theta \sim \theta_P$$

$$r_P \sim r - x \cos \theta$$

and as the distribution of the effective current for the same one is

$$I \sim (I_p / \sqrt{2}) \cos \beta x$$

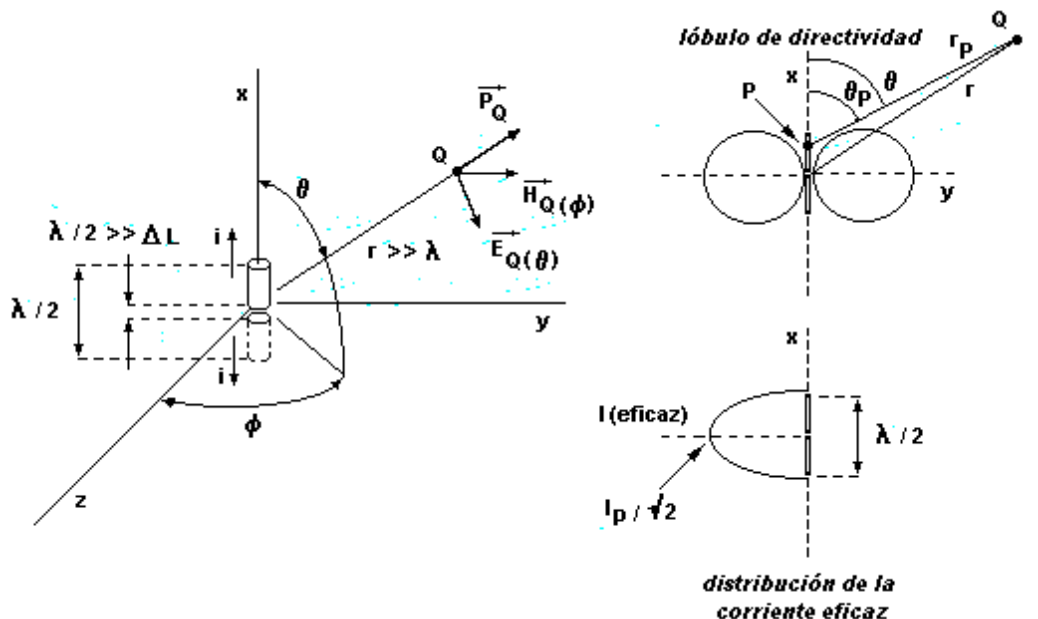
it is then that the electric field received in a point distant Q is practically the same one that in the case previous of a *small conductor*

$$E_Q = \int_{-\lambda/4}^{\lambda/4} \{ [Z_0 I_p (\lambda/2) \cos \beta x \sin \theta_P] / 2r\lambda \} e^{j(\omega t - \beta r_P)} \partial x \approx$$

$$\approx (Z_0 I_p \sin \theta_P / 4r) \int_{-\lambda/4}^{\lambda/4} [\cos \beta x \cos(\omega t - \beta r + \beta \cos \theta)] \partial x =$$

$$= \{ Z_0 I_p \cos[(\pi/2) \cos \theta] / 2r\lambda \sin \theta \} \cos(\omega t - \beta r) \sim \rightarrow$$

$$\sim \rightarrow [Z_0 I_p \sin \theta / 2r\pi] e^{j(\omega t - \beta r)} = E_{Qp} e^{j(\omega t - \beta r)}$$



To find the total radiated instantaneous power for the antenna it will be enough to integrate spherically the one received in those points Q

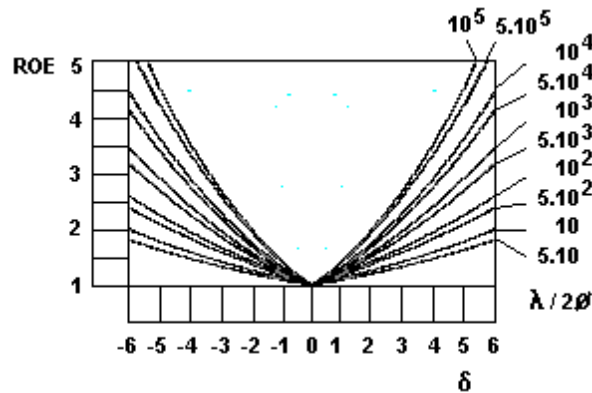
$$\begin{aligned}
 P_{\text{rad}} &= \int_s P_{(Q)} \rightarrow \partial s \rightarrow = \iiint [E_{Qp}^2 / Z_0 e^{j(\omega t - \beta r)}] \partial x \partial y \partial z = \\
 &= \iiint [E_{Qp}^2 / Z_0 e^{j(\omega t - \beta r)}] r_p^2 \cos(\pi/2 - \theta_p) \partial(\pi/2 - \theta_p) \partial \phi \partial x = \\
 &= (2Z_0 I_p^2 / 4\pi^2) \int_{-\pi/2}^{\pi/2} \left\{ \int_{-\pi/2}^{\pi/2} [(2/\lambda) \int_{-\lambda/4}^{\lambda/4} (r - x \cos \theta)^2 \partial x] \sin^2 \theta \cos(\theta - \pi/2) \partial(\pi/2 - \theta) \right\} \partial \phi \sim \\
 &\sim \rightarrow (Z_0 I_p^2 / 3\pi) e^{j(\omega t - \beta r)} = P_{\text{radp}} e^{j(\omega t - \beta r)}
 \end{aligned}$$

and to obtain finally

$$\begin{aligned}
 P_{\text{rad}} &= \int_0^{2\pi} P_{\text{radmed}} e^{j(\omega t - \beta r)} \partial(\omega t - \beta r) \rightarrow \int_0^{2\pi} (P_{\text{radp}}/2) \sin(\omega t - \beta r) \partial(\omega t - \beta r) = \\
 &= Z_0 I_p^2 / 3\pi \\
 R_{\text{rad}} &= P_{\text{rad}} / (I_p / \sqrt{2})^2 = (2Z_0 / 3\pi) \sim 80 \rightarrow \text{de la práctica} \rightarrow 75 [\Omega]
 \end{aligned}$$

If the conductor that we use of antenna has a diameter  $\emptyset$  and our wave longitude corresponds to a frequency « $f_0$ » of syntony in which the line is adapted (that is to say that it possesses  $Z_0 = R_0 = Z_{\text{rad}} = R_{\text{rad}}$ , ours ROE = 1), when we move from this frequency to another generic one « $f$ » it ROE it will worsen according to the following graph, where

$$\delta = (f - f_0) / f_0$$



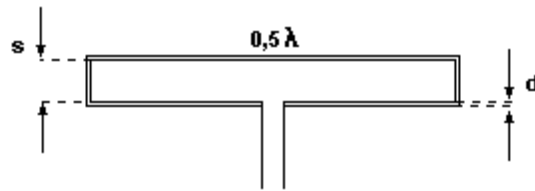
he following equation can be used to determine the effectiveness of this antenna if we treat her as of effective area

$$A_{\text{ef}} \sim 0,13 \lambda^2$$

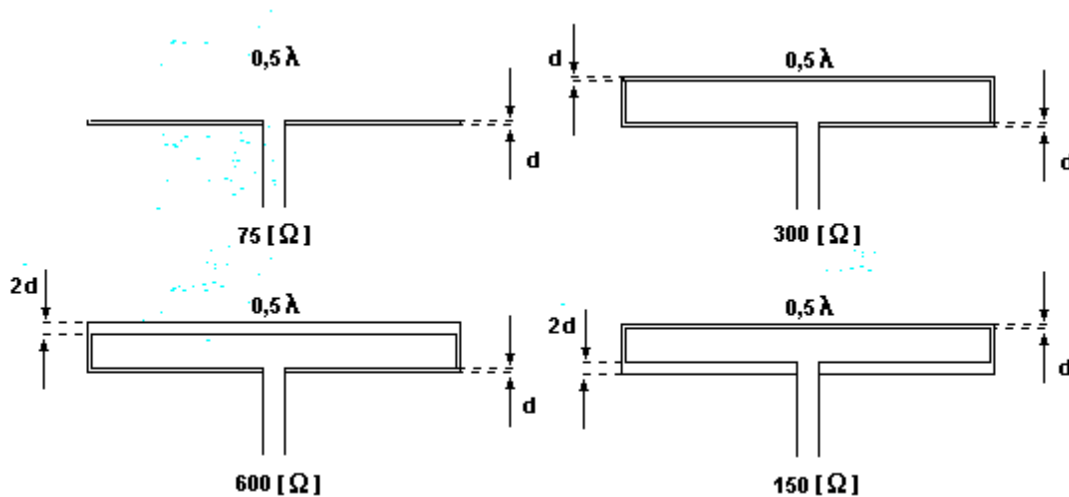
### Dipoles of half wave folded

We can use the abacus of it ROE it previous in the following implementation if we consider the correction

$$\emptyset = (2sd)^{1/2}$$

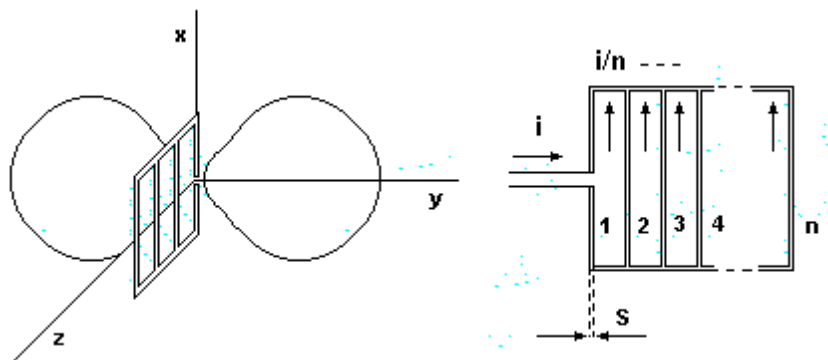


also



When putting «n» antennas dipoles of *half wave* in parallel of same section S like sample the figure, the radiation resistance or reception increases

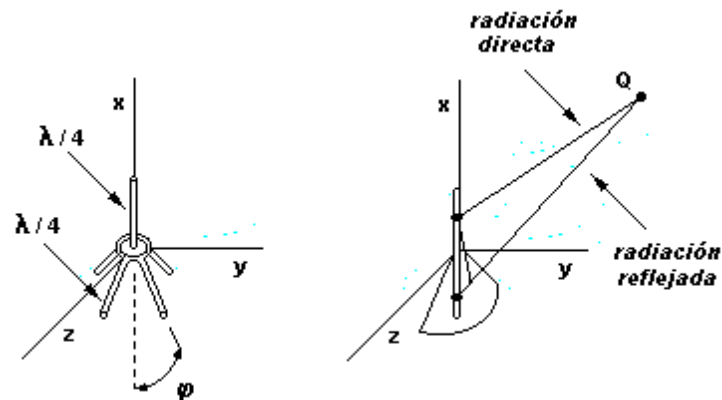
$$R_{\text{rad}} = P_{\text{rad}} / (I_p/n\sqrt{2})^2 = n^2 75 [\Omega]$$



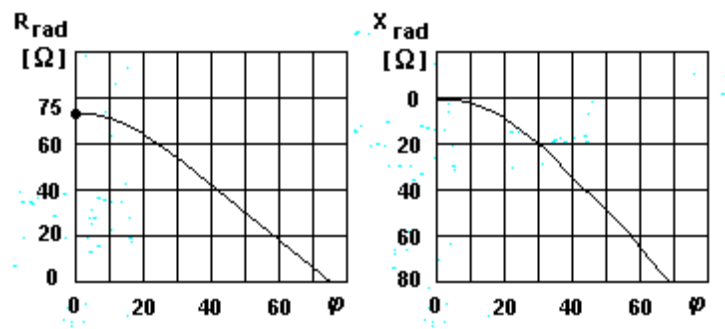
### Dipole of half wave with earth plane

Usually well-known as *Yagui*, it is an antenna type vertical mast of  $\lambda/4$  that it takes advantage

of their reflection in a plane of artificial earth created in their supply point. This plane is common that it is not horizontal.



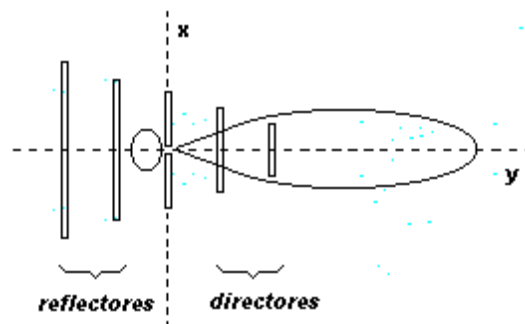
also



### Dipole of half wave with elements parasites

When connecting for before and from behind of the dipole bars in parallel, without electric connection, three effects of importance are observed

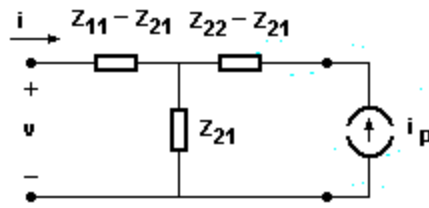
- decrease of the  $R_{\text{rad}}$  (or  $R_{\text{rec}}$ )
- the directivity of the lobe increases
- the spectral selectivity increases diminishing the band width (bigger  $Q_{\text{ef}}$ )



These selectivity principles and variation of the component activates they can be explained if we outline equations to the antenna considering it a symmetrical and passive network ( $Z_{12} = Z_{21}$ )

$$v = i Z_{11} + i_p Z_{21} = i (Z_{11} - Z_{21}) + (i + i_p) Z_{21}$$

$$0 = i Z_{21} + i_p Z_{22} = (i + i_p) Z_{21} + i_p (Z_{22} - Z_{21})$$



where

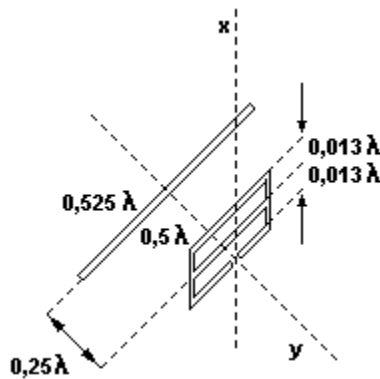
$$Z_{11} = Z_{\text{rad}} = Z_{\text{rec}} \quad (\text{without elements parasites})$$

$$i_p \quad (\text{circulating current for all the elements parasites})$$

### Dipole of half wave of wide band

This antenna you can use for wide spectra like they are it the channels of TV or the reception of MF. Their characteristic is

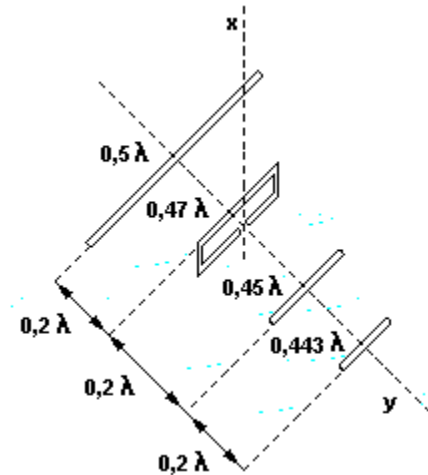
$$R_{\text{rad}} = R_{\text{rec}} \sim 300 [\Omega]$$



### Dipole of half wave of short band

This disposition presents the advantage of the selectivity of the directivity lobe, overcoming with it the rebounds and the interferences. It can be used for channels of TV or in MF. Their resistance is

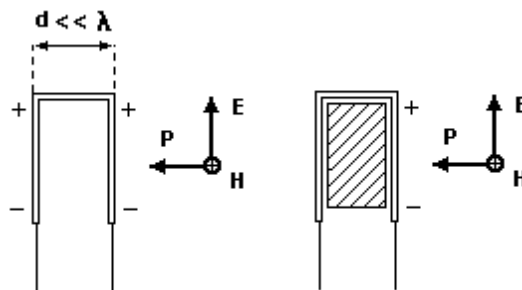
$$R_{\text{rad}} = R_{\text{rec}} \sim 50 [\Omega]$$



## SQUARE ANTENNA

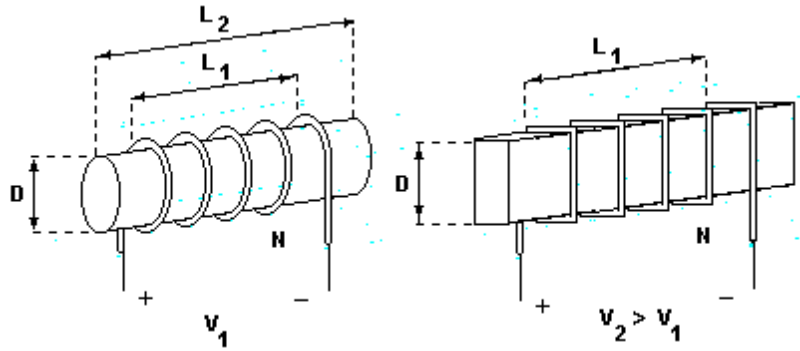
### Antenna with ferrite

In the following figure it is shown that the received electric field when nucleus of air is used it determines, due to the long longitude of the wave that the opposed driver receives an induction same and opposed that it cancels it. It won't pass the same thing in the enclosed case in that it has put on a mpermeability material to the step of the electromagnetic wave as it is the ferrite, since the induction now will be in a single conductor.



The effective induction will increase rolling several spires  $N$ . It is observed here that it will be bigger the induction the more parallel it is the conductor to the wave front. The effective voltage induced for these cases when it meets with a front of wave of frequency « $f$ » and effective electric field « $E$ » being the coil onelayer, it can approach to

$$V_1 \sim 6,86 \cdot 10^{-12} \mu_{ef} E N D^2 f [1 - 0,17 (L_1/L_2)]$$



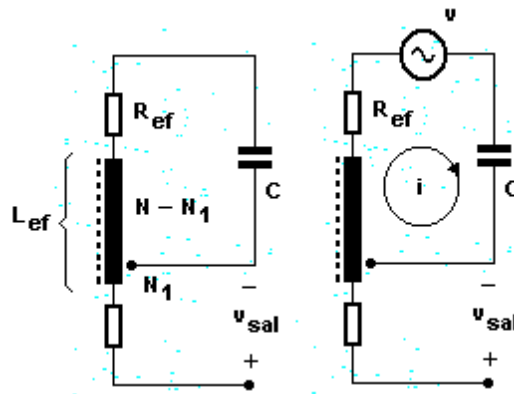
It is obtained with this antenna big output voltages if we syntonize it in series like it is shown, but so that the exit circuit doesn't load to the syntony it should attenuate with the relationship of spires ( $N \gg N_1$ )

$$\omega_c = 1 / [L_{ef} C (N - N_1)^2 / N^2]^{1/2} \sim 1 / [L_{ef} C]^{1/2}$$

$$Q_{ef} = [\omega_c L_{ef} (N - N_1)^2 / N^2] / R_{ef} \sim \omega_c L_{ef} / R_{ef} = 1 / \omega_c C R_{ef}$$

$$V_{salp} = I_p [\omega_c L_{ef} (N - N_1)^2 / N^2] N_1 / (N - N_1) \sim V_p Q_{ef} N_1 / N$$

$$B_{ef} = \omega_c / Q_{ef}$$

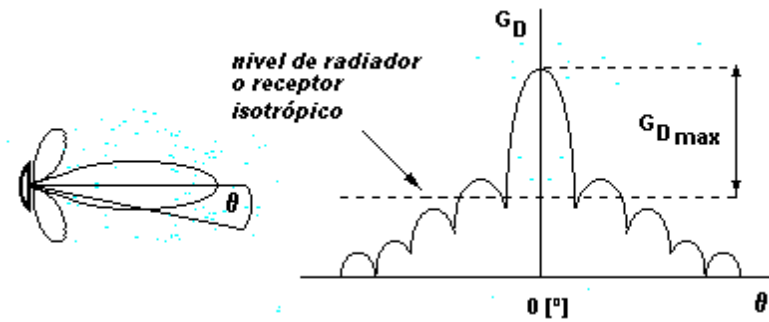


## PARABOLIC ANTENNA

Subsequently we show their diagram or radiation lobe, that it is commonly expressed as *directivity gain*  $G_D$

$$G_D = P / P_{ISOTRÓPICA} = P / P_{med}$$

$$G_{Dmax} \sim (4\pi/\lambda^2) A$$



and it differs of the *gain of power*  $G_P$  that has for the *efficiency*  $\eta$  (here  $A$  it is the physical area of the diameter antenna  $D$ )

$$G_P = \eta G_D$$

$$\eta = A_{\text{ef}} / A$$

of where

$$G_{P\text{max}} = \eta G_{D\text{max}} \sim (4\pi/\lambda^2) A_{\text{ef}}$$

It is also spoken FM of the *factor of merit* of the antenna like the relationship among their gain of power  $G_P$  and their equivalent *temperature of noise*  $T_{\text{eq}}$

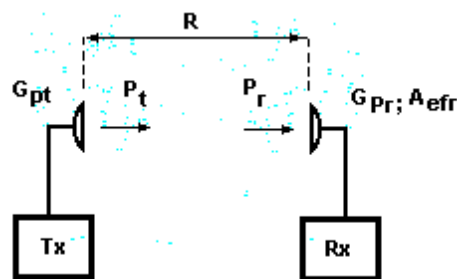
$$\text{FM [dB/°K]} = G_P / T_{\text{eq}}$$

For an optic connection, without noise and liberate in the area of Fresnel, the received power  $P_r$  is

$$L_P = 4\pi R^2 / \lambda \quad (\text{propagation loss in the free space})$$

$$P_{\text{eirp}} = G_{\text{pt}} P_t \quad (\text{equivalent power radiated of way isotropic})$$

$$P_r = P_{\text{eirp}} L_P A_{\text{efr}} = G_{\text{pt}} P_t G_{\text{Prmax}} / (4\pi \lambda R)^2$$





## Chap. 32 Electric and Electromechanical installations

### TARIFF

#### Generalities

#### Calculation

### CONDUCTORS

#### Voltage in a conductor

#### Calculation

### PROTECTION

#### Fusible

#### Termomagnetic

#### Insulation

#### Connection to ground

#### Lightning rod

### ASYNCHRONOUS MOTORS

#### Generalities

#### Calculation

#### Protection

#### Connection

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### TARIFF

#### Generalities

In three-phasic, given a consumption expressed by the line current  $I_L$  [A] and approximate  $\cos\phi \sim 0,8$  (factor of power), if we know the total energy cost  $E_T$  [KW.h] expressed as  $\alpha$  [\$/KW.h], the cost is determined by month in the following way

$$\begin{array}{ll} P_T = S_T \cos \phi = 660 I_L \cos \phi \sim 528 I_L & \text{total power permanently [W]} \\ \alpha & \text{energy cost per hour [$/KW.h]} \\ \beta & \text{energy cost per month [$/KW.mes]} \\ \alpha P_T & \text{total cost per hour [$/h]} \\ \beta P_T = 30 \text{ días} \cdot 24 \text{ h} \cdot \$/\text{h} = 720 \alpha P_T & \text{total cost per month [$/mes]} \end{array}$$

and we should add the monthly fixed cost that we call  $C_{FIJO}$  [\$/mes]. Consequently then can determine the monthly total cost with the following equation

$$C_{\text{TOTAL MENSUAL}} = C_{\text{FIJO}} + \beta P_T = C_{\text{FIJO}} + 720 \alpha P_T \sim C_{\text{FIJO}} + 380000 \alpha I_L$$

### Calculation

Be the data of the energy ticket that it sends us the Company of Electricity and we obtain of her

$$C_{\text{FIJO}} [\text{\$}] = \dots$$

$$\alpha [\text{\$/KW.h}] = \dots$$

We measure with a clip amperometer (or if there is him, with an installed instrument) the line current that it enters to the establishment

$$I_L [\text{A}] = \dots$$

and we obtain finally

$$C_{\text{TOTAL MENSUAL}} [\text{\$}] = C_{\text{FIJO}} + 380000 \alpha I_L = \dots$$

-  
-

### CONDUCTORS

#### Voltage in a conductor

We will call

$\Delta V$	voltage in the conductor in [V]
$L$	longitude of the conductor in [m]
$I$	effective or continuous current in [A]
$S$	section of the conductor [mm <sup>2</sup> ]

The voltage approached in copper drivers or aluminum can be with the following expressions

$$\Delta V \sim 0,0172 L I / S \quad (\text{copper})$$

$$\Delta V \approx 0,04 L I / S \quad (\text{aluminum})$$

### Calculation

Be the data

$$\Delta V_{\text{max}} = \dots \quad L = \dots \quad I = \dots$$

For the design of the installations it doesn't convenient that the effective voltage of phase is smaller than the 10 [%] of the normal one. That is to say that in uses of 220 [V] they should not diminish of the 200 [V].

We adopt a material according to the economic possibilities. We will choose that it is made preferably of copper, otherwise of aluminum for currents above dozens of amperes —let us keep in mind that the costs of the cable will be redeemed with the energy savings of heat that are avoided along their longitude..

For example then, if we have chosen copper we find

$$S = \dots > 0,0172 L I / \Delta V_{\max}$$

## PROTECTION

### Fusible

We will call

$I_n$	nominal current that the maker of the fuse indicates in his capsule in [A]
$I_f$	effective current to which melts in [A]
$\varnothing$	diameter of the copper wire in [mm]
$S$	section of the copper wire in [mm <sup>2</sup> ] ( $S = \pi \varnothing^2 / 4$ )

We have the correlations

$I_f \sim 1,8 I_n$	for $I_n$ among	00/ 10 [A]
$I_f \sim 1,57 I_n$	for $I_n$ among	15/ 25 [A]
$I_f \sim 1,45 I_n$	for $I_n$ among	35/ 60 [A]
$I_f \sim 1,45 I_n$	for $I_n$ among	80/ 200 [A]

or to remember

$$I_f \approx 1,5 I_n$$

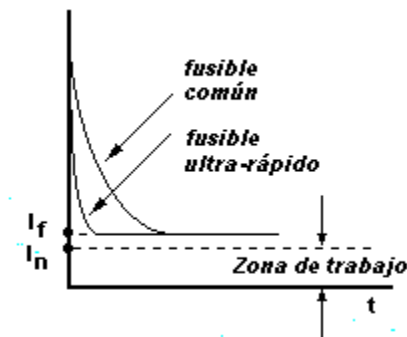
and in table

$\varnothing$	0,4	0,5	0,6	0,7	0,8	0,9	1	1,25	1,50
$S$	0,1	0,2	0,28	0,38	0,5	0,64	0,78	1,23	1,77
$I_f$	20	30	40	50	60	70	80	110	135

or equation

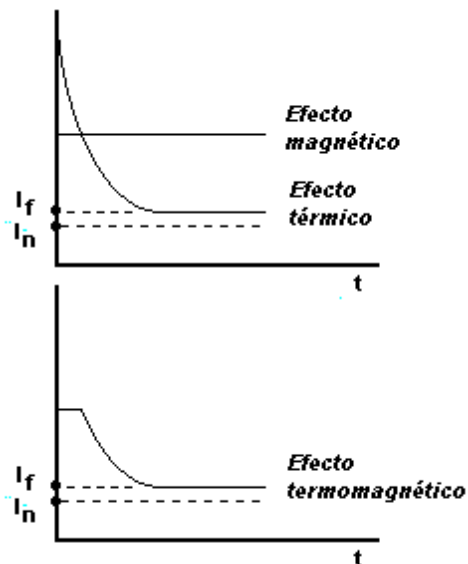
$$I_f [A] \approx 80 \text{ Diámetro [mm]}^{1,5}$$

A variant of the common fuses is those denominated *ultra-rapids*. These consist in fusible of ordinary copper but tensed by the force of a such spring that, in speed, they are very quick. They are usually used in applications of the electronics of the semiconductors.



## Termomagnetic

They are switches that act for the thermal and magnetic principles; the first one slow and the second rapid, they protect short circuits avoiding damage in the installations of the conductors. They don't protect to the equipment but to the installations.



There are them (or there was) of the type G and L. The first ones are applied at motors and the second to the illumination. They change among them for the slopes of the curves.

## Insulation

All installation should have connected a switch of differential protection. Usually adjusted at the 0,03 [A], it will be enough to prove it (if it is not trusted the test button that they bring) to connect between the alive one and ground a resistance of 6800 [ $\Omega$ ] ( $220 [V] / 0,03 [A] = 7333 [\Omega]$ ) and to check their instantaneous disconnection; it should never be proven with the own human body.

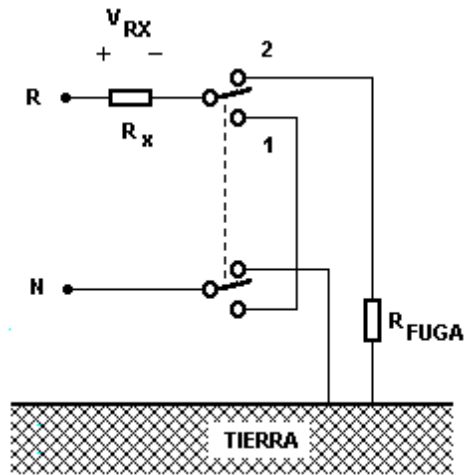
A correct installation (it doesn't in specific uses as laboratories electromedicine, electronic, etc.) it will determine an insulation of at least 1 [ $K\Omega/V$ ] (human currents of the order of the few

miliamperes:  $220 \text{ [V]} / 220 \text{ [K}\Omega\text{]} = 0,001 \text{ [A]}$ ). To verify this many times the use of a multimeter it is not enough, since the losses only appear with the circulation of the high current (mainly when there are heating loads); for such a reason the following method can be used where it is looked for that for the well-known resistance  $R_x$  circulate an intensity the nearest to the real one.

$$I_1 = V_F / R_x \quad \Rightarrow V_{Rx1} = \dots$$

$$I_2 = V_F / (R_x + R_{FUGA}) \quad \Rightarrow V_{Rx2} = \dots$$

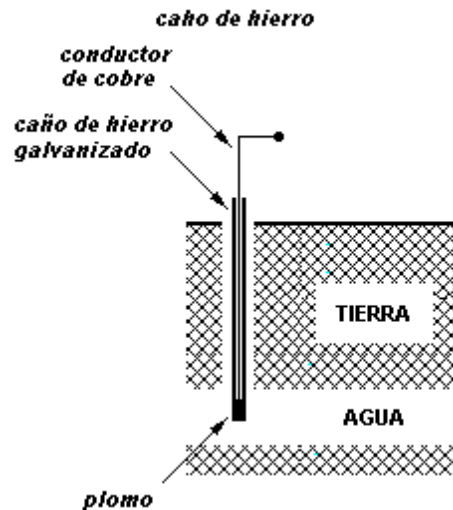
$$R_{FUGA} = R_x (V_{Rx1} - V_{Rx2}) / V_{Rx2} = \dots$$



### Connection to ground

The connection to ground are essential for all installation. They will protect people and they will grant a correct operation to the electronic equipment.

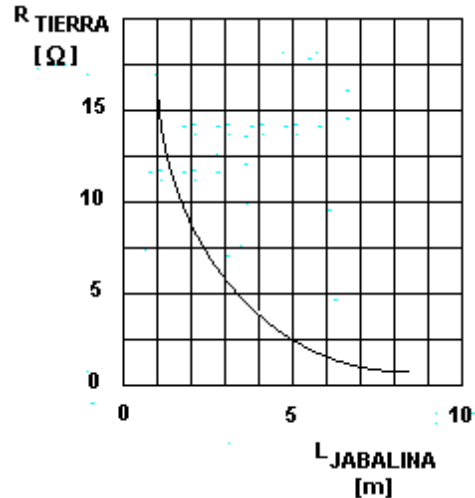
The frequent use is with the connection of copper javelins of galvanized iron, vertical plates of copper, horizontal meshes of copper, etc., but this many times it is insufficient or of insecure prevention. The author advises, for a connection to excellent ground, the following configuration, that is: to make a well of water until arriving to the next layer, then to place a pipe of previously prepared galvanized iron inside in their inferior extreme with lead and connected to him (submerged in the lead) a naked cable (or braid) of copper.



The resistance of the copper wire is remembered

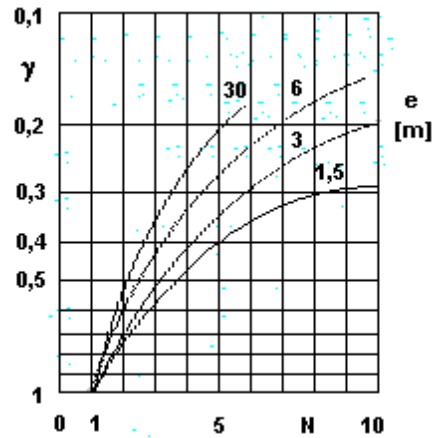
$$R [\Omega] = 0,0172 L [m] / S [\text{mm}^2]$$

to which it will be necessary to add him that of the terrestrial  $R_{\text{TIERRA}}$  that, very approximately, it can be obtained with the following abacus of vertical javelins of longitude  $L_{\text{JABALINA}}$



in which is observed that the depth won't improve the situation. To overcome this then it should be connected several in parallel. Subsequently we see an abacus for longitudinal disposition of «N» vertical javelins of same resistances  $R_{\text{TIERRA}}$  spaced among them by constant distances «e», and where the resistance effective total is (in the practice, although similar requirements are not completed, the effect will be equally approximate)

$$R_{\text{TOTAL TIERRA}} = \gamma R_{\text{TIERRA}}$$



## Lightning rod

The rays are loaded clouds that are discharged on atmospheres referred to the terrestrial potential. Their contour effluvioms make their ramified characteristic —and they are not, indeed, the enormous distances that usually represent the stories of fairies.

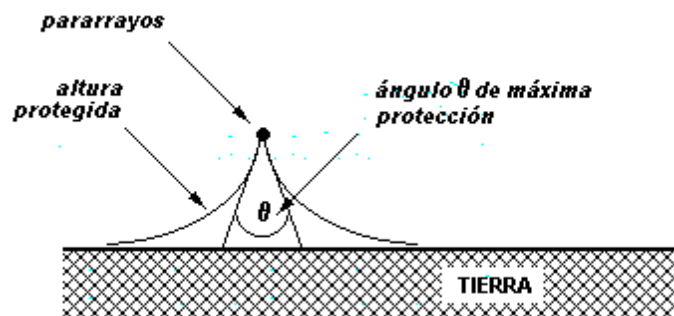
To begin, we should know that there is *not area sure hundred percent in the covering* —f.ex.: the church of Belleville in France was played by a ray to 8 meters of distance of a lightning rod of 10 meters of height, or that in 1936 trees were fulminated to the foot of the Torre Eiffel.

It is really this a complex topic. Difficult to try for their mathematical of propagation of waves and distribution of loads, as well as for the diverse information that is obtained of the different makers —if was not this way, there would not be so many hypothesis in the academic and commercial. They, basically, only design their extremes or tips.

We know that the charges in a metal accumulate to the edges; for this reason the tip Franklin is the better known —simple conical tip. They should always to be placed starting from the 4 meters above the highest part of the build to protect. The following statistic shows the covering

$\theta = 60^\circ$  the statistical security is of the  $\approx 98\%$

$\theta = 90^\circ$  the statistical security is of the  $\approx 92\%$



The erosion of the atmosphere, when also the electric impulsive discharge, goes deteriorating

the tips reason why they manufacture them to him disposables and of accessible cost, as they are it those of chromed bronze.

Each maker designs his own tip and, logically, it will have his own empiric content in the making of abacus and tables of protection covering. There are them different as much in their tips as in their base termination —f.ex.: with auto-valve shot to certain voltage.

## ASYNCHRONOUS MOTORS

### Generalities

We will study the industrial common motors and their installations. These machines are of the asynchronous type with rotor in short circuit.

Being balanced systems of 3 X 380 [V] either star or triangle, we will have in their study the following nomenclature (all subindex with the letter «n» it indicates «nominality» of the electric machine)

$V_L$	effective voltage of line [V]
$V_F$	effective voltage of phase [V]
$I_L$	effective current of line [A]
$I_F$	effective current of phase [A]
$Z_L$	line impedance or triangle [ $\Omega$ ]
$Z_F$	phase impedance or star [ $\Omega$ ]
$S_T$	total apparent power [VA]
$P_T$	total active power [W]
$W_T$	power reactivates total [VAR]
$\cos \phi$	factor of power [vecas]
$f_L$	line frequency (50 [Hz])
$\omega_L$	$2\pi$ line frequency ( $\sim 314$ [rad / seg])
$\omega_s$	$2\pi$ synchronous frequency [rad / seg]
$N_s$	frequency or synchronous speed [RPM]
$\omega_m$	$2\pi$ frequency of the rotor [rad / seg]
$N_m$	frequency or speed of the rotor [RPM]
$p$	number of even of poles [vecas]
$\xi$	constant of slip [vecas]
$C_T$	total torque in the rotor [N m]
	$Z_{eq}$ equivalent impedance for phase (the coil, is not that of line) [ $\Omega$ ]

The relationship between voltages and currents is

$$V_L = V_F \sqrt{3} \sim 380 \text{ [V]}$$

$$V_F = V_L / \sqrt{3} \sim 220 \text{ [V]}$$

$$I_L = I_F \sqrt{3} \text{ (motor in triangle)}$$



$$I_L = I_F \text{ (motor in star)}$$

$$\cos \phi \sim 0,8 \quad (\phi \sim 37 [^\circ], \text{sen } \phi \sim 0,6)$$

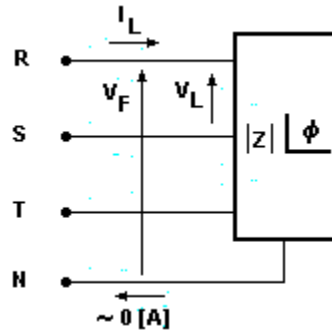
$$Z_L = |Z_L| e^{j\phi}$$

$$Z_F = |Z_F| e^{j\phi}$$

$$S_T = 3 V_F I_L = \sqrt{3} V_L I_L \sim 658 I_L$$

$$P_T = S \cos \phi$$

$$W_T = S \text{sen } \phi$$



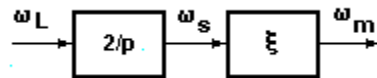
and like one observes, when being balanced their homo-polar voltage is practically null (that is to say that for the neuter one, if connects it, current would not circulate).

The revolving machine is presented with the following functional drawing. The speed in the mechanical axis of the motor  $\omega_m$  depends on its construction and of the friction (it loads useful) that has

$$\omega_m = 2\xi \omega_L / p$$

$$\xi = 1 - (\omega_m / \omega_s) \sim < 1$$

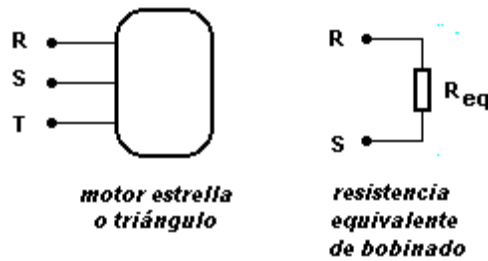
$$p = 2 \omega_L / \omega_s \sim < 2 \omega_L / \omega_m$$



When a motor says in its foil that it is of voltage 220/380 it is of connection in star; on the other hand, if it says 380/660 (and nobody truly knows the current reason about this) it will be of connection in triangle. For machines above 5 [HP] the outburst use is always suggested it star to triangle to protect the installations.

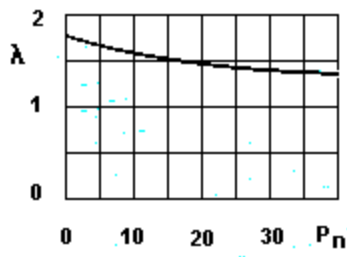
Each winding of the motor is as if was a transformer with a secondary resistive that its magnitude of the speed of the rotor depends. This way, from being null when it is braked until a maximum that is always smaller than the magnetic reactance, and therefore

$$Z_{eq} \sim R_{eq} + j 0 \approx R_{ROTOR} / [(\omega_s / \omega_m) - 1] = R_{ROTOR} \xi / (\xi - 1)$$



When we need more accuracy in the obtaining of the line current  $I_L$  that takes the motor it can be appealed to the following expression

$$I_L [A] = \lambda P_n \approx 1,5 P_n [CV \text{ o bien en HP}]$$



When we have an useful load that demands in a motor of nominal power  $P_{n1}$  an intensity  $I_{L1}$  with an excessive heating, and it wishes it to him to change for another motor of more power  $P_{n2}$ , it should not be expected that the new current diminishes, but rather it will be the same one practically (sometimes this is approximate due to the different productions and alineality of the electric machines), since the friction (it loads useful) it has not changed. The advantage that one will have, obviously, is that it won't heat, its life will be more useful and it will improve the energy efficiency of the system.

### Calculation

Be the characteristic data of the foil of the motor

$$P_n [CV \text{ o bien HP}] = \dots$$

$$N_{mn} [RPM] = \dots$$

$$3 \times V_{Ln} [V] = \dots$$

$$f_{Ln} [Hz] = \dots (50 [Hz])$$

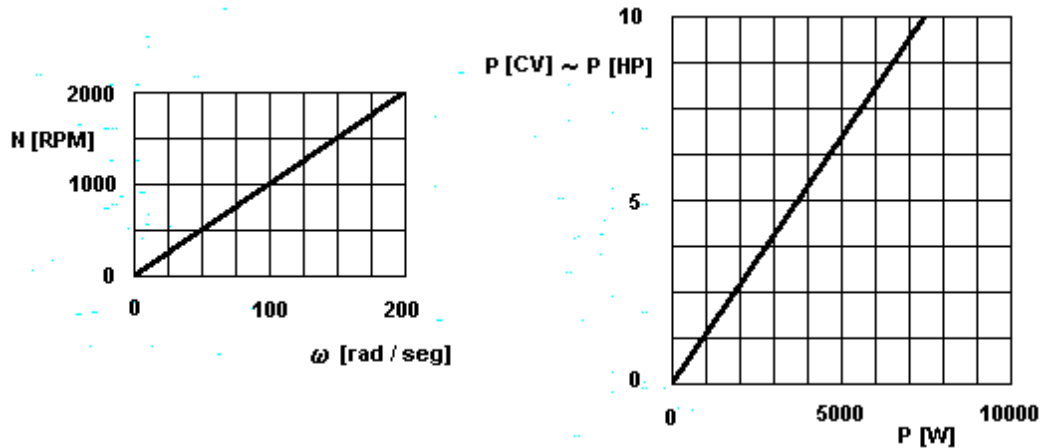
$$I_{Ln} [A] = \dots$$

$$\cos \phi_n = \dots (\sim 0,8)$$

where

$$1 [CV] \sim 0,99 [HP] \sim 736 [W]$$

$$1 [RPM] \sim 0,0166 [Hz] \sim 0,105 [rad/seg]$$



and that it considers them to him nominal. This "nominality" characteristic, although in their theory it is founded by the *functional optimization* of the machine, truly in the practice it is only of *commercial optimization*; this is, it will be more nominal so much adult it is their sale. This way, anything has to do their foil data with that waited technologically (unless the price is omitted and they are bought of first line), but rather it is this a point of work operation where the motor doesn't heat, its useful life is long and mainly, profitable to the maker. Therefore we conclude that, while the motor is to o.k. temperature (this is that we put our hand and we should not take out it because it burns us) it will be working well in its so much nominality of voltage like of current.

We continue with our calculations

$$P_T = P_n = \sqrt{3} V_L I_{Ln} \cos \phi_n [ \approx 526 I_L \text{ (theoretical, to see above) } ] = \dots$$

$$N_m = N_{mn} = \dots$$

$$V_L = V_{Ln} = \dots$$

$$f_L = f_{Ln} = \dots$$

$$I_L = I_{Ln} = \dots$$

$$\cos \phi = \cos \phi_n = \dots$$

and for other work points the case will be studied —question that is not convenient because this always indicates over-heating and, therefore, little time of useful life to the machine. These data will allow to be finally

$$p = \dots \leq 2 \omega_L / \omega_m = 120 f_L / N_{mn} = 6000 / N_{mn} [\text{RPM}]$$

$$N_s [\text{RPM}] = \omega_s = 2 \omega_L / p = 120 f_L / p = 6000 / p = \dots$$

$$\xi = 1 - (N_{mn} / N_s) = \dots$$

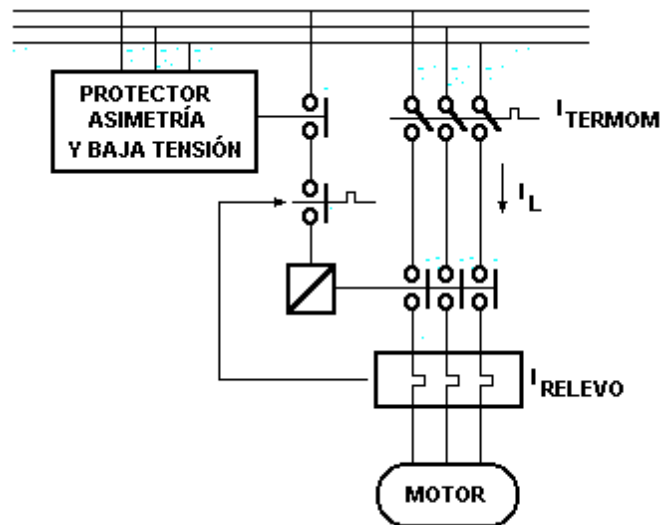
$$C_{Tn} [\text{Kg cm}] = P_T \omega_{mn} = 526 I_L [\text{A}] / N_{mn} [\text{RPM}] = 7028 P_n [\text{CV}] / N_{mn} [\text{RPM}] = \dots$$

## Protection

All protection should consist of three things, that is:

- guard-motor
- switch termomagnetic

- thermal relay
- protective of asymmetry and low voltage



and never to use fusible common, since one breaks of them and the motor is without a phase.

If we have a circulation for the motor  $I_L$ , the *thermal relay*, dedicated to avoid the over-heating of the winding, will adjust it to him preferably in an experimental way (of high  $I_{RELEVO}$  to low until giving with the no-court point) previously measured or calculated its magnitude that will be for the order of the 10 [%] more than this  $I_L$  —this depends on the load type, because it is not the same fans of constant consumption that extrusors that change its consumption continually.

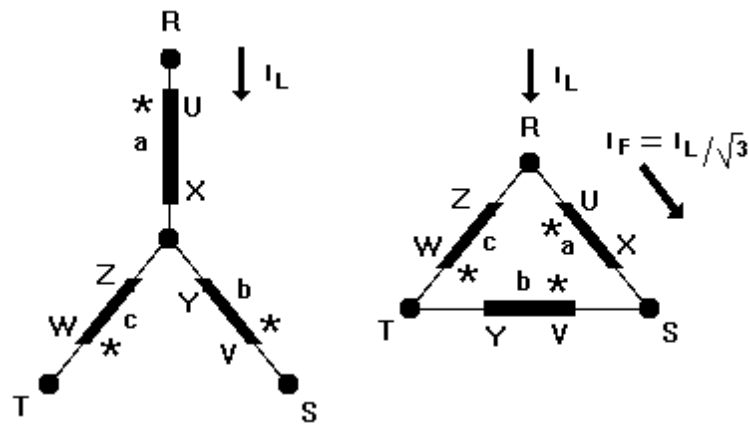
The switch termomagnetic, preferably of the type G, will only protect short circuits avoiding over-heatings of cables or ignitions. Their magnitude  $I_{TERMOM}$  won't have to do with the  $I_L$  of the motor but only so that in the outburst it doesn't disconnect it for the high consumption, and for it approaches it to it in the order of the 50 [%] or more than this. It is recommended here, for an efficient installation, to appeal to the manuals and leaves of data of the switch.

The protector of asymmetry or low voltage of the phases of the line is very necessary because it avoids periodic over-heatings that they harm to the motor although the system guard-motor works. It is seen that, and mainly in powder atmospheres with humidity that to the few years, when also few months, periodic courts of the relays of the guard-motors finish burning the motor.

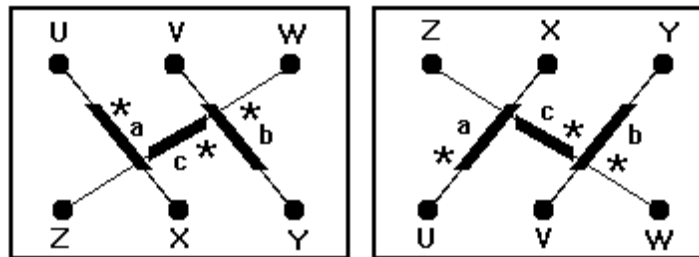
Clearing will be that the mentioned group will over-value its cost with respect to which have the small motors, but if one thinks of the losses that produce its inactivity while it is replaced, it is necessary in occasions this investment like future gain.

## Connection

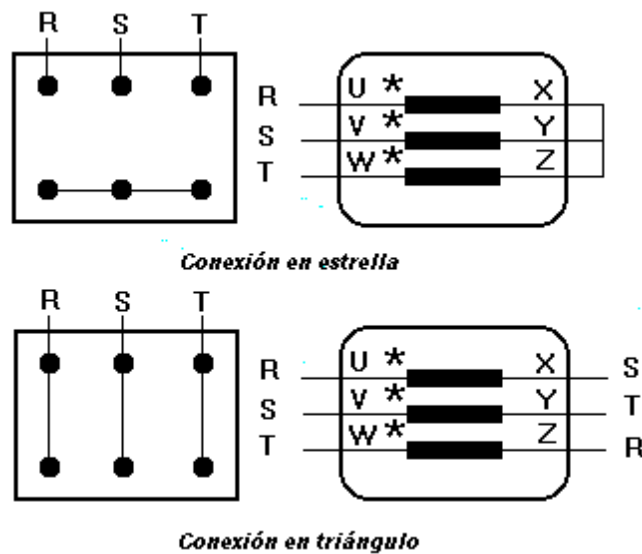
As for the disposition of the coils in the motor are



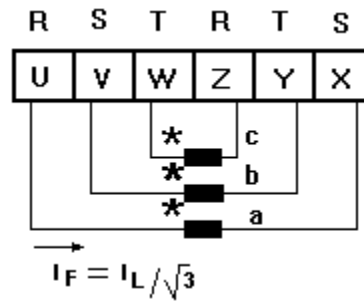
and the connection of the terminals that is given by the maker of the machine having it the following two possible ways



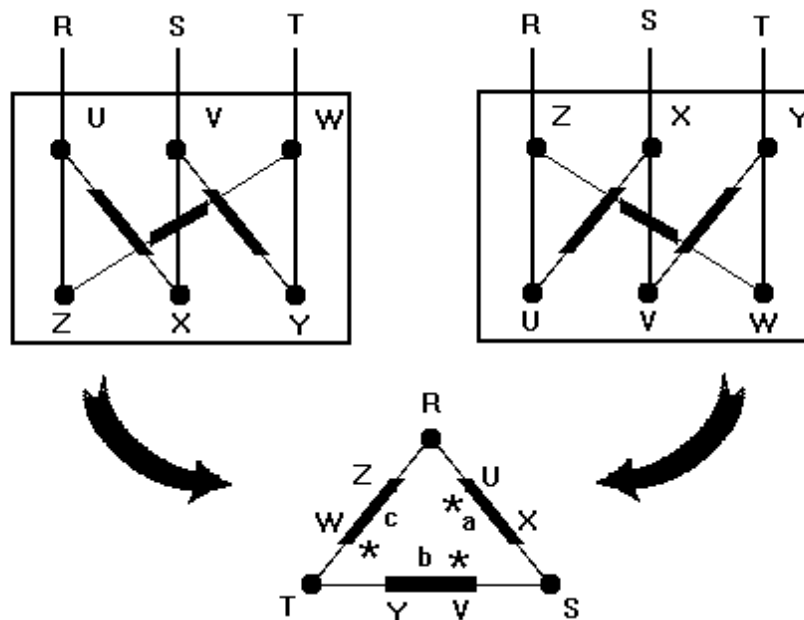
since their armed one should be



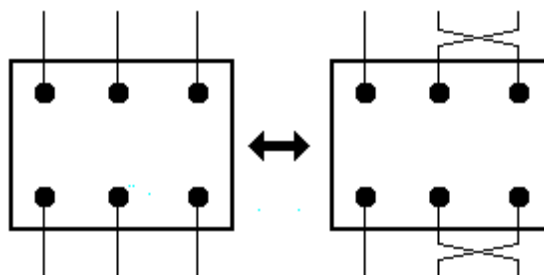
and we can suggest this other of output for the box



On the other hand, we can see that in this disposition (common in the practice) both connections are the same thing



To change the rotation sense it will be enough with investing any couple of alimentation conductors



## Chap. 33 Control of Power (I Part)

### GENERALITIES

#### CONTROL FOR REGULATION OF PHASE

##### Generalities

##### Manual control

##### Design

#### TYPES OF LOADS

##### Passive loads

##### Heating

##### Illumination

##### Inductive

##### Resonances

##### Active loads

##### Generalities

##### Motors of continuous

##### Universal motors

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### GENERALITIES

We can differentiate the energy controls in two types

- proportional
  - analogical
    - lineal (controls for sources continuous)
    - not lineal (controls for phase regulation)
  - digital (for wide of pulse or other techniques)
  - mixed (for whole cycles of line, modulated, etc.)
- not proportional (on-off)

#### CONTROL FOR REGULATION OF PHASE

##### Generalities

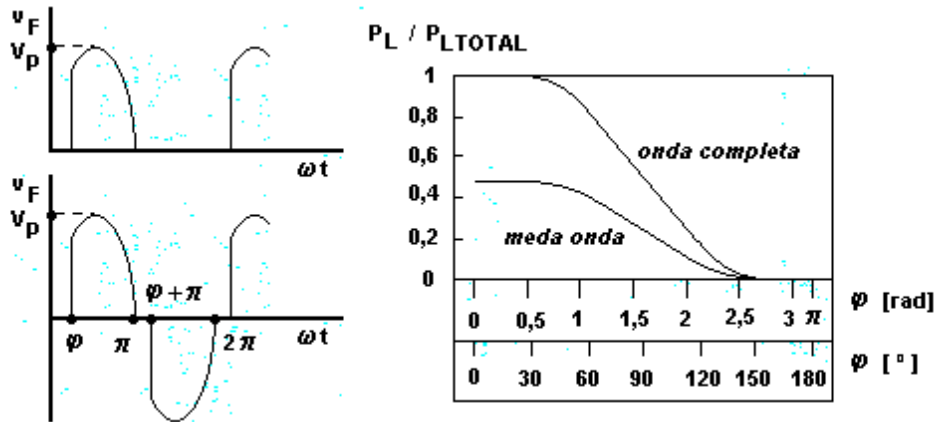
The figure following sample the disposition for half wave or it completes. Their behavior equations are for a load resistive the following

$$v_F = V_F \sin \omega t$$

$$P_{L\text{TOTAL}} = V_F^2 / R_L$$

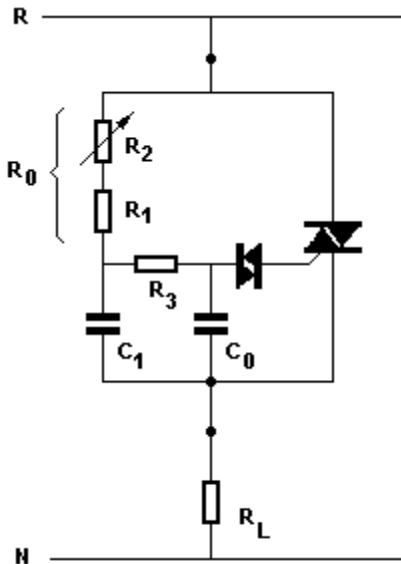
$$P_{L(\text{MEDIA ONDA})} = \left[ (1/2\pi) \int_{\varphi}^{\varphi+\pi} v_F^2 \omega dt \right]^{1/2} = P_{L\text{TOTAL}} \{ 1 - [\varphi - \sin(2\varphi)] / 2 \} / \pi \}$$

$$P_{L(\text{ONDA COMPLETA})} = 2 P_{L(\text{MEDIA ONDA})} = P_{L\text{TOTAL}} \{ 1 - [\varphi - \sin(2\varphi)] / 2 \} / \pi \}$$



### Manual control

The following implementation has spread for its effectiveness and simplicity. The problem that has is that condensers of mark grateful and high tension should be used, because with the time they change its value and not only they take out of polarization the work point (notable in low  $\varphi$ ) but rather they make unstable to the system.



If we design to simplify



$$R_3 \ll R_0$$

$$C_1 \ll C_0$$

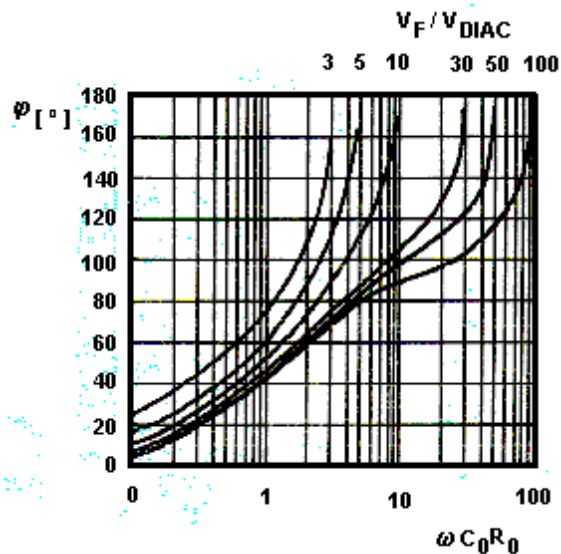
their operation equations are the following

$$v_F = v_{RN} \text{ (it can also be for line } v_{RS})$$

$$v_0 \sim v_F (1/sC_0) / (R_0 + 1/sC_0) \rightarrow V_p [1 + (\omega R_0 C_0)^2]^{-1/2} e^{j(\omega t - \arctan \omega R_0 C_0)}$$

$$v_{0(\varphi)} = V_{DIAC} = V_p [1 + (\omega R_0 C_0)^2]^{-1/2} \sin(\varphi - \arctan \omega R_0 C_0)$$

$$\varphi = (\arctan \omega R_0 C_0) + \{ \arcsin (V_p [1 + (\omega R_0 C_0)^2]^{-1/2} / V_{DIAC}) \}$$



## Design

Be the data

$$P_{Lmax} = \dots \quad P_{Lmin} = \dots$$

$$V_F = \dots \text{ (monophasic 311 [V] o threephasic 536 [V]; or monophasic 165 [V] or threephasic 285 [V])}$$

$$f = \dots \text{ (50 [Hz]; o bien 60 [Hz])}$$

We choose a diac and components of low losses

$$V_{DIAC} = \dots \text{ (typical 30 [V])}$$

$$C_0 = \dots$$

$$C_1 = \dots \ll C_0 \text{ (typical } C_1 \sim 10 \text{ [nF] X 600 [V])}$$

and of the abacus of powers we obtain

$$\Phi_{Lmax} = \dots$$

$$\Phi_{Lmin} = \dots$$

for after that of phase

$$(\omega R_0 C_0)_{\max} = \dots$$

$$(\omega R_0 C_0)_{\min} = \dots$$

and with it

$$R_1 = (\omega R_0 C_0)_{\min} / \omega C_0 = \dots$$

$$R_2 = (\omega R_0 C_0)_{\max} - R_1 = \dots$$

and we verify that the adoptions have not altered the calculations

$$R_3 = \dots \ll (\omega R_0 C_0)_{\max} \text{ (typical } R_3 \sim 1 \text{ [K}\Omega\text{]} \times 0,25\text{[W])}$$

Subsequently we obtain the requirements of the triac

$$P_{\text{TRIAC}} = I_{\text{efmax}} V_{T2T1} \sim I_{\text{efmax}} (1 \text{ [V]}) \sim (1 \text{ [V]}) P_{L\max} / (V_F / \sqrt{2}) \sim 1,4 P_{L\max} / V_F = \dots$$

and of the manual

$$T_{\text{JADM}} = \dots$$

$$P_{\text{TRIACADM}} = \dots > P_{\text{TRIAC}}$$

$$\theta_{\text{JC1}} = (T_{\text{JADM}} - 25) / P_{\text{TRIACADM}} = \dots$$

what will allow us to calculate the dissipator (to see the chapter of *dissipation of heat*)

$$\text{surface} = \dots$$

$$\text{position} = \dots$$

$$\text{thickness} = \dots$$

## TYPES OF LOADS

### Passive loads

#### Heating

They are usually coils of nichrome wire (nickel alloy, chromium and iron) over mica or *miquelina* (ceramic of compact mica powder) trapped by their metal capsule for their dissipation of heat in a metallic mass to heat. Designed in general for alimentation of monophasic voltage, they usually possess plane forms, in hairspring, tubular (right or helical) or of brackets, as well as they are manufactured to order.

If the horizontal exposed conductor *is outdoors windless and to habitable ambient temperature*, the following table presents the resistance and necessary current approximately for over-heating in its immediate surface

SECCIÓN [mm <sup>2</sup> ]	RESISTENCIA [Ω/m]	CORRIENTE EFICAZ [A]		
		TEMPERATURA [°C]		
		200	500	1000
0,05	22	0,9	1,8	3,8
0,07	15,3	1,1	2,1	4,6
0,1	11,3	1,4	2,7	5,9
0,12	8,63	1,6	3,1	6,9
0,2	5,52	2,2	4,1	9,4
0,28	3,84	2,7	5,1	11,5
0,38	2,82	3,3	6,4	14,5
0,5	2,16	3,8	7,6	17,4
0,64	1,7	4,2	8,7	20,4
0,78	1,38	4,8	10,1	23,7
1,13	0,98	6,3	13	32
1,77	0,62	7,2	16,1	39,2
3,14	0,34	11,6	26,7	65,5
4,9	0,22	16,3	37,7	92
7,07	0,15	18,8	44	108

## Illumination

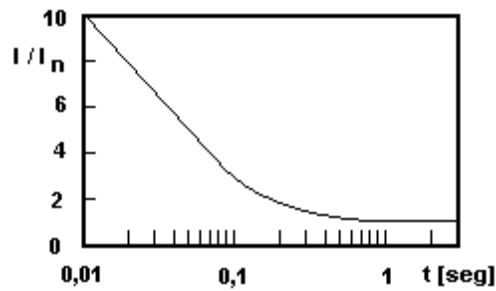
The incandescent lamps possess an approximate efficiency according to the following abacus and valid equation for the environment of effective voltage «V» that is specified

$$\phi \approx \phi_n (V / V_n)^{3,5}$$

$$0,3 V_n \leq V \leq 1,2 V_n$$

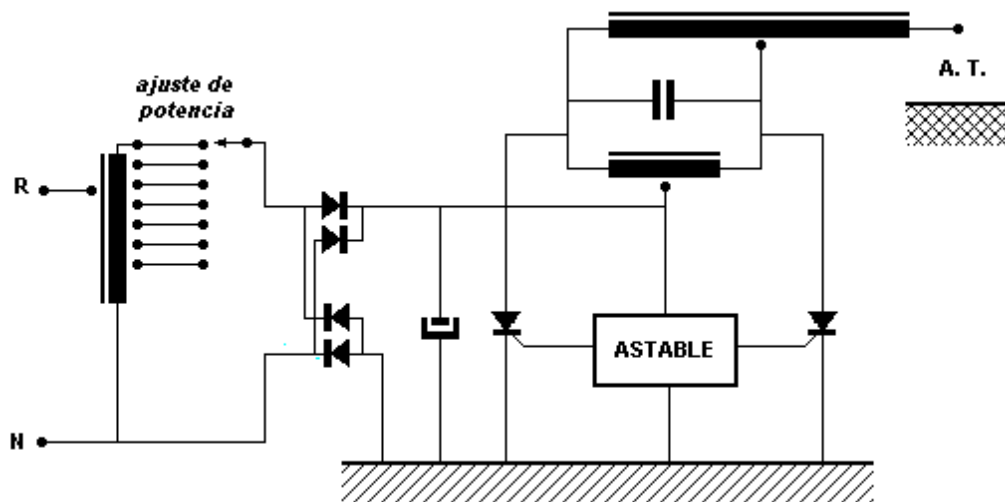
where « $\phi$ » it is the luminous flow in [lumen = lux.m<sup>2</sup>], the «V» they are all effective magnitudes and the subindex «n» it indicates nominality (work voltage specified by the maker).

We know that the resistance in cold is always smaller than when it takes temperature for illumination. The graph following sample an approach of the current circulates with respect to the nominal one. For inferior lamps at the 150 [W], although only sometimes but it is to keep in mind, the disruptive current can arrive until the 200 times the nominal one.



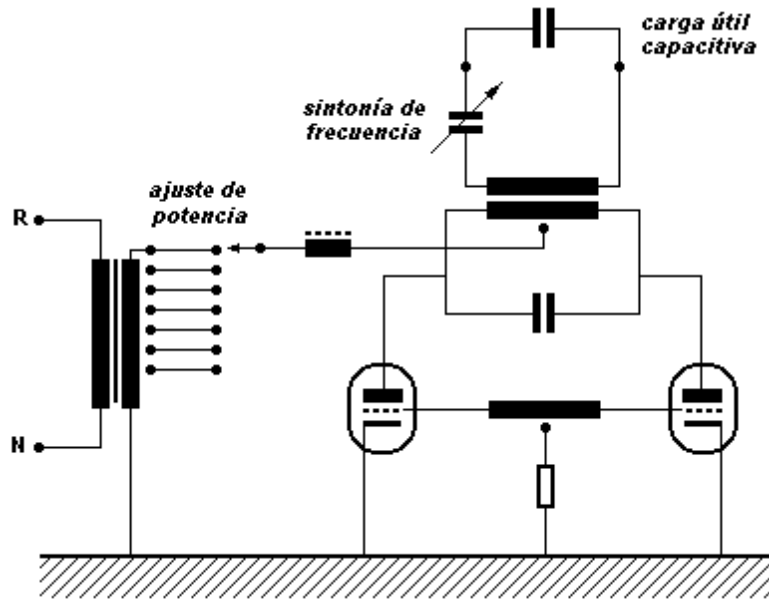
## Inductive

Prepared the following circuit to generate high variable tension, it is usually used in the *treaties of polarization of film of polymers*. It consists basically on an oscillator multivibrator that two RCS commutes (or TBJ) on an inductor syntonized with the purpose of obtaining great voltage to send to the secondary of the transformer of high voltage.



## Resonances

Usually of syntony series, these implementations are taken advantage of to generate the denominated *short waves*. Their use is so much industrial as medical — plastic welders, physiologic treatmentss, etc. The circuit that follows sample a typical configuration where, in each opportunity that is presented, the syntony series of the secondary should be adjusted. It consists on an oscillator auto-polarized in class C that possesses a transformer of nucleus of air in their output, freeing with it the syntonies: parallel to their input and series to their output —to see the chapters of *amplifiers of RF in class C* and of *harmonic oscillators*.



## Active loads

### Generalities

Under the "active" name we either mean those loads that possess a *voltage* or *current electromotive*, being opposed or favoring the incoming current that gives it. Inside this group of loads they are the motors of continuous that, being true back-electromotive "forces" for their induced voltage, they leave aside to the transformers that are not it. It is denied with this that the transformers possess back-electromotive "force"; that error that it has come per decades confusing our studies.

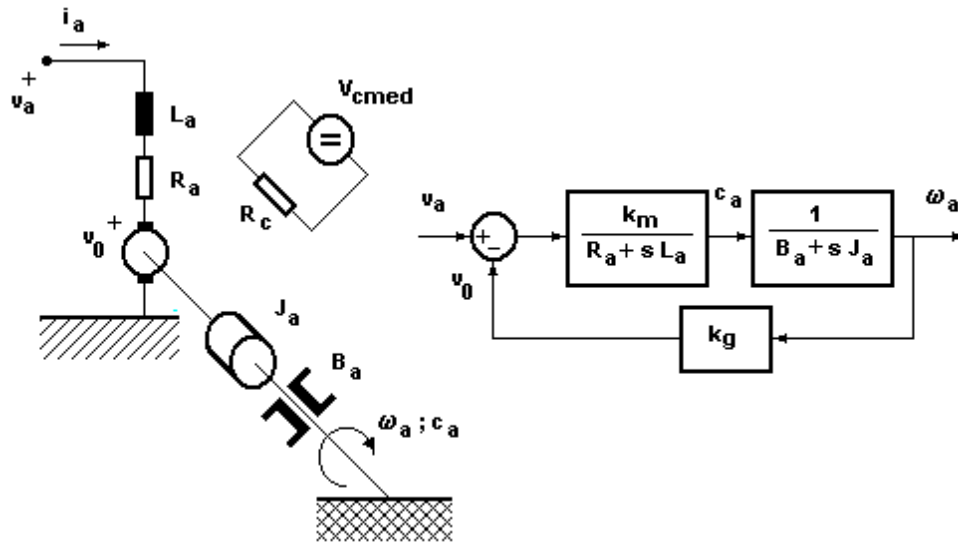
### Motors of continuous

The following drawing represents its diagram in blocks for the configuration *without load* of independent excitement of fields (that is the usual one), and where the small machines respond in a same way but with permanent imams in their fields. Their equations are

$v_a = i_a Z_a + v_0$	<i>vltage (electromotive) applied to the rotor</i>
$v_0 = k_g \omega_a$	<i>voltage (or "force") back-electromotive</i>
$c_a = k_m i_a$	<i>couple (or torque) in the rotor [N m]</i>
$Z_a = R_a + sL_a$	<i>impedance of the winding of the rotor</i>
$B_a$	<i>friction of the rotor [N m seg / rad]</i>
$J_a$	<i>moment of inertia of the rotor [N m seg<sup>2</sup> / rad]</i>
$k_g$	<i>constant as generator [V seg / rad]</i>
$k_m$	<i>constante motriz [Kg m / A]</i>
$R_c$	<i>resistance of the fields of the stator</i>

being

$$L_a / R_a \gg B_a / J_a \rightarrow 0$$



and consequently, when it is applied continuous or a rectified signal of value averages  $V_{amed}$  having a physical load  $B_L$  and  $J_L$  appreciable

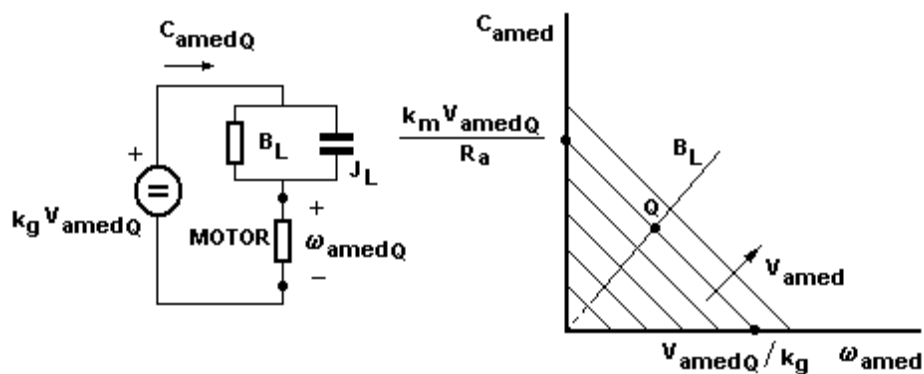
$$V_{amed} = I_{amed} R_a + V_{0med} \sim V_{0med}$$

$$C_{amed} = (B_a + B_L) \omega_{amed} + (J_a + J_L) \frac{\partial \omega_{amed}}{\partial t} \sim B_L \omega_{amed} + J_L \frac{\partial \omega_{amed}}{\partial t}$$

being simplified the expression for stationary state ( $\frac{\partial \omega_{amed}}{\partial t} = 0$ ) in a point work Q

$$C_{amed} = B_L \omega_{amed}$$

$$\begin{aligned} C_{amed} &= k_m I_{amed} = k_m (V_{amed} - V_0) / R_a = k_m [V_{amed} - (k_g \omega_{amed})] / R_a = \\ &= (k_m V_{amed} / R_a) - (k_m k_g / R_a) \omega_{amed} \end{aligned}$$



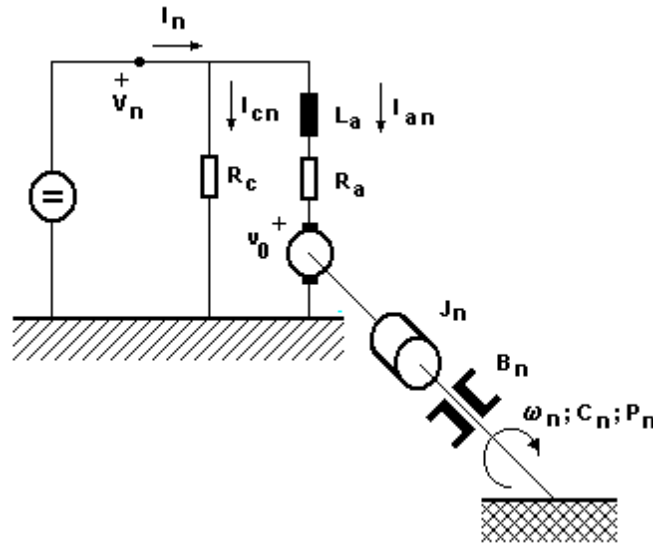
When the maker of the machine obtains his foil characteristics, the configuration that uses is usually in derivation (for its simplicity), and it presents its product in a nominal way as it is detailed (for the conversions of units to go to the chapter of *electric installations*)

$$V_n = \dots$$

$$I_n = \dots$$

$$\omega_n = \dots$$

$$P_n = \dots$$



reason why, if we want to obtain the parameters of the machine first we should experience and to measure. If we know or we approach the energy efficiency meetly (around ninety percent)

$$\eta = P_n / V_n I_n \sim 0,9$$

we can determine the resistance of the rotor with the following empiric equation (or, like it was said, to measure it)

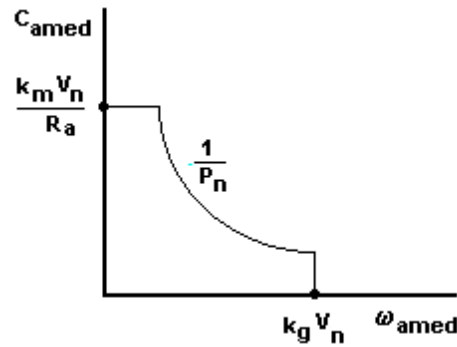
$$R_a = \dots \sim V_n (1 - \eta) / 2 I_n$$

and to continue with our deductions (we reject  $I_{cn} \ll I_{an}$ )

$$k_g = \omega_n / V_{0n} \sim \omega_n / V_n = \dots$$

$$k_g = C_n / I_n = (P_n / \omega_n) / I_n = P_n / \omega_n I_n = \dots$$

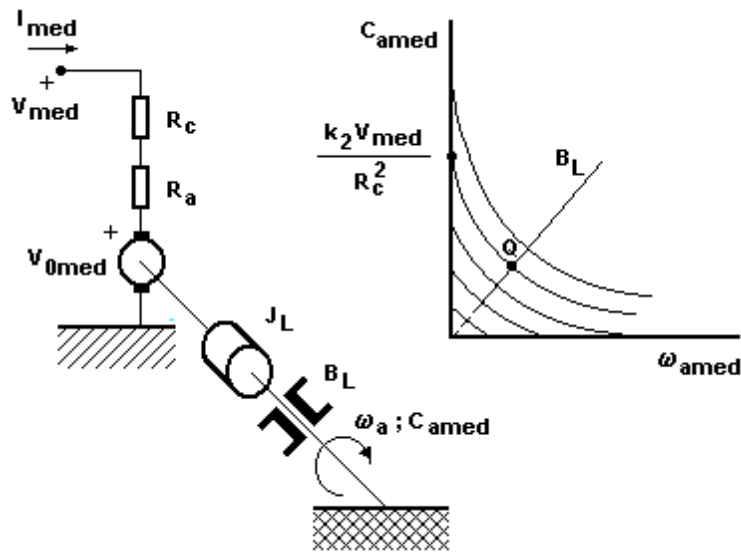
being defined an *area of sure operation* as sample the figure (truly this has more margin), because it is not convenient to overcome the  $P_n$  since the maker it always considered their magnitude in function of the possibility of giving merit to their product



## Universal motors

Of practical utility and domestic (f.ex.: drills), they are motors of continuous with windings rotor and stator in series. The rectification takes place geometrically for the disposition of their collector. Following the nomenclature and precedent studies, then their equations are

$$I_{med} = I_{amed} = I_{cmed}$$



and therefore

$$V_0 = k_1 \omega_a I_{cmed} = k_1 \omega_a I_{med}$$

$$V_{med} = I_{med} (R_a + R_c) + V_0 \sim I_{med} R_c + k_1 \omega_a I_{med} = I_{med} (R_c + k_1 \omega_a)$$

$$C_{amed} = B_L \omega_{amed}$$

$$C_{amed} = k_2 I_{amed} I_{cmed} = k_2 I_{med}^2 = k_2 [V_{med} / (R_c + k_1 \omega_a)]^2$$



## Chap. 34 Control of Power (II Part)

### CONTROL

#### Regulator of motor speed of continuous

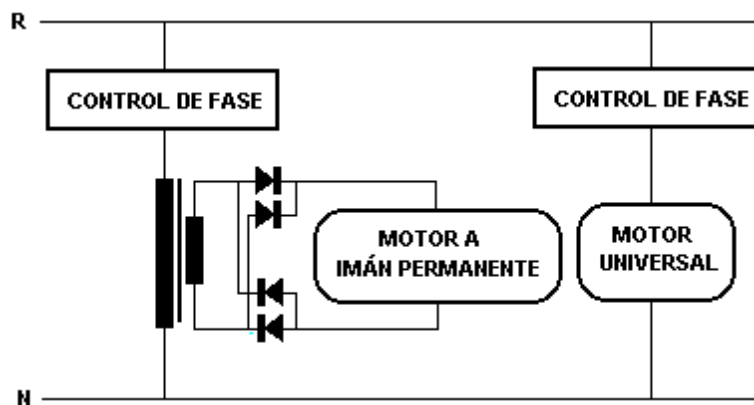
#### Regulator of speed of motor asynchronous three-phasic

### CONTROL

#### Regulator of motor speed of continuous

Although dedicated to the old libraries of the electronics, these regulators continue being used in small motors of permanent imam. We will see a general case of power until practically the 10 [HP].

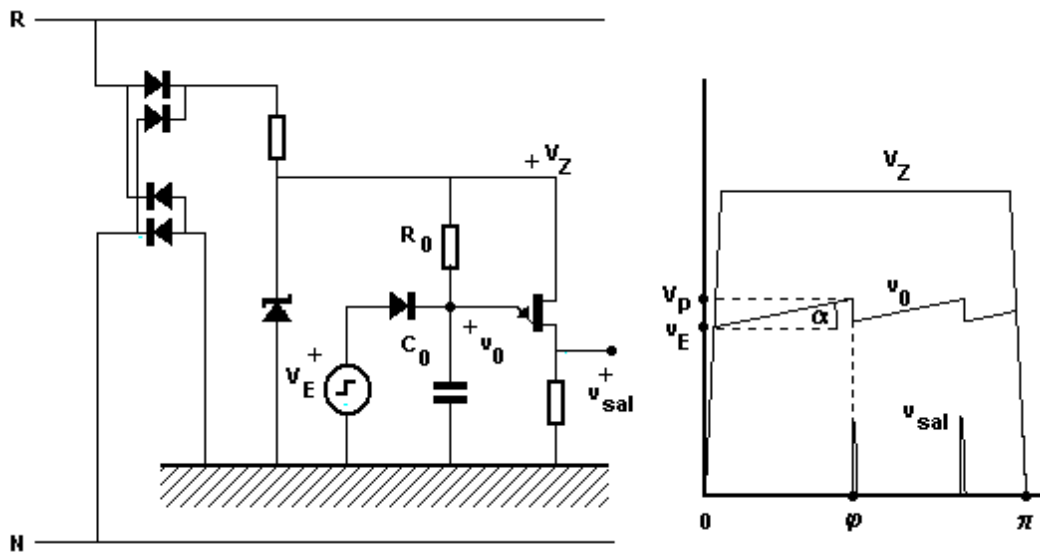
The first implementation that we see is simple, of bad stabilization of speed for not maintaining the torque since is a system without feedback, but of economic and efficient cost for lows mechanical loads. The circuit consists in the phase control explained previously, and it doesn't usually apply above the 1/4 [HP].



For these powers or bigger, when we want constant the speed, is to produce a feedback in the circuit. The reference of speed can be taken with a tachometer, with an optic detector, with sensors of proxitimy, etc. We will see that simple implementation that taking of sample to the same back-electromotive voltage  $V_{0med}$  (recuérdese que  $V_{amed} \sim V_{0med} = k_g \omega_{amed}$ ).

The technique here employee calls herself of *ramp and pedestal*. The first one refers a control of the gain for displacement of the phase  $\phi$ , and the second to a pedestal  $V_E$  of polarization of this phase —for the reader not familiarized with the TUJ he can go to the chapter of *relaxation oscillators*.

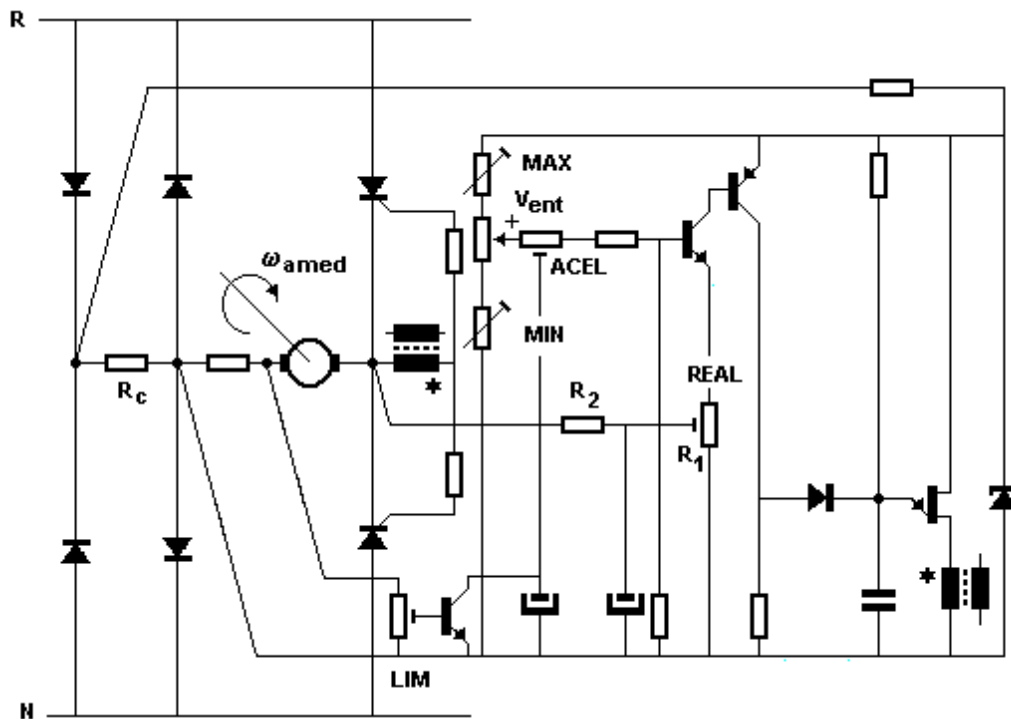
The operation of the system consists on loading with a constant current to the condenser and that it follows a ramp. The manual regulation will come given by the change of the amplitude of the pedestal.



The implementation following sample a circuit regulator of speed. It possesses five adjustment controls

- MÁXIMA y MÍNIMA to adjust the range of resolution of the potentiometer
- LIMITADOR of current for the rotor
- REGULADOR of the negative feedback
- ACELERACIÓN of the speed protecting abrupt changes in the potentiometer

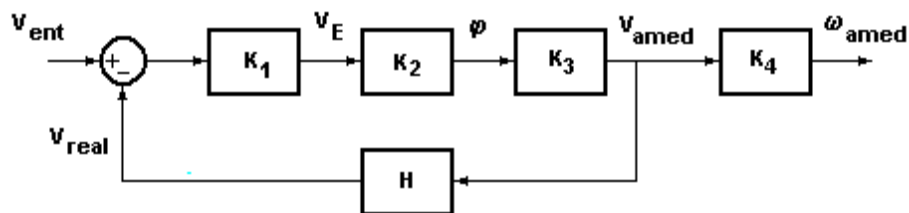
being omitted that of adjustment of the un-lineality of the current of the rotor in low speeds usually called IXR (voltage in an external resistance R of the current of the rotor) for not being very appreciable their effect.



Due to the great negative feedback the output speed is practically constant, because it depends on the dividing resistive of the potentiometer and it is not in its equation the load  $B_L$

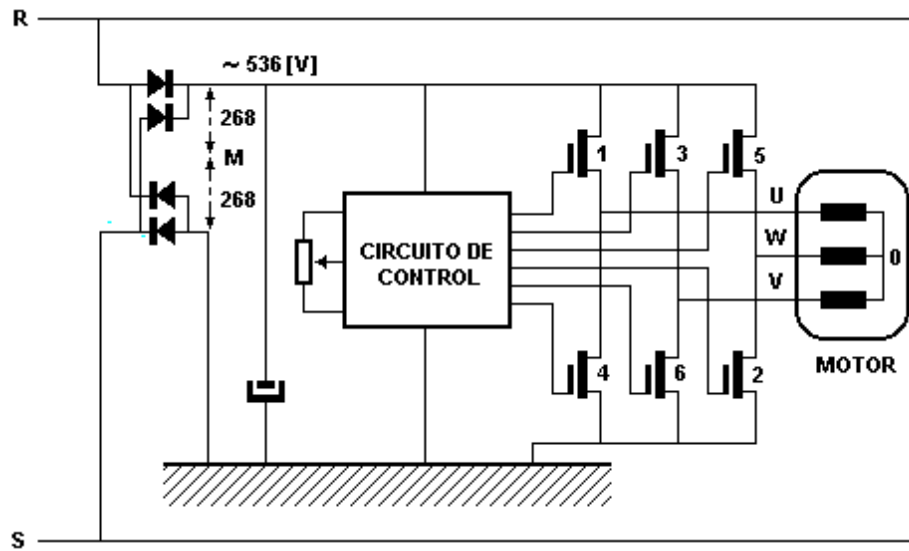
$$H = R_1 / (R_1 + R_2) \sim R_1 / R_2$$

$$\omega_{amed} \sim (K_4 / H) V_{ent} \sim (K_4 R_1 / R_2) V_{ent}$$

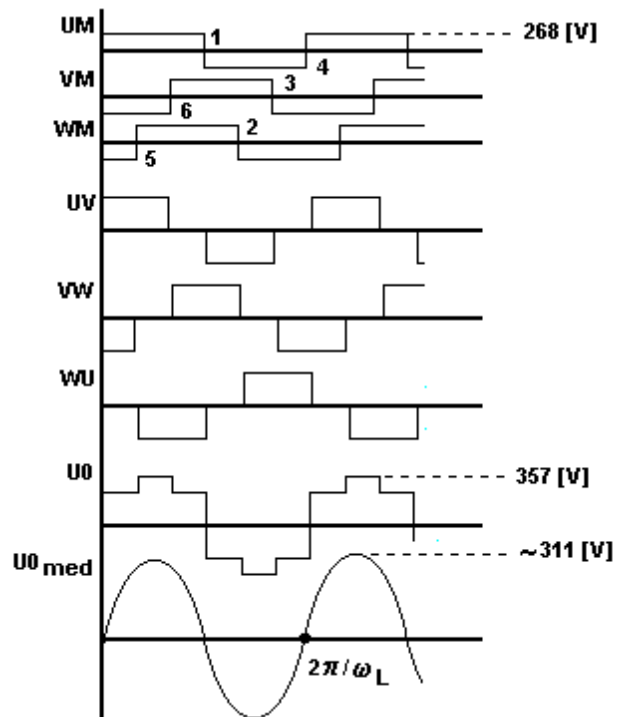


### Regulator of speed of motor asynchronous three-phasic

The normal use of these regulators up to about 5 [HP] it is in motors of the type 220/380, this is, with windings of supply 220 [V] effective; for such a reason it connects them to him in triangle when the regulator is of input mono-phasic (RN) and in star if it is it bi-phasic (RS), and it has the following circuit so that approximately it is obtained on them a voltage pick of 311 [V] as maximum —as for the connection if it is star or triangle, it can have exceptions for the different uses and dispositions of each production. For more big, and very bigger powers (f.ex.: 300 [HP]), the supply is three-phasic and the rectify-filtrate is obviated commuting the three-phasic directly on the machine, determining with it another analytic approach and ecuations to the respect that we don't see.

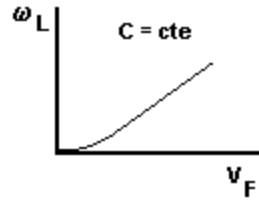


The disposition of the inverter can be with TEC like it is shown, with RCS, or modernly with transistors IGBT or TBJ. The following figures are representative of the sequence order and result of the commutations



To maintain the torque constant  $C$  in these motors the relationship among the frequency applied to the machine and their effective voltage it will practically also be a constant. The following expressed graph that said

$$\omega_L \sim k_1 V_F$$

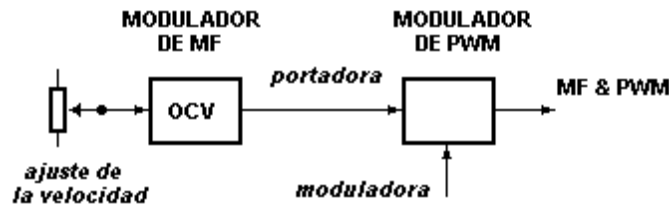


The technique that is described in this circuit is to change the frequency of input  $\omega_L$  with the purpose of varying the speed  $\omega_m$  of the axis. For such a reason it is regulated the frequency like pulses that arrive to their windings. Being  $v_F$  the  $U_0$  in the graph, to slide constant  $\xi$  because it is supposed that the machine is not demanded, it is then (to go to the chapter of *electric installations*)

$$\omega_m = 2\xi \omega_L / p \sim k_2 \omega_L$$

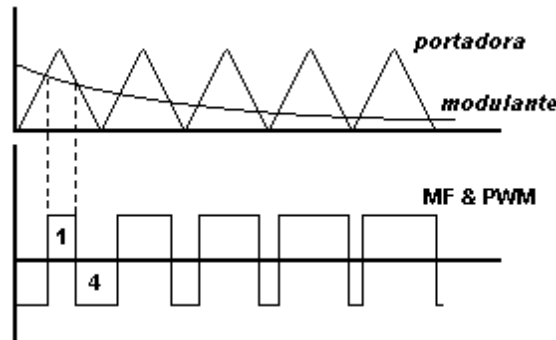
and to maintain the torque the width of the pulses it is modulated with the purpose of varying the  $V_F$ .

In summary, the system consists on a modulator of frequency (MF) followed by a modulator of wide of pulses (PWM)

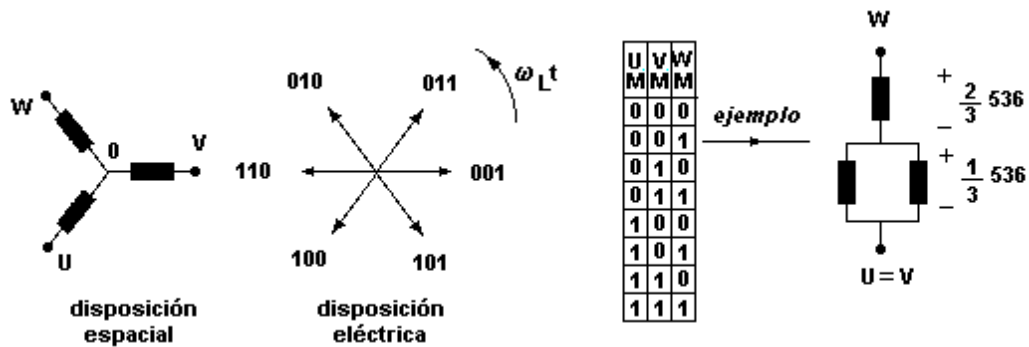


Claro estará que este sistema es de lazo abierto y por lo tanto no garantiza la manutención de cambios de cupla; es decir que sólo sirve para pequeñas cargas fijas (bajos y constantes rozamientos  $B_L$ ) donde el  $\xi$  se mantiene como se dijo. Para superar esto se sensará la corriente por uno de los bobinados y realimentará convenientemente, y donde se jugará con el ancho de los pulsos y con esto consecuentemente sobre la tensión eficaz en  $U_0$ , es decir  $V_F$ , logrando mantener la recta de la gráfica anterior.

There would be two ways to generate this modulation for wide of pulses, that is: a first one that compares the level of continuous of reference (or modulating) with a generation in triangular ramp (or carrier) according to the following diagram



and other second that explains to you in the *vectorial diagram of the space states* of the windings of the motor, and that it goes producing a sequence of commutations moving to a wL it conforms to it explains in the drawing



Of more to say two things will be, that is: the problem of the harmonics that generates this circuit type, and second the electronic complexity, so much of hardware as software and their mathematical sustenance with which these systems are implemented. They are usually prepared to measure protection, to estimate internal states of the motor, etc.

# Chap. 35 Introduction to the Theory of the Control

## THE LAPLACE CONVOLUTION

### DECOMPOSITION IN SIMPLE FRACTIONS

#### Denominator with simple poles

#### Denominator with multiple poles

### AUTO-VALUES AND AUTO-VECTORS

#### Generalities

#### Determination of the auto-values

#### Determination of the auto-vector

### ORDER AND TYPE OF A SYSTEM

#### Order of a system with feedback Glc

#### System type with feedback Glc

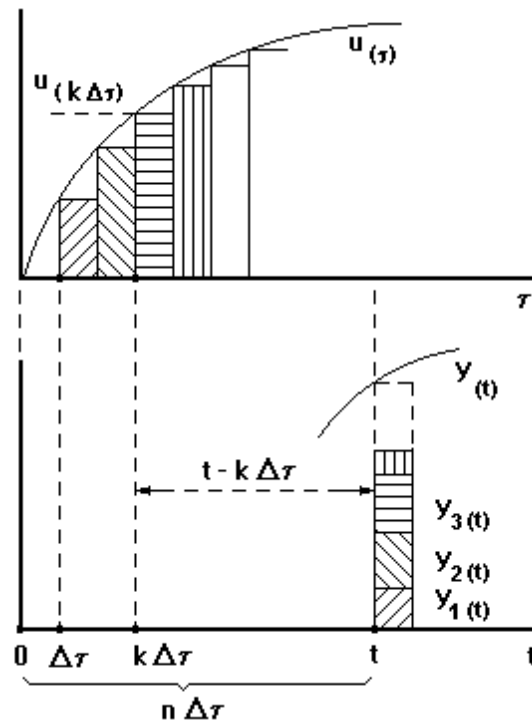
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## THE LAPLACE CONVOLUTION

Be a transfer  $G(s) = y(s) / u(s)$  to the one that is applied a certain signal temporary  $u(\tau)$  that will also determine an output temporary  $y(t)$ .

The equations will be

$$g(\tau) = \mathcal{L}^{-1}[G(s)] \rightarrow \text{response to the impulse}$$
$$0 \leq k \leq n$$



$$y_k(t) = \text{Área} \cdot g_{(t-k\Delta\tau)} = u_{(k\Delta\tau)} \cdot \Delta\tau \cdot g_{(t-k\Delta\tau)}$$

$$y(t) = \sum_{k=1}^n y_k(t) = \sum_{k=1}^n u_{(k\Delta\tau)} \cdot \Delta\tau \cdot g_{(t-k\Delta\tau)}$$

$$y(t) = u_{(\tau)} \cdot g_{(t)} = u_{(t)} * g_{(t)} =$$

$$= \int_0^t u_{(\tau)} \cdot g_{(t-\tau)} \cdot \partial\tau \equiv \int_0^t u_{(t-\tau)} \cdot g_{(\tau)} \cdot \partial\tau$$

## DECOMPOSITION IN SIMPLE FRACTIONS

### Denominator with simple poles

$$G_{(s)} = G_0(s) / [(s+p_1)(s+p_2) \dots] = [A_1 / (s+p_1)] + [A_2 / (s+p_2)] + \dots$$

$$A_i = [(s+p_i) \cdot G_{(s)}]_{s=-p_i}$$

### Denominator with multiple poles

$$G_{(s)} = G_0(s) / [(s+p_1)(s+p_2) \dots (s+p_k)^r (s+p_n)] =$$

$$= [A_1 / (s+p_1)] + [A_2 / (s+p_2)] + \dots [A_n / (s+p_n)] + \rightarrow \text{simple}$$



$$+ [ B_1 / (s+p_k) ] + [ B_2 / (s+p_k)^2 ] + \dots [ B_r / (s+p_k)^r ] \rightarrow \text{multiple}$$

$$A_i = [ (s+p_i) \cdot G(s) ]_{s=-p_i} \rightarrow \text{simple}$$

$$B_r = [ (s+p_k)^r \cdot G(s) ]_{s=-p_k} \rightarrow \text{multiple}$$

$$B_{r-1} = [ \partial / \partial s ] \cdot [ (s+p_k)^r \cdot G(s) ]_{s=-p_k}$$

$$B_{r-2} = [ 1 / 2! ] \cdot [ \partial^2 / \partial s^2 ] \cdot [ (s+p_k)^r \cdot G(s) ]_{s=-p_k}$$

...

$$B_1 = [ 1 / (r-1)! ] \cdot [ \partial^{r-1} / \partial s^{r-1} ] \cdot [ (s+p_k)^r \cdot G(s) ]_{s=-p_k}$$

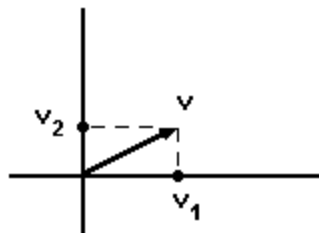
## AUTO-VALUES AND AUTO-VECTORS

### Generalities

Be a matrix A multi-dimensional

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

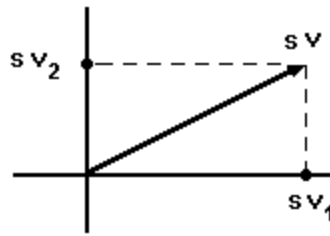
and a bi-dimensional vector



$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [v_1 \ v_2]^T = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

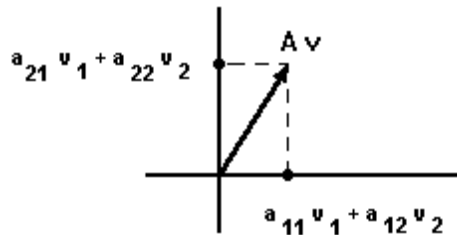
with  $v_1 = [v_{11} \ v_{12}]^T$  y  $v_2 = [v_{21} \ v_{22}]^T$

We will be able to change their module without changing their angle if we multiply it for a scalar (real or complex) «s»



$$s.v = \begin{bmatrix} s.v_1 \\ s.v_2 \end{bmatrix}$$

and also their module and angle if we multiply it for the matrix A

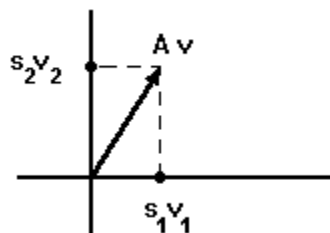


$$A.v = \begin{bmatrix} a_{11}v_1 & a_{12}v_2 \\ a_{21}v_1 & a_{22}v_2 \end{bmatrix}$$

If now it is completed that «s» it is a matrix line of elements scalers (real or complex)

$$s = \begin{bmatrix} s_1 & s_2 \end{bmatrix}$$

and we make coincide their products in the way



$$A.v = s.v = \begin{bmatrix} s_1.v_1 \\ s_2.v_2 \end{bmatrix}$$

it is said that «v» it is the own vector or *auto-vector* of the matrix A, and where

$$s_1 v_1 = a_{11} v_1 + a_{12} v_2$$

$$s_2 v_2 = a_{21} v_1 + a_{22} v_2$$

### Determination of the auto-values

If we wanted to find these scalars of «s» we make

$$0 = s v - A v = (s I - A) v = \begin{bmatrix} s-a_{11} & -a_{12} \\ -a_{21} & s-a_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$0 = (s I - A) v = \begin{bmatrix} s-a_{11} & -a_{12} \\ -a_{21} & s-a_{22} \end{bmatrix}$$

$$\begin{aligned} \text{Det } 0 &= \text{Det } (s I - A) = s^2 - s(a_{11} + a_{22}) + (a_{11}a_{22} - a_{12}a_{21}) = (s + s_1) + (s + s_2) = \\ &= s^2 + a_1 s + a_2 = 0 \end{aligned}$$

$$s_1, s_2 = \{ (a_{11} + a_{22}) \pm [(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})]^{1/2} \} \cdot 1/2$$

where we observe that they are the same scalars that determine the *roots of the characteristic equation* or *roots of the characteristic polynomial* of the matrix A.

On the other hand, for the case peculiar of a matrix A diagonal ( $a_{12} = a_{21} = 0$ )

$$s_1, s_2 = a_{11}, a_{22}$$

In summary

$$\text{Det } (s I - A) = (s + s_1) + (s + s_2) = s^2 + a_1 s + a_2 = 0 \quad \text{ec. characteristic}$$

$$a_1 = \dots, a_2 = \dots \quad \text{coefficients}$$

$$s_1 = \dots, s_2 = \dots \quad \text{auto-values (- poles)}$$

$$s = [s_1 \ s_2 \ s_3] \quad \text{matrix line of the auto-values}$$

### Determination of the auto-vector

If we wanted to find these vectors of «v» we make

$$A \cdot v = s \cdot v$$

$$A \cdot v = (s \cdot I) \cdot v = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \cdot v$$

$$0 = (A - s I) \cdot v$$

then now

$$0 = (A - s_1 I) \cdot v_1 = \begin{pmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_1 \end{bmatrix} - A \end{pmatrix} \cdot v_1 = \begin{pmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{pmatrix} \cdot v_1$$

$$0 = (A - s_2 I) \cdot v_2 = \begin{pmatrix} \begin{bmatrix} s_2 & 0 \\ 0 & s_2 \end{bmatrix} - A \end{pmatrix} \cdot v_2 = \begin{pmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{pmatrix} \cdot v_2$$

and with it

$$\begin{aligned} a_{11} \cdot v_{11} + a_{12} \cdot v_{12} &= 0 \\ a_{21} \cdot v_{11} + a_{22} \cdot v_{12} &= 0 \end{aligned} \rightarrow v_{11} = \dots \quad v_{12} = \dots$$

$$\begin{aligned} a_{11} \cdot v_{21} + a_{12} \cdot v_{22} &= 0 \\ a_{21} \cdot v_{21} + a_{22} \cdot v_{22} &= 0 \end{aligned} \rightarrow v_{21} = \dots \quad v_{22} = \dots$$

## ORDER AND TYPE OF A SYSTEM

### Order of a system with feedback Glc

It is the «order» or «degree» of the polynomial denominator of Glc; that is to say, of the quantity of inertias or poles that it has.

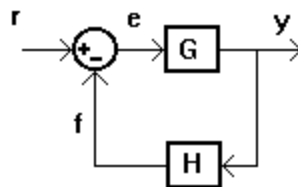
### System type with feedback Glc

For systems with feedback H without poles in the origin —we will study in those H that are constant—, it is denominated «type» to the quantity of «n» poles in the origin that has G—integrations of the advance.

Let us see their utility.

As it was said, be then

$$\begin{aligned} G(s) &= K_G \cdot [(1+s/z_1)(1+s/z_2) \dots] / [s^n (1+s/s_1)(1+s/s_2) \dots] \\ H(s) &= K_H \\ F(s) &= 1 + G(s)H(s) = K_F \cdot [(1+s/w_1)(1+s/w_2) \dots] / [s^n (1+s/s_1)(1+s/s_2) \dots] \end{aligned}$$



Now we will find the error «and» of the system in permanent state using the *theorem of the final value*

$$\begin{aligned}
e_{(\infty)} &= \lim_{s \rightarrow 0} s \cdot e(s) = \lim_{s \rightarrow 0} s \cdot [y(s) / G(s)] = \\
&= \lim_{s \rightarrow 0} s \cdot [r(s) G(s) / G(s)] = \lim_{s \rightarrow 0} s \cdot [r(s) / F(s)] = \\
&= \lim_{s \rightarrow 0} s \cdot [r(s) / (K_F / s^n)] = (1/K_F) \cdot \lim_{s \rightarrow 0} s^{n+1} \cdot r(s)
\end{aligned}$$

and the output «y» also in permanent state

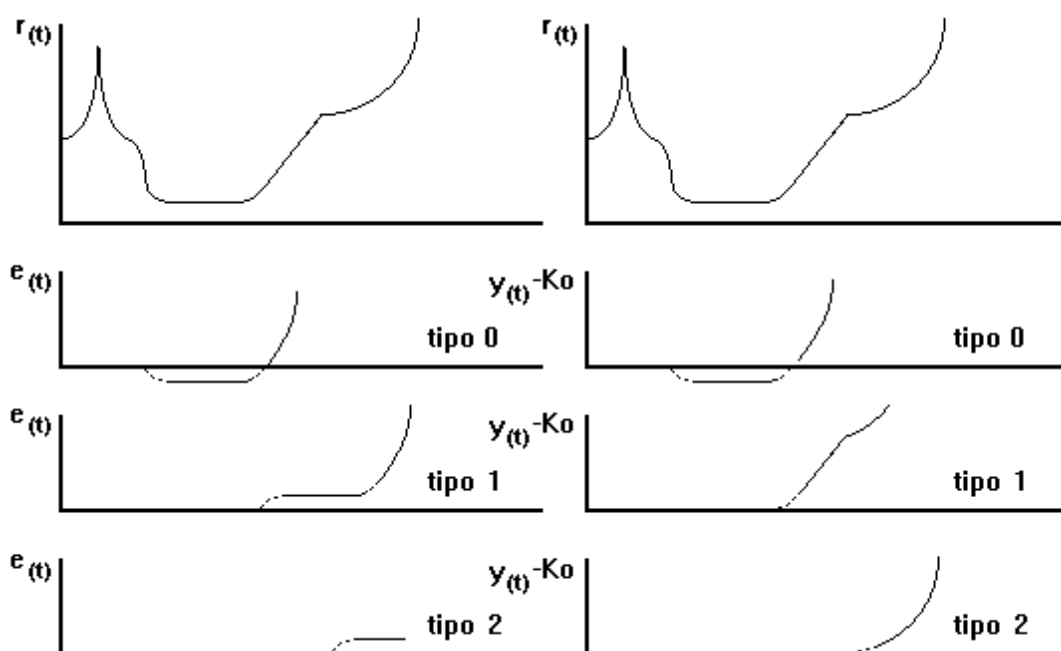
$$\begin{aligned}
e_{(\infty)} &= \lim_{s \rightarrow 0} s \cdot e(s) = \lim_{s \rightarrow 0} s \cdot [y(s) / G(s)] = (1/K_G) \cdot \lim_{s \rightarrow 0} s^{n+1} \cdot y(s) = \\
&= (1/K_G) \cdot \lim_{s \rightarrow 0} s \cdot [s^n \cdot y(s)] = \lim_{t \rightarrow \infty} \partial^n y(t) / \partial t^n \\
y_{(\infty)} &= K_G \cdot \int^n e_{(\infty)} \cdot \partial t^n + K_0
\end{aligned}$$

with  $K_0$  an integration constant —more big, same or smaller than zero, and that it will depend on the system.

Now these equations detail the following table for different excitements «r»

	impulso (Kronecker)	escalón	rampa	parábola
	$\delta^*$	1	t	$t^2$
Error « $e_{(\infty)}$ »				
tipo 0	0	$1/K_F$	$\infty$	$\infty$
tipo 1	0	0	$1/K_F$	$\infty$
tipo 2	0	0	0	$1/K_F$
Salida « $y_{(\infty)}-K_0$ »				
tipo 0	0	$K_G/K_F$	$\infty$	$\infty$
tipo 1	0	0	$(K_G/K_F) \cdot t$	$\infty$
tipo 2	0	0	0	$(K_G/2K_F) \cdot t^2$

that it expresses the approximate tracking of the error and of the output.



## Chap. 36 Discreet and Retained signals

RELATIONSHIP AMONG THE ONE DERIVED AND THE LATER SAMPLE

GENERALITIES OF THE SAMPLING

TRANSFER OF THE «SAMPLING AND R.O.C.» («M-ROC»)

SYSTEMS WITH «M-ROC»

1<sup>o</sup> CASE

2<sup>o</sup> CASE

3<sup>o</sup> CASE

4<sup>o</sup> CASE

5<sup>o</sup> CASE

RESOLUTION OF TEMPORARY GRAPHS

Application example

EQUATIONS OF STATE

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RELATIONSHIP AMONG THE ONE DERIVED AND THE LATER SAMPLE

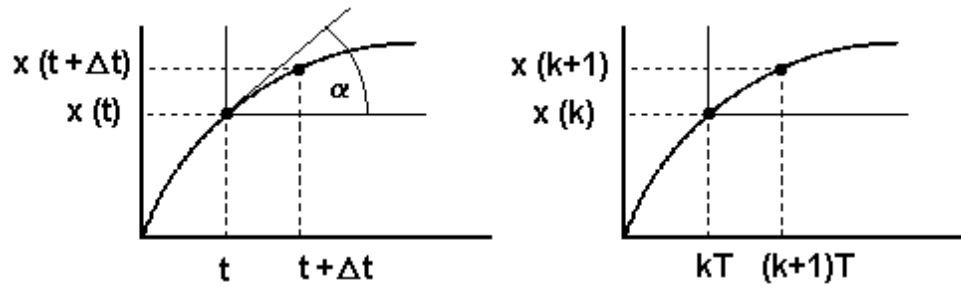
We will work on a sampling retained of order zero.



The speed of the sampling will be much bigger than the maximum speed of the signal «x»; or, said otherwise by the theorem of the sampling:

$$T < 1 / 2 f_{\max}$$

Subsequently we show that, for a dynamic consideration, the amplitude of the next sample is proportional to the one derived in the point of previous sample:



$$x' = \operatorname{tg} \alpha = [x_{(t+\Delta t)} - x_{(t)}] / \Delta t = [x_{(kT+T)} - x_{(kT)}] / T = [x_{(k+1)} - x_{(k)}] / T$$

As for the transformed of Laplace

$$L[x_{(t-kT)} - x_{(0)}] = e^{-skT} [x_{(s)} - x_{(0)}]$$

or

$$L[x_{(t+kT)} - x_{(0)}] = e^{skT} [x_{(s)} - x_{(0)}]$$

And for the transformed z

$$Z[x_{(k+1)}] = z \cdot [x_{(z)} - x_{(0)}] = L^{-1} \{ e^{sT} [x_{(s)} - x_{(0)}] \}$$

It is

$$x' \cdot T = x_{(k+1)} - x_{(k)} = L^{-1} \{ e^{skT} [x_{(s)} - x_{(0)}] \} = Z[x_{(k+1)}]$$

Finally

$$x' \propto Z[x_{(k+1)}]$$

A simple demonstration would be

$$x' = \operatorname{tg} \alpha = x_{(k+1)} / T = [x_{(t+\Delta t)} - x_{(t)}] / \Delta t$$

$$\lim_{\Delta t \rightarrow 0} \operatorname{tg} \alpha = x_{(t)}' = x_{(k+1)} / T$$

$$\Delta t \rightarrow 0$$

$$x_{(k+1)} = x_{(t)}' \cdot T \propto x'$$

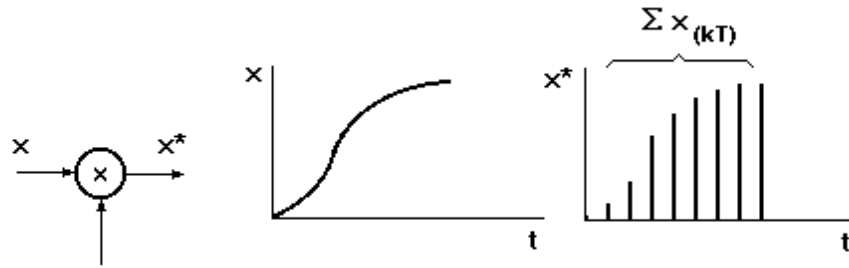
## GENERALITIES OF THE SAMPLING

As



$$z = e^{sT}$$

$$x_{(kT)} = x_{(k)}$$

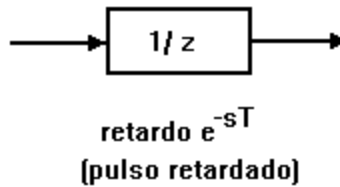


it is for the one transformed of Laplace

$$x(z) = x^*(t) = \sum_{k=0}^{\infty} x_{(k)} \cdot e^{-ksT} = \sum_{k=0}^{\infty} x_{(k)} \cdot z^{-k} = x_{(0)} + x_{(T)} \cdot z^{-1} + x_{(2T)} \cdot z^{-2} + \dots$$

where an equivalence is observed among the «integrative» with the «retarder»:

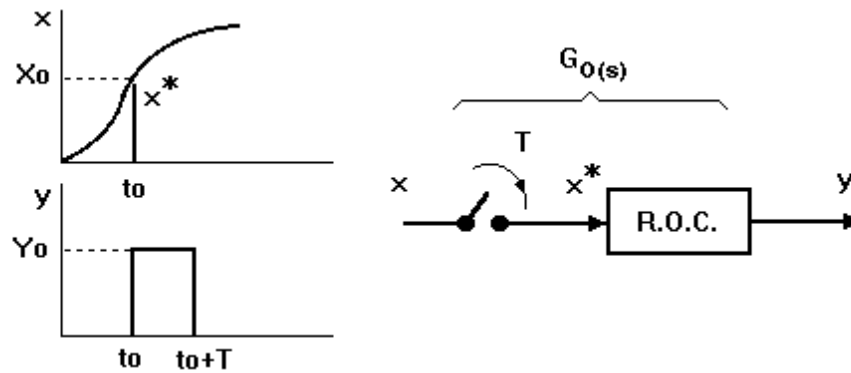
$$e^{-sT} = z^{-1} = 1/z$$



### TRANSFER OF THE «SAMPLING AND R.O.C.» («M-ROC»)

Be «X<sub>0</sub>» a pedestal and «δ\*» the impulse of Kronecker. Then:

$$\begin{aligned} x_{(t)} &= \delta^* X_0 \\ x_{(s)} &= X_0 \\ y_{(t)} &= Y_0 \cdot [X_{(t_0)} - X_{(t_0+T)}] \\ y_{(s)} &= Y_0 \cdot [ (1/s) - (e^{-sT}/s) ] \end{aligned}$$



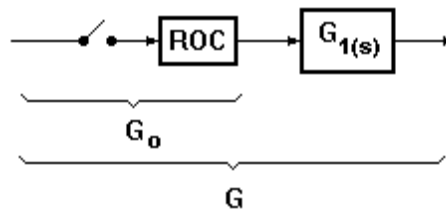
de donde

$$G_{O(s)} = y(s) / x(s) = (Y_0/X_0) \cdot [(1 - e^{-sT}) / s] = G_0 \cdot [(1 - e^{-sT}) / s]$$

$$\begin{aligned} G_{O(z)} &= y(z) / x(z) = Z[G_{O(s)}] = Z[(1 - e^{-sT}) \cdot Z^{-1}(1/s)] = \\ &= (1 - z^{-1}) \cdot Z[1/s] = (1 - z^{-1}) \cdot (1 - z^{-1})^{-1} = 1 \end{aligned}$$

## SYSTEMS WITH «M-ROC»

### 1º CASE

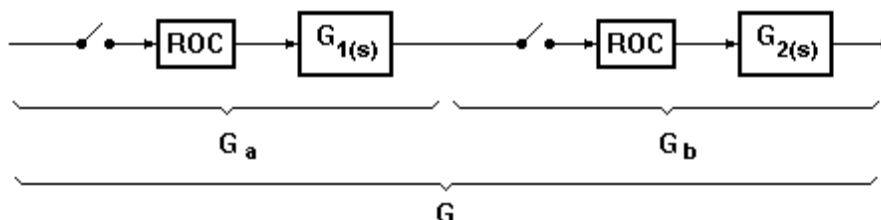


$$G_{0(s)} = (1 - e^{-sT}) / s$$

$$G(s) = G_{0(s)} \cdot G_{1(s)} = (1 - e^{-sT}) \cdot G_{1(s)} / s$$

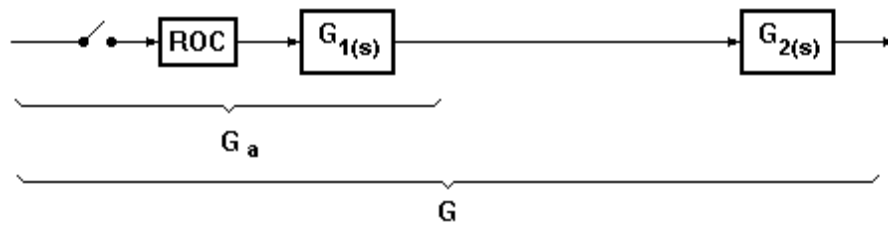
$$G(z) = Z[(1 - e^{-sT}) \cdot Z^{-1}[G_{1(s)} / s]] = (1 - z^{-1}) \cdot Z[G_{1(s)} / s] \neq 1 \cdot Z[G_{1(s)}]$$

### 2º CASE



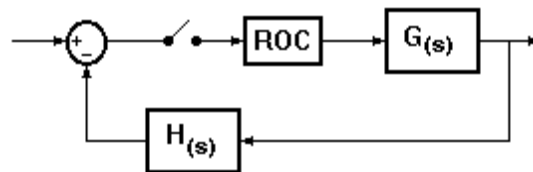
$$G_{(z)} = G_{a(z)} \cdot G_{b(z)}$$

### 3º CASE



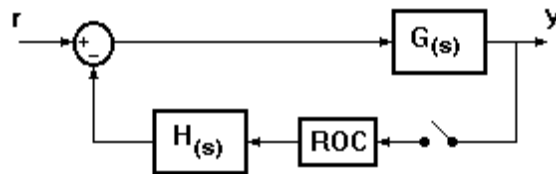
$$G_{(z)} = Z [ G_{a(s)} \cdot G_{2(s)} ] \neq G_{a(z)} \cdot G_{2(z)}$$

### 4º CASE



$$G_{lc(z)} = \{ (1 - e^{-z}) \cdot Z [ G_{(s)} / s ] \} / \{ 1 + (1 - e^{-z}) \cdot Z [ G_{(s)} H_{(s)} / s ] \}$$

### 5º CASE



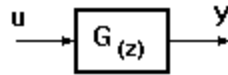
$G_{lc(z)} \rightarrow$  no se puede hallar

$$y_{(z)} = Z [ r \cdot G_{(s)} ] / \{ 1 + (1 - e^{-z}) \cdot Z [ G_{(s)} H_{(s)} / s ] \}$$

### RESOLUTION OF TEMPORARY GRAPHS

Be

$$\begin{aligned} G_{(z)} &= K_1 \cdot [ (z+z_1) (z+z_2) \dots ] / [ (z+p_1) (z+p_2) \dots ] = \\ &= K_2 \cdot [ (1+z_1 z^{-1}) (1+z_2 z^{-1}) \dots ] / [ (1+p_1 z^{-1}) (1+p_2 z^{-1}) \dots ] \end{aligned}$$



and decomposing it in simple fractions

$$G(z) = K_2 \cdot \{ [A / (1+p_1 z^{-1})] + [B / (1+p_2 z^{-1})] + \dots \}$$

it can be

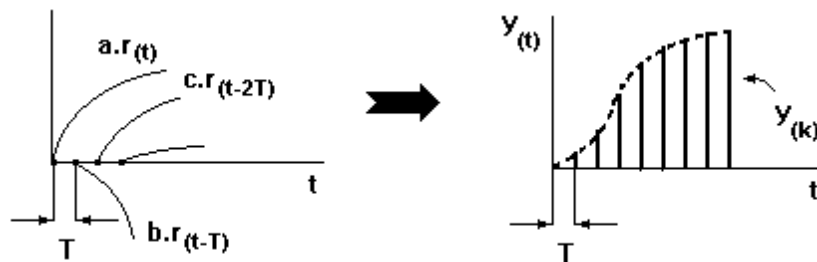
$$G(s) = K_2 \cdot (a + b \cdot e^{-sT} + c \cdot e^{-s2T} + \dots)$$

and for the one transformed of Laplace it is

$$\mathcal{L}[e^{-snT} \cdot r(s)] = r(t-nT)$$

it is finally

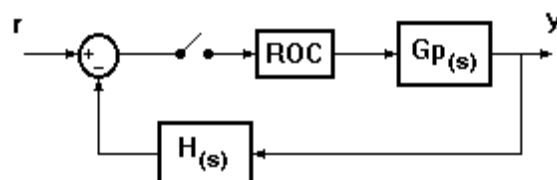
$$\begin{aligned} y(t) &= \mathcal{L}^{-1}[G(s) \cdot r(s)] = \\ &= K_2 \cdot [a \cdot r(t) + b \cdot r(t-T) + c \cdot r(t-2T) + \dots] \end{aligned}$$



### Application example

Be the following plant system  $G_p$  with feedback

$$\begin{aligned} G_p(s) &= K / (s + a) \\ H(s) &= 1 / s \\ r(t) &\rightarrow \text{unitary pedestal} \end{aligned}$$



$$(1 - z^{-1}) \cdot \sum [G_{p(s)} / s] = (1 - z^{-1}) \cdot \sum [K / s(s + a)] = [(1 - e^{-aT}) / (2 - e^{-aT})] \cdot K/a$$

$$(1 - z^{-1}) \cdot \sum [G_{p(s)} H_{(s)} / s] = \{ [z(e^{-aT} - 1 + aT) + (1 - e^{-aT} - aTe^{-aT})] / [(z-1)(z - e^{-aT})] \} \cdot K/a^2$$

and if we simplify  $A=1$ ,  $T=1$  and  $K=1$

$$\begin{aligned} \text{Glc}_{(z)} &= \{ (1 - z^{-1}) \cdot \sum [G_{(s)} / s] \} / \{ 1 + (1 - z^{-1}) \cdot \sum [G_{(s)} H_{(s)} / s] \} = \\ &= 0,63 (z-1) / (z^2 - z + 0,63) = 0,63 (z-1) / [(z-z_1)(z-z_1^*)] \\ z_1 &= 0,5 + j 0,62 = 0,796 \cdot e^{j51,1^\circ} \quad (0,796 < 1 \rightarrow \text{estable}) \end{aligned}$$

We outline the excitement now

$$\begin{aligned} r_{(s)} &= 1 / s \\ r_{(z)} &= z / (z-1) \end{aligned}$$

to find the output

$$\begin{aligned} y_{(z)} &= r_{(z)} \cdot \text{Glc}_{(z)} = 0,63 z / [(z-0,5-j0,62)(z-0,5+j0,62)] = \\ &= z \cdot [A/(z-z_1) + B/(z-z_1^*)] \end{aligned}$$

with

$$\begin{aligned} A &= -j 0,51 \\ B &= j 0,51 \end{aligned}$$

of where

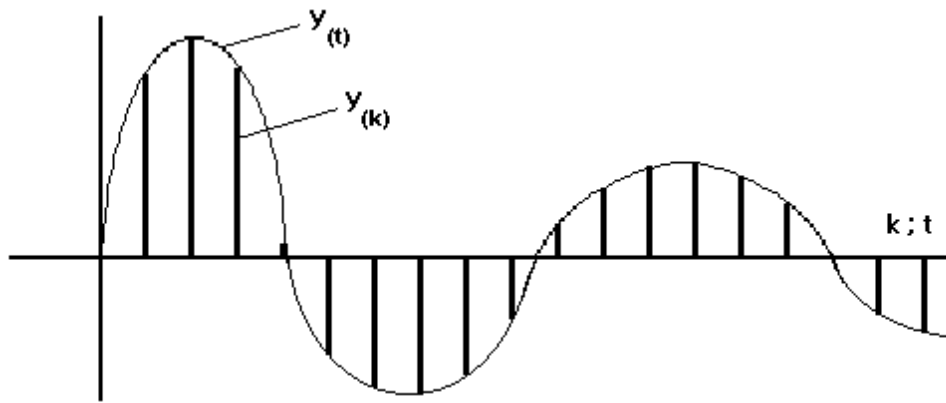
$$y_{(z)} = [-j 0,51 z/(z-z_1)] + [j 0,51 z/(z-z_1^*)]$$

and as those transformed (pedestal in this case) they correspond

$$U_{(t)} \equiv 1_{(k)} \equiv 1 / s \equiv 1 / (1 - z^{-1}) = z / (z - 1)$$

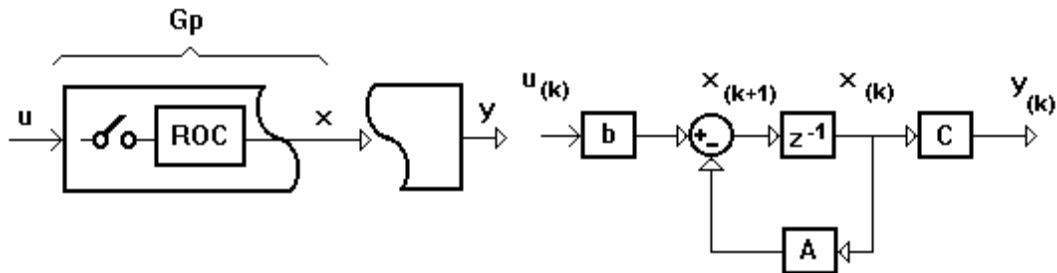
it is finally

$$\begin{aligned} y_{(k)} &= [-j 0,51 z_1^k] + [j 0,51 z_1^{*k}] = \\ &= [-j 0,51 (0,5 + j 0,62)^k] + [j 0,51 (0,5 - j 0,62)^k] \equiv \\ &\equiv 1,02 \cdot z_1^k = 1,02 \cdot 0,796^k \cdot \sin(k 51,1^\circ) \end{aligned}$$



## EQUATIONS OF STATE

We present the system and their behavior equations subsequently



$$\begin{cases} x_{(k+1)} &= A x_{(k)} &+ b u_{(k)} \\ y_{(k)} &= C x_{(k)} &+ d u_{(k)} \end{cases}$$

$$G_{p(z)} = y_{(z)} / u_{(z)} = G_{p0} / [(z + z_1)(z + z_2) \dots]$$

$$x_{(z)} / u_{(z)} = b \cdot [G / (1 + G.H)] = b \cdot \{(1/z) / [1 + (A \cdot 1/z)]\} = b / (z + A)$$

## Chap. 37 Variables of State in a System

### GENERALITIES

Lineal transformation of A

DETERMINATION OF THE STATES

CONTROLABILITY

OBSERVABILITY

VARIABLES OF STATE OF PHASE

In continuous systems

State equation

Exit equation

Example 1

Example 2

In discrete systems

TRANSFER, RESOLVENT AND TRANSITION

TRANSFER [ $\phi_{(s)}$ ]

Example

RESOLVENT [ $\Psi_{(s)}$ ]

TRANSITION [ $\phi_{(t)}$ ]

If the system is SI-SO

If the system is MI-MO

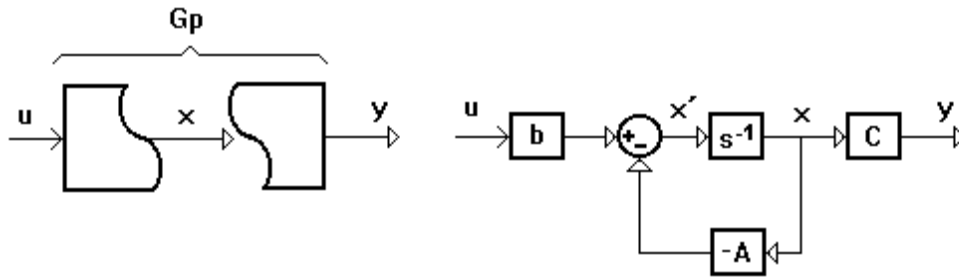
Example

Example

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### GENERALITIES

Each transfer reactivates inertial it will determine a state «  $x_i$  », because there is not a lineal correspondence (in the time) between its entrance and the output.



$$\begin{cases} \dot{x} = A x + b u \\ y = C x + d u \end{cases}$$

$x$  vector of the state of the plant Gp of dimension  $n \times 1$   
 $u$  vector of input of control of the plant Gp of dimension  $r \times 1$   
 $y$  vector of the output of the plant Gp of dimension  $m \times 1$

$$G_p = y / u = G_{po} / [(s + s_1)(s + s_2) \dots]$$

$$x / u = b \cdot [G / (1 + G.H)] = b \cdot \{(1/s) / [1 + (A \cdot 1/s)]\} = b / (s + A)$$

$A$  matrix of speed of the plant Gp [1/seg] of dimension  $n \times n$   
 $B$  matrix of control of the state through the input «u» of the plant Gp of dimension  $n \times r$   
 $C$  output matrix «y» of the plant dimension Gp of dimension  $m \times n$   
 $D$  matrix of direct joining of the input «u» of the plant Gp of dimension  $m \times r$

$$A = \begin{bmatrix} -s_1 & a_{12} \\ a_{21} & -s_2 \end{bmatrix}$$

$$\text{Det}(sI - A) = \begin{vmatrix} s + s_1 & 0 \\ 0 & s + s_2 \end{vmatrix} = (s + s_1)(s + s_2) \dots = s^2 + a_1 s + a_2 = 0$$

$$a_1 = a_{11} + a_{22} = (-s_1) + (-s_2) = -(s_1 + s_2)$$

$$a_2 = a_{11}a_{12} - a_{12}a_{21}$$

### Lineal transformation of A

For a plant matrix A diagonalized like  $A^*$  for a lineal transformation T (*modal* matrix, denominated this way by the «mode» in that transforms, and that it has their columns made with the auto-vectors —adopted— of A):

$$T = [v_1 \ v_2] = \begin{bmatrix} v_{11} & v_{12} \\ \vdots & \vdots \end{bmatrix}$$



$$\begin{bmatrix} v_{21} & v_{22} \end{bmatrix}$$

$$A^* = T^{-1} \cdot A \cdot T$$

$$b^* = T^{-1} \cdot b$$

$$c^{*T} = c^T \cdot T$$

$$x = T \cdot x^*$$

under the form non *general* Jordan but *canonical*—because their auto-values are different, and called this way to be a «legal particular convention»— the poles of the plant  $G_p$  that are  $s_1$  and  $s_2$  is similar to the poles of the  $G_p^*$ —that would be  $s_1^*$  and  $s_2^*$ —, because the auto-values of a diagonal matrix is their elements, and because the auto-values don't change for a lineal transformation.

Then:

$$\begin{bmatrix} -s_1^* & 0 \\ 0 & -s_2^* \end{bmatrix} = \begin{bmatrix} -s_1 & 0 \\ 0 & -s_2 \end{bmatrix} \quad A^* = \begin{bmatrix} -s_1 & 0 \\ 0 & -s_2 \end{bmatrix}$$

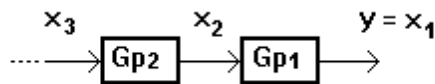
what determines that both characteristic equations of  $G_p$  and of  $G_p^*$  they are same

$$\text{Det}(sI - A) = \text{Det}(sI - A^*) = \begin{vmatrix} s+s_1 & 0 \\ 0 & s+s_2 \end{vmatrix} = (s+s_1)(s+s_2)$$

## DETERMINATION OF THE STATES

In the spectrum field

$$G_p = y/u = G_{p1} \cdot G_{p2} \dots = (y/x_2) \cdot (x_2/x_3) \dots = [G_{p01}/(s+s_1)] \cdot [G_{p02}/(s+s_2)] \dots$$



$$x_{i-1}/x_i = G_{p0i}/(s+s_i)$$

$$x_i \cdot G_{p0i} = x_{i-1} \cdot (s+s_i) = x_{i-1} \cdot s + x_{i-1} \cdot s_i$$

In the temporary field

$$x_i \cdot G_{p0i} = x_{i-1}' + x_{i-1} \cdot s_i$$

$$x_{i-1}' = (-s_i) \cdot x_{i-1} + G_{p0i} \cdot x_i$$

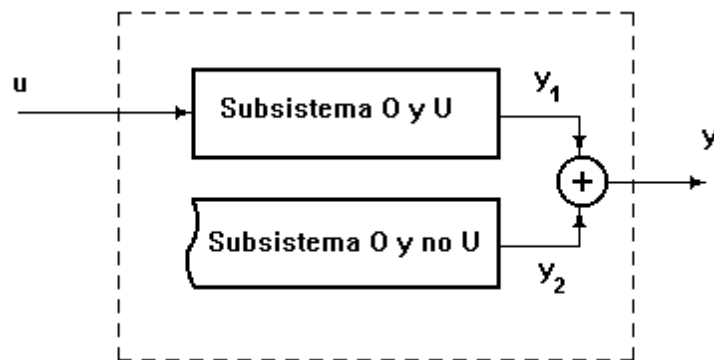
that is to say, generalizing for 3 inertias

$$\begin{aligned}
 \dot{x}_1 &= (-s_1) \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 \\
 \dot{x}_2 &= a_{21} \cdot x_1 + (-s_2) \cdot x_2 + a_{23} \cdot x_3 \\
 \dot{x}_3 &= a_{31} \cdot x_1 + a_{32} \cdot x_2 + (-s_3) \cdot x_3
 \end{aligned}$$

## CONTROLABILITY

The transfer  $G(s)$  it is only partially a report.

$$G(s) = y(s) / u(s) \mid_{x(0)=0} = G_{(y1)} \neq G_{(y2)}$$



It specifies this concept the possibility *to control*—or to govern—the state variables «x» from the input «u».

$$U = [B \quad A \cdot B \quad \dots \quad A^{n-1} \cdot B] \quad \text{matrix of controlability of } n \times (n \times r)$$

$$\text{Rango } U = \dots$$

$$\text{Quantity of «x» not controllable} = n - \text{Rango } U$$

## OBSERVABILITY

It specifies this concept the possibility *to observe*—to have access to their translation and mensuration—the state variables «x». If it is not this way, it will be necessary to estimate them as «x^».

$$O = [C^T \quad A^T C^T \quad (A^T)^2 C^T \quad \dots \quad (A^T)^{n-1} C^T] \quad \text{matrix of observability}$$

$$\text{Rango } O = \dots$$

$$\text{Quantity of «x» non observabilities} = n - \text{Rango } O$$

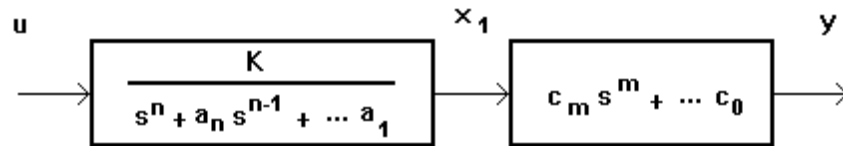
## VARIABLES OF STATE OF PHASE

## In continuous systems

Be a plant  $G_p$  in the transformed field of Laplace, where there are  $m$  zeros and  $n$  poles and that, so that it is inertial it requires logically that  $m \leq n$

$$G_p = y / u = K \cdot [c_m s^m + \dots c_0] / [s^n + a_n s^{n-1} + \dots a_1] =$$

$$= [x_1 / u] \cdot [y / x_1] = [K / (s^n + a_n s^{n-1} + \dots a_1)] \cdot [c_m s^m + \dots c_0]$$



## State equation

In the spectrum

$$x_1 / u = K / (s^n + a_n s^{n-1} + \dots a_1)$$

$$K u = x_1 s^n + x_1 a_n s^{n-1} + \dots x_1 a_1$$

and in the time

$$K u = x_1^n + a_n x_1^{n-1} + \dots a_1 x_1$$

and being

$$\begin{array}{ll} x_2 = x_1' & x_1' = x_2 \\ x_3 = x_2' & x_2' = x_3 \\ \dots & \dots \\ x_n = x_{n-1}' & x_{n-1}' = x_n \end{array}$$

it is

$$K u = x_{n+1} + a_n x_n + \dots a_2 x_2 + a_1 x_1$$

and finally (here it is exemplified  $n = 3$ )

$$x' = \begin{bmatrix} x_1' \\ \dots \\ x_n' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & \dots & -a_n \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} u$$

## Putput equation

In the spectrum

$$y / x_1 = c_m s^m + \dots c_0$$

$$y = c_m s^m x_1 + c_{m-1} s^{m-1} x_1 + \dots c_0 x_1$$

and in the time

$$y = c_m x_1^m + c_{m-1} x_1^{m-1} + \dots c_1 x_1' + c_0 x_1$$

because

$$x_{m+1} = x_1^m$$

$$x_m = x_1^{m-1}$$

...

$$x_2 = x_1'$$

and finally (here it is exemplified  $m = 3$ )

$$y = \begin{bmatrix} y_1 \\ \dots \\ y_{m+1} \end{bmatrix} = \begin{bmatrix} c_1 & \dots & c_m & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \dots \\ x_{m+1} \end{bmatrix}$$

donde la cantidad de elementos de  $\begin{bmatrix} c_1 & \dots & c_m & 0 & 0 \end{bmatrix}$  es  $n$ .

### Example 1

Be

$$m = n - 1$$

$$Gp = y / u = K \cdot [c_{n-1} s^{n-1} + \dots c_1] / [s^n + a_n s^{n-1} + \dots a_1]$$

then

$$x' = \begin{bmatrix} x_1' \\ \dots \\ x_n' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & \dots & -a_n \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix} \begin{bmatrix} K \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ \dots \\ y_n \end{bmatrix} = \begin{bmatrix} c_1 & \dots & c_m & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix}$$

### Example 2

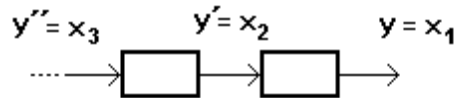
In the transformed field

$$G_p = (y.s^n + y.s^{n-1} . k_{n-1} + \dots y . k_0) / u$$

In the temporary field

$$G_p . u = y^n + y^{n-1} . k_{n-1} + \dots y . k_0$$

$$x_1 = y, x_2 = x_1' = y', \dots x_n = x_{n-1}' = y^{n-1}$$



For 3 inertias (n = 3)

$$G_p = (y.s^3 + y.s^2 . k_2 + y.s . k_1 + y . k_0) / u$$

$$G_p . u = y''' + y'' . k_2 + y' . k_1 + y . k_0$$

$$x_1' = y' = x_2$$

$$x_2' = y'' = x_3$$

$$x_3' = y''' = G_p . u - y'' . k_2 - y' . k_1 - y . k_0 = (-x_3) . k_2 + (-x_2) . k_1 + (-x_1) . k_0 + G_p . u$$

$$\begin{aligned} x_1' &= 0.x_1 + 1.x_2 + 0.x_3 + 0.u \\ x_2' &= 0.x_1 + 0.x_2 + 1.x_3 + 0.u \\ x_3' &= (-k_0)x_1 + (-k_1)x_2 + (-k_2)x_3 + G_p.u \end{aligned}$$

$$x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_0 & -k_1 & -k_2 \end{bmatrix} . x + \begin{bmatrix} 0 \\ 0 \\ G_p \end{bmatrix} . u$$

$$\text{Det}(sI - A) = s^3 + s^2 . k_2 + s . k_1 + k_0$$

### In discrete systems

Be a plant  $G_p$  in the transformed field  $z$ , where there are  $m$  zeros and  $n$  poles and that, so that it is inertial it requires logically that  $m \leq n$

$$G_p = y / u = K . [ c_m z^m + c_{m-1} z^{m-1} + \dots c_0 ] / [ z^n + a_n z^{n-1} + a_{n-1} z^{n-2} + \dots a_1 ]$$

and where the equivalence is given « $k \leftrightarrow z$ »

$$y_{(k+n)} + a_n y_{(k+n-1)} + a_{n-1} y_{(k+n-2)} + \dots + a_1 = K [c_m u_{(k+m)} + c_{m-1} u_{(k+m-1)} + \dots + c_0]$$

$$y_{(z)} z^n + a_n y_{(z)} z^{n-1} + a_{n-1} y_{(z)} z^{n-2} + \dots + a_1 = K [c_m u_{(z)} z^m + c_{m-1} u_{(z)} z^{m-1} + \dots + c_0]$$

it is deduced finally

$$x_{(k+1)} = \begin{bmatrix} x_{1(k+1)} \\ \vdots \\ x_{n(k+1)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ & \ddots & \ddots \\ -a_1 & \dots & -a_n \end{bmatrix} \begin{bmatrix} x_{1(k)} \\ \vdots \\ x_{n(k)} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ K \end{bmatrix} u_{(k)}$$

$$y_{(k)} = \begin{bmatrix} y_{1(k)} \\ \vdots \\ y_{n(k)} \end{bmatrix} = \begin{bmatrix} c_0 & c_1 & \dots & c_m & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{1(k)} \\ \vdots \\ x_{n(k)} \end{bmatrix} + \dots$$

where the quantity of elements of  $[c_1 \dots c_m 0 0]$  it is n.

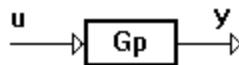
## TRANSFER, RESOLVENT AND TRANSITION

### TRANSFER $[\phi_{(s)}]$

It is denominated matrix of transfer  $\phi_{(s)}$  of a plant Gp to the following quotient with conditions null initials

$$\phi_{(s)} = y_{(s)} / u_{(s)} \quad ]_{x(0)=0}$$

$$y_{(s)} = [y_{1(s)} \ y_{2(s)} \ \dots]^T = \phi_{(s)} \cdot u_{(s)}$$



and it is observed that

$$\phi_{(s)} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = K \cdot [(s+z_1)(s+z_2) \dots] / [(s+s_1)(s+s_2) \dots]$$

$$\text{Det}(sI - A) = (s + s_1)(s + s_2) \dots = s^3 + a_1 s^2 + a_2 s + a_3 = 0 \quad \rightarrow \text{polos de } \phi_{(s)}$$

If we outline in the time

$$\begin{cases} \dot{x} = A x + b u \\ y = C x + d u \end{cases}$$

it is in the spectrum field

$$\begin{cases} s x = A x + b u \\ y = C x + d u \end{cases}$$

$$\begin{aligned} s I \cdot x - A x &= B u \\ x &= (s I - A)^{-1} B u \\ y &= C (s I - A)^{-1} B u + D u = [C (s I - A)^{-1} B + D] u \\ \phi(s) &= y(s) / u(s) = C (s I - A)^{-1} B + D = C \Psi B + D \end{aligned}$$

and finally

$$\phi(s) = C_{(s)} \Psi_{(s)} B_{(s)} + D_{(s)}$$

For discrete systems

$$\phi(z) = C_{(z)} \Psi_{(z)} B_{(z)} + D_{(z)}$$

In a general way, for continuous or discrete systems, as «C, B, D» they don't have poles, the characteristic equation of « $\phi$ » and of « $\Psi$ » they are the same ones.

### Example

Be the following system and that we want to find the outputs  $y(t)$  in permanent state for an excitement  $u(t)$  in unitary pedestal.

$$\dot{x} = \begin{bmatrix} -0,011 & 0,001 \\ 0,1 & -0,1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0,001 & -0,001 \end{bmatrix} x$$

They are then

$$\begin{bmatrix} s+0,011 & -0,001 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} | & \\ -0,1 & s+0,1 \end{bmatrix}$$

$$\text{Det}(sI - A)^{-1} = s^2 + 0,111s + 0,001 = (s + s_1)(s + s_2) = 0$$

$$s_1; s_2 = 0,0099; 0,101$$

$$\text{Cof}(sI - A)^{-1} = \begin{bmatrix} s+0,1 & 0,1 \\ 0,001 & s+0,011 \end{bmatrix}$$

$$\text{Adj}(sI - A)^{-1} = \begin{bmatrix} s+0,1 & 0,001 \\ 0,1 & s+0,011 \end{bmatrix}$$

of where

$$\begin{aligned} \phi_{(s)} &= y_{(s)} / u_{(s)} = C_{(s)} \Psi_{(s)} b_{(s)} = C(sI - A)^{-1} b = \\ &= C [\text{Adj}(sI - A)^{-1} / \text{Det}(sI - A)^{-1}] b = \\ &= C [\text{Adj}(sI - A)^{-1}] b / \text{Det}(sI - A)^{-1} = \\ &= \begin{bmatrix} (s + 0,1) / \text{Det}(sI - A)^{-1} \\ 0,1 / \text{Det}(sI - A)^{-1} \\ 0,001 \cdot s / \text{Det}(sI - A)^{-1} \end{bmatrix} = \begin{bmatrix} (s + 0,1) / (s + s_1)(s + s_2) \\ 0,1 / (s + s_1)(s + s_2) \\ 0,001s / (s + s_1)(s + s_2) \end{bmatrix} \end{aligned}$$

$$u_{(s)} = 1/s$$

$$y_{1(\infty)} = \lim_{s \rightarrow 0} s \cdot y_{1(s)} = \lim_{s \rightarrow 0} s \cdot (1/s) \cdot [(s + 0,1) / (s + s_1)(s + s_2)] = 100$$

$$y_{2(\infty)} = \lim_{s \rightarrow 0} s \cdot y_{2(s)} = \lim_{s \rightarrow 0} s \cdot (1/s) \cdot [0,1 / (s + s_1)(s + s_2)] = 100$$

$$y_{3(\infty)} = \lim_{s \rightarrow 0} s \cdot y_{3(s)} = \lim_{s \rightarrow 0} s \cdot (1/s) \cdot [0,001s / (s + s_1)(s + s_2)] = 0$$

## RESOLVENT [ $\Psi_{(s)}$ ]

We call  $\Psi_{(s)}$  to the function that allows «to solve» the transfer function  $\phi_{(s)}$

$$\Psi_{(s)} = [sI - A_{(s)}]^{-1}$$

The poles of  $\Psi_{(s)}$  they are the auto-values of A, or, the poles of  $\phi_{(s)}$  of the plant Gp.



$$\text{Det } (sI - A)^{-1} = (s + s_1)(s + s_2) \dots$$

→ poles of  $\phi(s)$  and of  $\Psi(s)$

For discrete systems

$$\Psi(z) = [zI - A(z)]^{-1}$$

### TRANSITION $[\phi(t)]$

It is the matrix of transfer  $\phi(s)$  in the time

$$\phi(t) = L^{-1}[\phi(s)]$$

and it differs conceptually because it considers the initial state  $x_{(0)}$ .

### If the system is SI-SO

$$x' = ax + bu$$

$$sx - x_{(0)} = ax + bu$$

$$x = [x_{(0)} / (s-a)] + [bu / (s-a)]$$

with

$$x_{(0)} / (s-a) \rightarrow \text{transitory response of «x»}$$

$$bu / (s-a) \rightarrow \text{respuesta permanente de «x»}$$

and anti-transforming

$$x = e^{at} x_{(0)} + \int_0^t e^{a(t-\tau)} b u_{(\tau)} d\tau = \phi_{(t)} x_{(0)} + \phi_{(t)} * b u_{(t)}$$

being

$$\phi_{(t)} = [x / x_{(0)}]_{u=0} = e^{at}$$

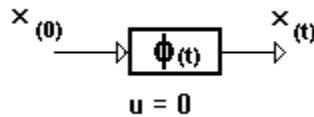
that is to say that, conceptually, the transition of the state  $\phi_{(t)}$  it allows to determine the state « $x_{(t)}$ » like it adds of their previous state « $x_{(0)}$ » and what accumulates —convolution— for the excitement « $u_{(t)}$ ».

### If the system is MI-MO

$$\phi_{(t)} = [x / x_{(0)}]_{u=0} = e^{At}$$

$$x = e^{At} x_{(0)} + \int_0^t e^{A(t-\tau)} B u_{(\tau)} d\tau = \phi_{(t)} x_{(0)} + \phi_{(t)} * b u_{(t)}$$

$$y = C x + D u = C [\phi_{(t)} x_{(0)} + \phi_{(t)} * b u_{(t)}] + D u_{(t)}$$



Also, it can demonstrate himself that there is a coincidence among  $\phi_{(s)}$  and  $\Psi_{(s)}$

$$\Psi_{(s)} = \phi_{(s)}$$

For discrete systems

$$\begin{cases} sx = A x + b u \\ y = C x + d u \end{cases}$$

$$x_{(k=1)} = A x_{(0)} + B u_{(0)}$$

$$x_{(2)} = A x_{(1)} + B u_{(1)} = A^2 x_{(0)} + A B u_{(0)} + B u_{(1)}$$

...

$$x_{(k)} = A^k x_{(0)} + \sum_{i=0}^{k-1} A^{k-i-1} B u_{(i)} = \phi_{(k)} x_{(0)} + \phi_{(k)} * B u_{(k)}$$

then

$$\phi_{(z)} = \phi_{(k)} = A^k$$

$$z \cdot \Psi_{(z)} = \phi_{(z)}$$

### Example

Be a system of plant of two poles

$$x' = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot x$$

then we find

$$sI - A_{(s)} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$[sI - A_{(s)}]^{-1} = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} / (s^2+3s+2)$$

$$\phi_{(t)} = L^{-1}[\phi_{(s)}] = L^{-1}[\Psi_{(s)}] = L^{-1}\{[sI - A_{(s)}]^{-1}\} =$$

$$= \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$$

$$x_{(t)} = \phi_{(t)} x_{(t_0)} + \phi_{(t)} * b u_{(t)} = \phi_{(t)} x_{(t_0)} + \int_0^t \phi_{(t-t_0)} [0 \ 1]^T u_{(t_0)} dt_0 =$$

$$= \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix} \begin{bmatrix} x_{1(t_0)} \\ x_{2(t_0)} \end{bmatrix} + \begin{bmatrix} 0,5u_{(t)}-e^{-t}+0,5e^{-2t} \\ e^{-t}-e^{-2t} \end{bmatrix} =$$

$$= \begin{bmatrix} x_{1(t)} \\ x_{2(t)} \end{bmatrix}$$

## Chap. 38 Stability in Systems

### CONTINUOUS SYSTEMS

#### Concept of the spectrum domain

In signals

In transfers

#### Stability

### CRITERIAL OF THE CALCULATION OF STABILITY

#### Approach of stability of Routh

#### Approach of stability of Nyquist

Example

Simplification for inertial simple systems

Gain and phase margins

Calculation of the over-peak

Systems MI-MO

Systems with delay

### DISCREET SYSTEMS

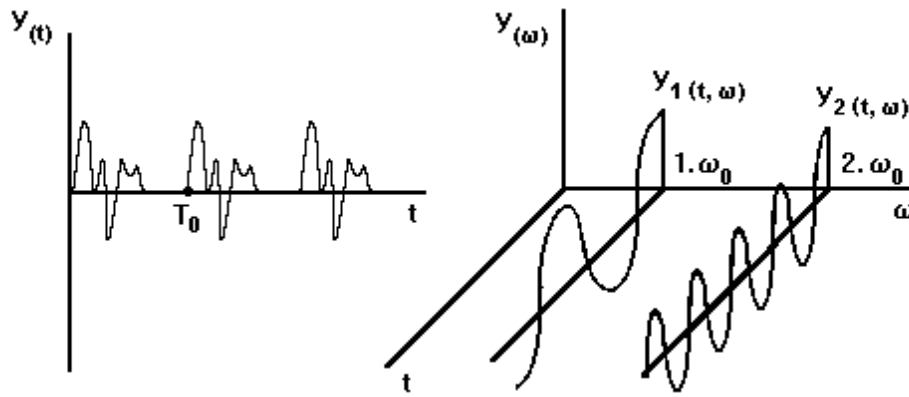
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### CONTINUOUS SYSTEMS

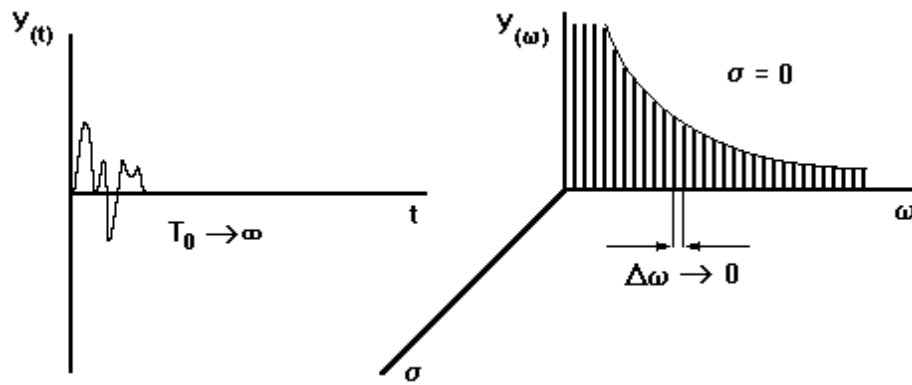
#### Concept of the spectrum domain

In signals

Given a temporary signal « $y_{(t)}$ » repetitive to a rhythm « $\omega_0=2\pi f_0=2\pi/T_0$ », it will have their equivalent one in the spectrum domain for their harmonics determined by the «series of Fourier» in « $y_{(\omega)}$ ».



When the temporary signal « $y(t)$ » it is not repetitive, it is said that it is isolated, and it will have their equivalent one in the spectrum domain for their harmonics determined by the «transformed of Laplace» in « $y(s = \sigma + j\omega)$ ». The real part of the complex variable « $\sigma$ », that is to say « $s$ », it is proportional to the duration of the damping of the transitory temporary transitions, and the imaginary one « $\omega$ » it is proportional to the frequency or speed of the permanent state. When « $\sigma = 0$ » the encircling of Laplace coincides with the encircling of the series of Fourier.



### In transfers

It is defined transfer to the quotient

$$G(s) = \frac{y(s)}{u(s)} \Big|_{y(0)=0}$$

that is to say, the output on the input, in the spectrum, for null initial conditions.

We know that the same taking the form of quotient of polynomials, or of expressed quotients their polynomials like product of their roots

$$\begin{aligned} G(s) &= K_1 \cdot [c_m s^m + \dots c_0] / [s^n + a_n s^{n-1} + \dots a_1] = \\ &= K_2 \cdot [(s+z_1)(s+z_2) \dots (s+z_m)] / [(s+s_1)(s+s_2) \dots (s+s_n)] \end{aligned}$$

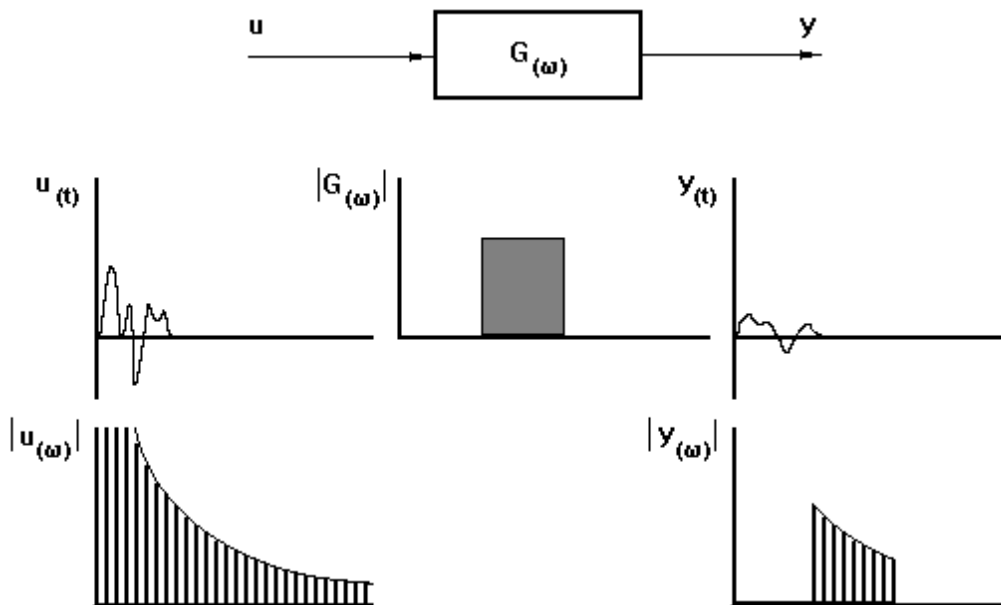
where  $m \leq n$  so that it is an inertial system; that is to say, so that it is real and don't respond to infinite speeds—that is to say that the  $G_{(\omega = \infty)} = 0$ .

We observe here that the following questions are given

$$\begin{aligned} G_{(s=-z_1)} &= 0 \\ G_{(s=-s_1)} &= \infty \end{aligned}$$

and consequently we call «zeros» of the equation  $G_{(s)}$  to the values of «s» —these are:  $-z_1, z_2, \dots$ — that annul it, and «poles» of the equation  $G_{(s)}$  to the values of «s» —these are:  $-s_1, s_2, \dots$ — that make it infinite.

The transfer concept —either in the complex domain « $s=\sigma+j\omega$ » or in the *fasorial* « $\omega$ »—, it consists on the «transmission» of the encircling spectrum of Laplace explained previously. The following graph it wants to serve as example and explanatory



## Stability

We define stability in a system when it is governable, that is to say, when their output it doesn't go to the infinite and it can regulate, or, it is annulled alone.

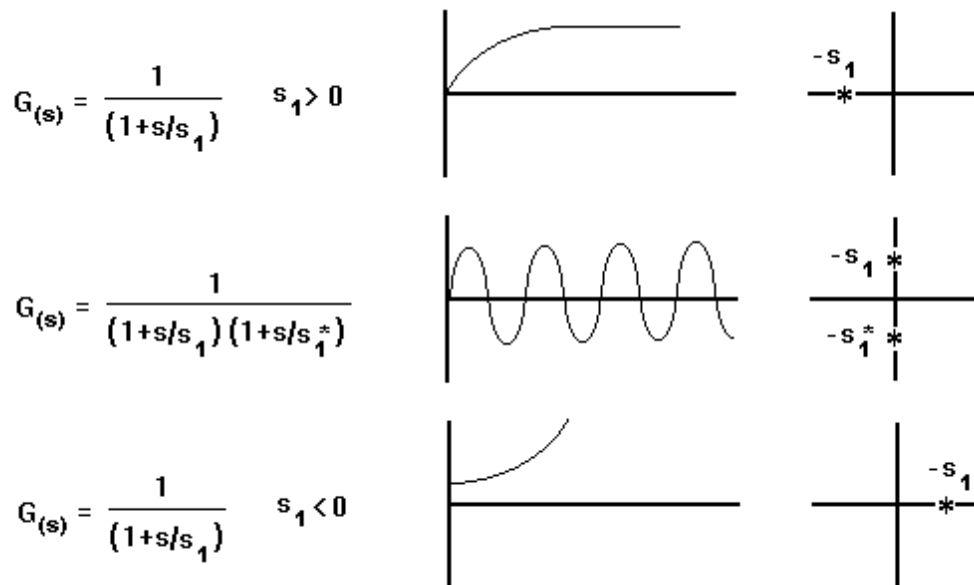
Let us see the matter mathematically.

Let us suppose that a pedestal is applied « $1/s$ » to a transfer  $G_{(s)} = y_{(s)} / u_{(s)}$  of a pole « $1/(1+s/s_1)$ ».

The output will be

$$\begin{aligned} y_{(s)} &= 1 / [ s \cdot (1+s/s_1) ] \\ y_{(t)} &= 1 - e^{-ts_1} \end{aligned}$$

for what we will obtain three possible temporary graphs, as well as their combinations, and that they are drawn next as  $y_{(t)}$  and the locations of the poles in the complex plane «s».



of where it is observed finally that the first case is only stable. This way, we redefine our concept of stability like «that system that doesn't have any pole in the right semi-plane», since it will provide some exponential one growing and not controllable to their output.

In summary, it is denominated to the equation of the denominator of all transfer like «characteristic equation», either expressed as polynomial or as the product of their roots, and it is the one that will determine, for the location of this roots, the stability or not of the system—transfer.

### CRITERIAL OF THE CALCULATION OF STABILITY

We know that the location of the poles of a total transfer —f.ex.: with feedback  $G_{lc}$ — it defines their stability. For that reason, at first sight, it seemed very simple to solve this topic: we experience  $G$  and  $H$  and we approach a polynomial of  $G_{lc}$ , and we find their roots then, and of there we see the location of the poles.

If as much  $G$  as  $H$  are inertial systems without zeros and with a dominant pole; or, all that can approach to the effect, can find experimentally in a simple way the transfers if we act in the following way, that is: applying a pedestal to the input of the system

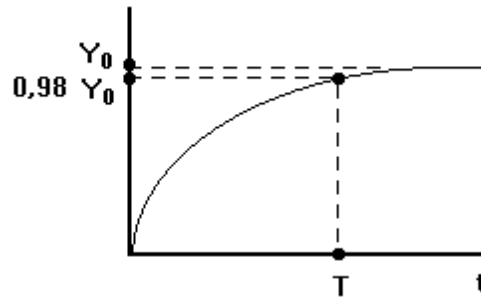
$$\begin{aligned}
 u(t) &= U_{(0)} = \dots \\
 u(s) &= U_{(0)} / s \\
 G(s) &= K / (1 + s \tau) \\
 Y(s) &= u(s) \cdot G(s) \\
 Y(t) &= [K U_{(0)}] \cdot (1 - e^{-t/\tau})
 \end{aligned}$$

where

$$Y(3\tau) = [K U_{(0)}] \cdot (1 - e^{-3\tau/\tau}) \cong 0,98 \cdot [K U_{(0)}]$$

and if we measure the output then until practically it stays

$$\begin{aligned} T &= \dots \\ Y_0 &= \dots \end{aligned}$$



we can determine

$$\begin{aligned} \tau &\cong 3 \cdot T = \dots \\ K &= Y_0 / U_{(0)} = \dots \end{aligned}$$

The inconvenience comes in the practice when the denominator of Glc is not of second degree, but bigger. To find the roots it should be appealed to algebraic or algorithmic annoying methods by computer. It is not dynamic. For this reason, if we are interested only in the question of stability, they are appealed to different denominated practical methods «criterial of stability». they are some of them: of Bode, of Routh, of Nyquist, of Nichols, of Liapunov, etc.

We will study that of Nyquist fundamentally for their versatility, wealth and didactics.

### Approach of stability of Routh

Given the characteristic equation «F» of the system with feedback «Glc», it is equaled to zero and they are their coefficients. Then a table arms with the elements that indicate the equations, and it is observed if there is or not changes of signs in the steps of the lines of the first column. If there are them, then there will be so many poles of Glc in the right semiplane —determining unstability— as changes they happen.

Let us see this:

$$F(s) = a_0 s^n + a_1 s^{n-1} + \dots a_n = 0$$

that for a case of sixth order

$$F(s) = a_0 s^6 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s + a_6 = 0$$

$s^6$	$a_0$	$a_2$	$a_4$	$a_6$
$s^5$	$a_1$	$a_3$	$a_5$	0
$s^4$	A	B	C	0
$s^3$	D	E	F	0
$s^2$	G	H	I	0
s	J	K	L	0



1      M      N      O      0

where each element of the table has been determined by the following algorithm

$$A = (a_1 \cdot a_2 - a_0 \cdot a_3) / a_1 = \dots$$

$$B = (a_1 \cdot a_4 - a_0 \cdot a_5) / a_1 = \dots$$

$$C = (a_1 \cdot a_6 - a_0 \cdot 0) / a_1 = \dots$$

$$D = (A \cdot a_3 - a_1 \cdot B) / A = \dots$$

$$E = (A \cdot a_5 - a_1 \cdot a_6) / A = \dots$$

$$F = (A \cdot 0 - a_1 \cdot 0) / A = \dots$$

$$G = (D \cdot B - A \cdot E) / D = \dots$$

$$H = (D \cdot C - A \cdot F) / D = \dots$$

$$I = (D \cdot 0 - A \cdot 0) / D = \dots$$

$$J = (G \cdot E - D \cdot H) / G = \dots$$

$$K = (G \cdot F - D \cdot I) / G = \dots$$

$$L = (G \cdot 0 - D \cdot 0) / G = \dots$$

$$M = (J \cdot H - G \cdot K) / J = \dots$$

$$N = (J \cdot I - G \cdot L) / J = \dots$$

$$O = (J \cdot 0 - G \cdot 0) / J = \dots$$

and it is observed the possible sign changes that leave happening in

$$a_0 \rightarrow a_1 \rightarrow A \rightarrow D \rightarrow G \rightarrow J \rightarrow M$$

For particular cases it should be helped with the reference bibliography.

### Approach of stability of Nyquist

Given the transfers of a system with feedback expressed as quotients of polynomials of zeros «Z» and of poles «P»

$$G_{(s)} = Z_G / P_G$$

$$H_{(s)} = Z_H / P_H$$

$$G_{(s)} H_{(s)} = Z_{GH} / P_{GH} = (Z_G / P_G) (Z_H / P_H) = Z_G Z_H / P_G P_H$$

$$F_{(s)} = Z_F / P_F = 1 + G_{(s)} H_{(s)} = 1 + Z_{GH} / P_{GH} = (P_{GH} + Z_{GH}) / P_{GH}$$

$$\begin{aligned} \text{Glc}(s) &= Z_{\text{Glc}} / P_{\text{Glc}} = G_{(s)} / [1 + G_{(s)} H_{(s)}] = G_{(s)} / F_{(s)} = \\ &= (Z_G / P_G) / [(P_{GH} + Z_{GH}) / P_{GH}] = (Z_G P_H) / (P_{GH} + Z_{GH}) \end{aligned}$$

where is seen that the stability of the closed loop depends on the zeros of the characteristic equation

$$P_{Glc} = Z_F = (P_{GH} + Z_{GH})$$

This way, our analyses will be centered on  $F(s)$ . We suppose that it has the form

$$\begin{aligned} F(s) &= K_F \cdot [(s+z_1)(s+z_2) \dots] / [(s+s_1)(s+s_2) \dots] = \\ &= K_F \cdot [(M_1 e^{j\alpha_1})(M_2 e^{j\alpha_2}) \dots] / [(s+s_1)(s+s_2) \dots] \end{aligned}$$

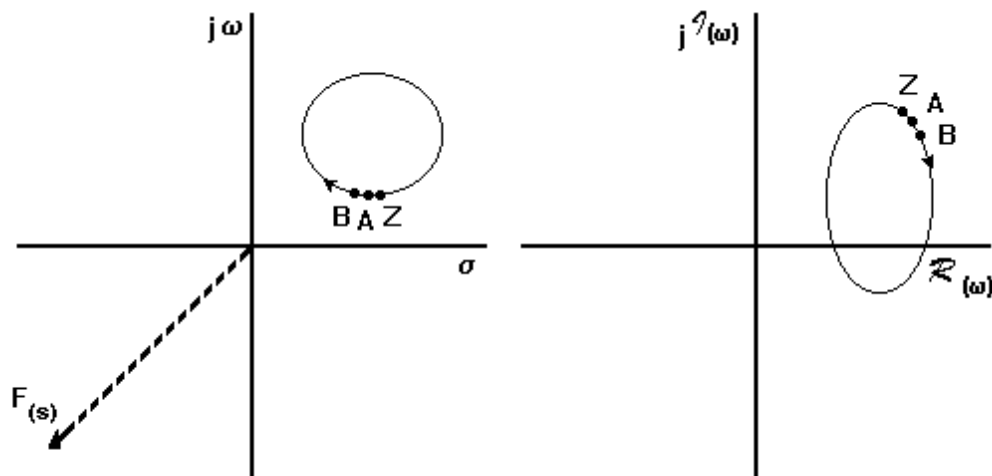
and in the domain *fasorial*

$$F(\omega) = F(s) ]_{\sigma \rightarrow 0} = R(\omega) + j I(\omega)$$

what will determine two corresponding correlated planes one another: the «s» to the « $\omega$ »

$$F(s) \leftrightarrow F(\omega)$$

that is to say, that a certain value of « $s=s_A = \sigma_A + j\omega_A$ » it determines a point «A» in the plane of  $F(s=s_A)$  and other « $\omega=\omega_A$ » in that of  $F(\omega=\omega_A)$ ; another contiguous point «B» the effect will continue, and so forth to the infinite that we denominate point «Z», forming a closed line then with principle and end so much in  $F(s)$  like in  $F(\omega)$ .

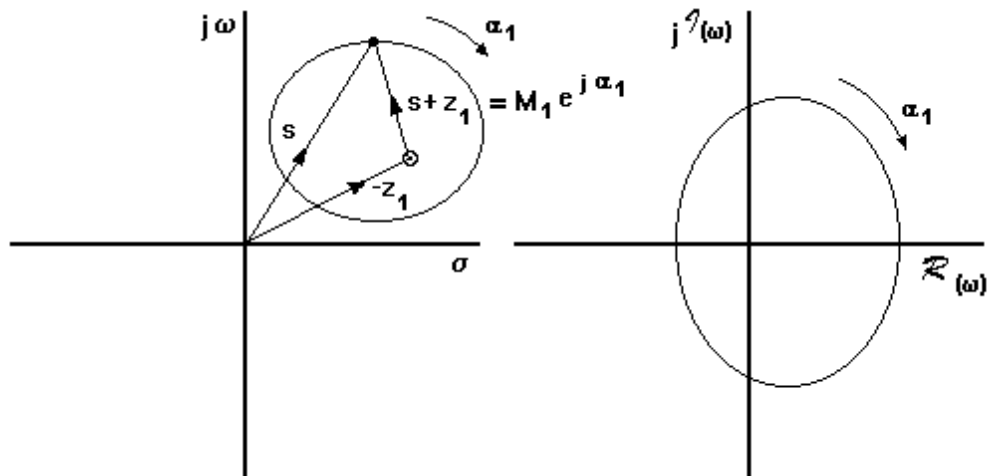


If now we have present that a curve closed in  $F(s)$  it contains a zero, then this curve will make a complete closing of the center of coordinated in  $F(\omega)$ , since

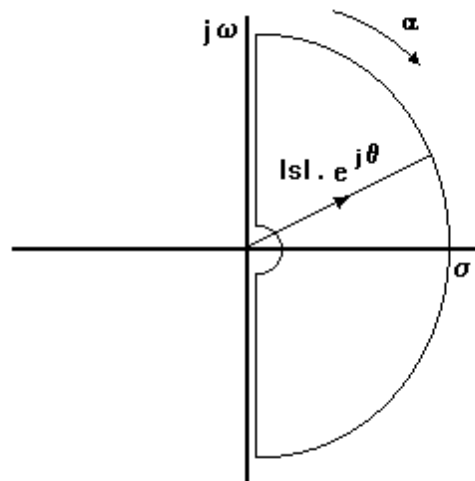
$$\begin{aligned} \alpha_1 &\rightarrow \text{gira 1 vuelta completa} \\ F(s) &= (M_1 e^{j\alpha_1}) \cdot \text{algo} \rightarrow \text{gira 1 vuelta completa} \\ F(\omega) &\rightarrow \text{gira 1 vuelta completa} \end{aligned}$$

and if what contains is a pole the turn it will be in opposed sense; and if the quantity of zeros and contained poles are the same one it won't rotate; but yes it will make it in the first way when there is a bigger quantity of zeros that of poles, that is to say when

$$P_F < Z_F$$



This property will be used to contain the whole right semiplane —that is the one that presents the difficulty in the study of the stability of Glc— of  $F(s)$  and with it to know in  $F(\omega)$ , that if the center is contained of coordinated, there is at least a zero —pole of Glc— introducing uncertainty.



As of the practice  $G(s)$  and  $H(s)$  are obtained, it is more comfortable to work in the graph of Nyquist with the gain of open loop  $G(s)H(s)$  and not with the characteristic equation  $F(s)$ . For such a reason, and as the variable «s» it is common to the graphs of both, one will work then on the first one and not the second. In other words, as

$$G(\omega)H(\omega) = F(\omega) - 1$$

the curve in the domain  $G(\omega)H(\omega)$  it will be observed if it contains or not the point «-1+j0». If exists the «uncertainty of Nyquist» as for that one doesn't know if it has some pole that is hiding the situation, to leave doubts it can be appealed to Ruth's technique. For it, with Nyquist can only have

the security of when a system is unstable, but not to have the security of when it is not it.  
 The graph of Nyquist is not only useful to know if a system is or not unstable —or rather to have the security that it is it—, but rather it is very useful for other design considerations, of calculation of over-peaks, of gain and phase margins, etc.

### Example

Be the transfer of open loop

$$G(s)H(s) = 1 / s^2 (s+1)$$

To trace the graph we can appeal to different technical. We will propose that that divides it in four tracts: lines I, II, III and IV. We use for it the following tables and approaches:

Line I ( $s = 0 + j\omega$ ,  $0^+ \leq \omega \leq \infty$ )

$IGHI$	$= 1 / \omega^2 (\omega^2 + 1)^{1/2}$	$\infty$	0,7	$10^{-3}$	0
$\phi_{GH}$	$= -\pi - \arctan \omega/1$	$-\pi$	$-1,25\pi$	$-1,47\pi$	$-1,5\pi$
$\omega$		0	1	10	$\infty$

Line II ( $s = \infty e^{j\theta}$ )

$$GH = 1 / (\infty e^{j\theta})^2 (\infty e^{j\theta} + 1) = 0 e^{-j3\theta}$$

That is to say that when rotating «s» half turn with radio infinite in address of clock, GH makes it with radio null three half turn in address reverse.

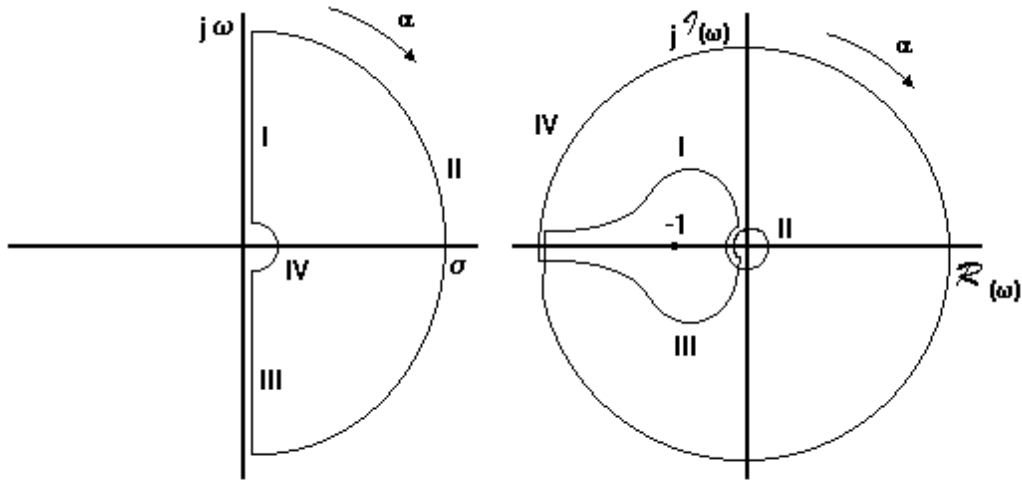
Line III ( $s = 0 - j\omega$ ,  $-\infty \leq -\omega \leq 0^-$ )

$IGHI$	$= 1 / \omega^2 (\omega^2 + 1)^{1/2}$	0	$10^{-3}$	0,7	$\infty$
$\phi_{GH}$	$= -\pi - \arctan (-\omega/1)$	$-0,5\pi$	$-0,53\pi$	$-0,75\pi$	$-\pi$
$\omega$		$\infty$	10	1	0

Line IV ( $s = 0 e^{j\theta}$ )

$$GH = 1 / (0 e^{j\theta})^2 (0 e^{j\theta} + 1) = \infty e^{-j2\theta}$$

That is to say that when rotating «s» half turn with null radio in reverse address, GH makes it with radio infinite two half turn in clock address.



It is observed in this example that has been given two turns containing to the point «-1+j0», indicating this that no matter that GH is stable since it has its poles in «0» and in «-1» —none in the right—, when closing the loop the Glc it will be unstable since there will be, at least, two poles of this —due to twice of confinement— in right semiplane.

We can verify this case with the algebraic following

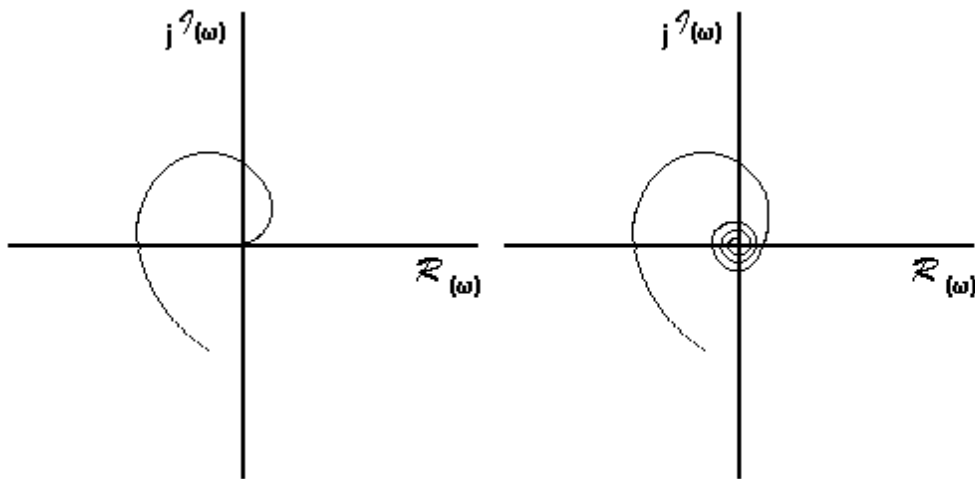
$$\begin{aligned}
 P_{Glc} &= P_{GH} + 1 = s^2 (s+1) + 1 = (s+a) (s+b) (s+c) \\
 s^2 (s+1) + 1 &= s^3 + s^2 + s.0 + 1 \\
 (s+a) (s+b) (s+c) &= s^3 + s^2 (a+b+c) + s (ab+bc+ac) + abc \\
 (a+b+c) &= 1 \\
 (ab+bc+ac) &= 0 \rightarrow \text{some should be negative} \\
 abc &= 1
 \end{aligned}$$

### Simplification for inertial simple systems

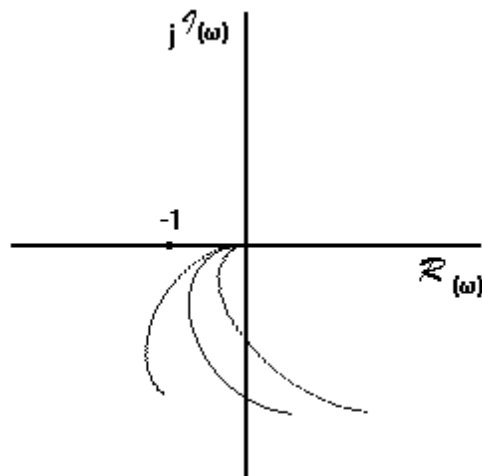
When the transfers G and H are simple, that is to say, when the loop systems Glc responds to inertial simple servomechanism —f.ex.: types 0 and 1—, then it is enough the analysis only of the first line of the graph of Nyquist.

They can in this to be observed different other aspects of interest.

The first is that they will only be kept in mind the dominant poles, and the other ones don't affect practically, like it is presented in the figure.



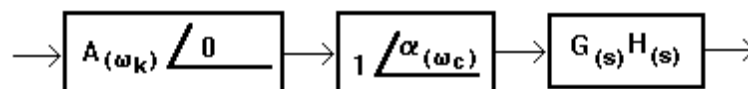
In second place that if the open loop is of not more than two poles, it will never be unstable to be the curve far from the critical point «-1+j0».

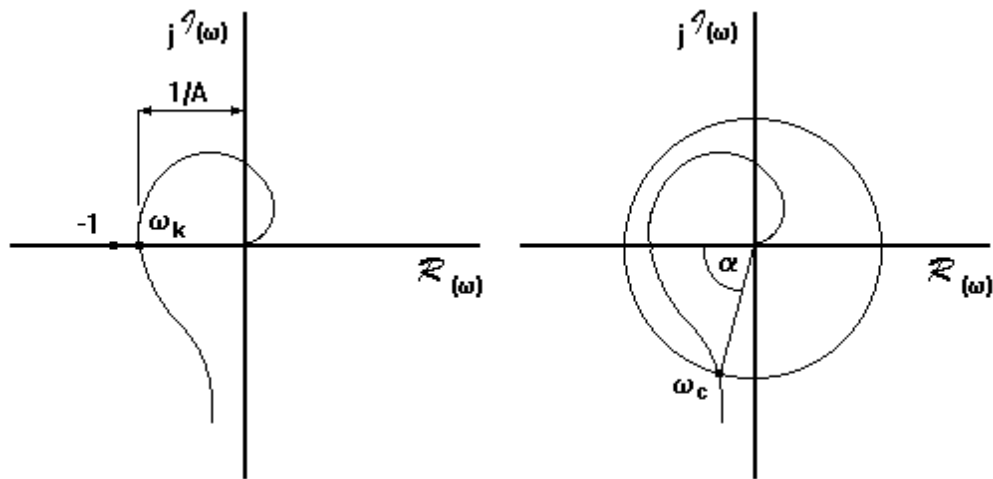


In third place that this first line is the one that corresponds to the graphs of Bode —not studied here.

### Gain and phase margins

The gain margins «A» and of phase « $\alpha$ » they are illustrative factors of the amplitude that we can increase the gain of the open loop GH and to displace its phase, for a given critical frequency « $\omega_k$  y  $\omega_c$ » respectively, without unstability takes place —in closed loop Glc. These are defined for convention in the following way





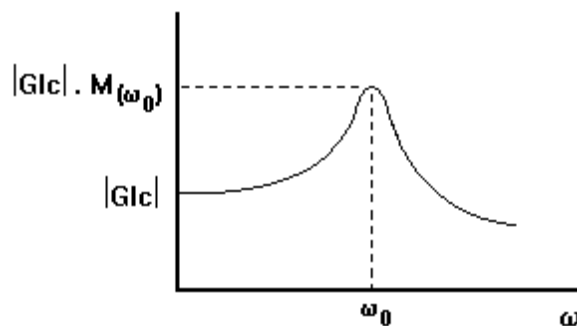
The way to calculate analytically one and another is the following

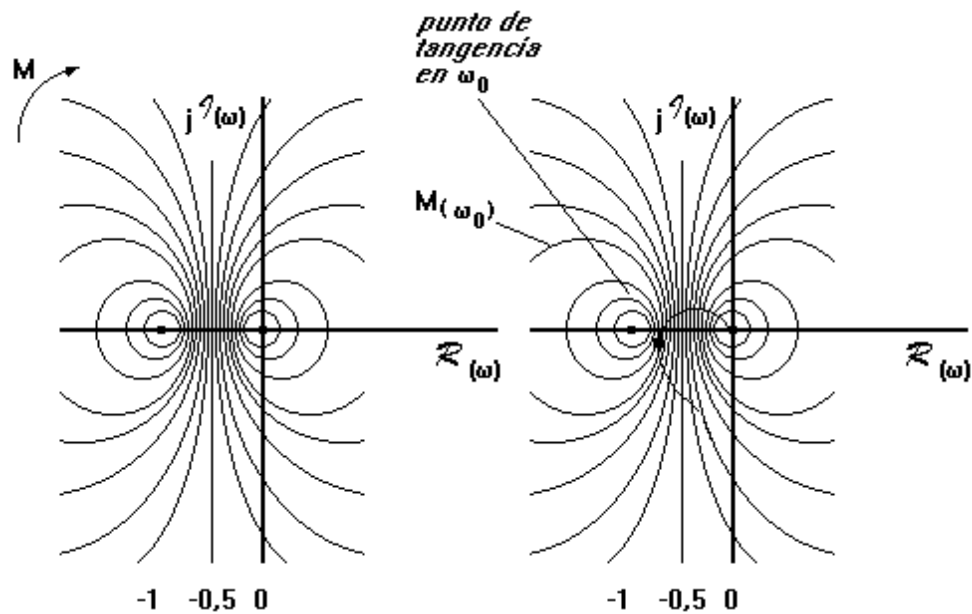
Calculation of A)	With	$G_{(\omega)}H_{(\omega)} =  GHI_{(\omega)}  \cdot e^{j\phi(\omega)} = \dots$
	we propose	$\phi_{(\omega_k)} = -\pi$
	we find	$\omega_k = \dots$
	and with it	$A_{(\omega_k)} = 1 /  GHI_{(\omega_k)}  = \dots$

Calculation of $\alpha$ )	With	$G_{(\omega)}H_{(\omega)} =  GHI_{(\omega)}  \cdot e^{j\phi(\omega)} = \dots$
	we propose	$ GHI_{(\omega_c)}  = 1$
	we find	$\omega_c = \dots$
	and with it	$\alpha_{(\omega_c)} = \pi -  \phi_{(\omega_c)}  = \dots$

### Calculation of the over-peak

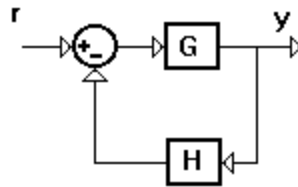
In the bibliography that is given like reference is shown universal abacus that allow to find the percentage of the module of the closed loop graphically  $|G|_c$ , according to the approach that has the curve of Nyquist with the critical point «-1+j0».





### Systems MI-MO

When they are many the inputs and the outputs to the system, the stability of each individual transfer will be analyzed, or its group in matrix given by the roots of the polynomial of the total characteristic equation



$$\text{Det}(sI - Af) = \text{Det } G(s) \cdot \text{Det } H(s) \cdot \text{Det} [I + G(s)H(s)] = (s+s_{f1})(s+s_{f2}) \dots = 0$$

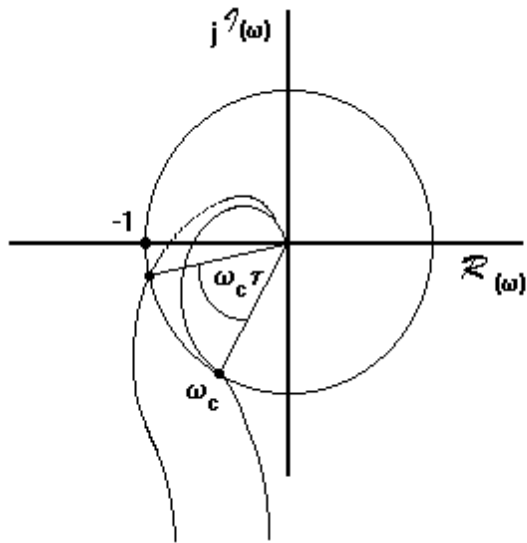
### Systems with delay

When we have a delay of « $\tau$ »seconds in an open loop, this effect should be considered in the total transfer of loop like a transfer  $e^{-s\tau}$  in cascade with the open loop GH, that is to say

$$G(s)H(s)]_{\text{efectivo}} = G(s)H(s)e^{-s\tau}$$

and the graph of Nyquist should be corrected in this value, modifying the cartesian axes in an angle  $\omega_c\tau$ , where  $\omega_c$  is denominated critical frequency to be the next to the critical point..





## DISCREET SYSTEMS

We know the correspondence among the variables of Laplace and «z» for a sampling of rhythm «T»

$$z = e^{sT}$$

or

$$z = e^{sT} = e^{(\sigma + j\omega)T} = e^{\sigma T} \cdot e^{j\omega T}$$

where we observe the correspondence

$$\begin{aligned} \sigma &\leftrightarrow |z| = e^{\sigma T} \\ \omega &\leftrightarrow \text{angle of } z = \omega T \end{aligned}$$

that is to say, that given the only condition of stability of the poles « $-s_i = -(\sigma_i + j\omega_i)$ » of a continuous closed loop  $Glc(s)$

$$Glc(s) = Z_{Glc} / P_{Glc} = Z_{Glc} / [(s+s_1)(s+s_2)(s+s_i) \dots]$$

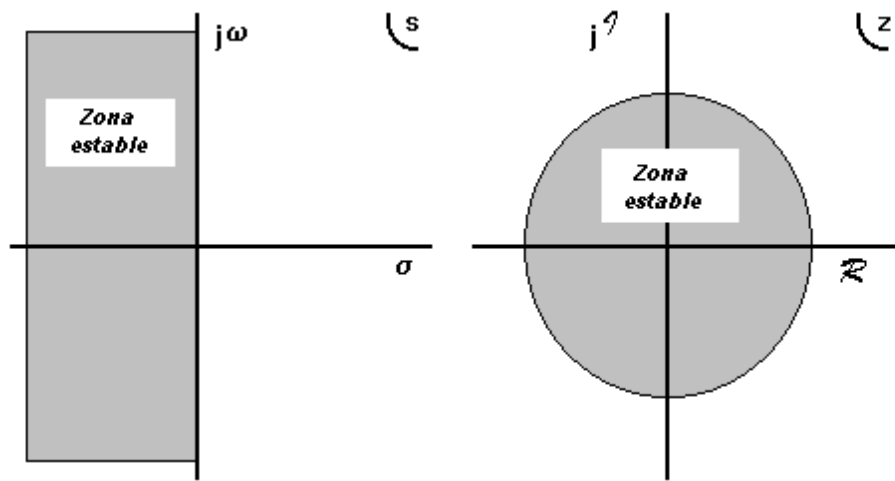
$$\sigma_i > 0$$

it is for the discrete system  $Glc(z)$  that the only condition of their poles « $-z_i$ » it is

$$Glc(z) = Z_{Glc} / P_{Glc} = Z_{Glc} / [(z+z_1)(z+z_2)(z+z_i) \dots]$$

$$-z_i = e^{(-s_i)T} = e^{-(\sigma_i + j\omega_i)T} = e^{-\sigma_i T} \cdot e^{-j\omega_i T}$$

$$|z| = e^{-\sigma T} < 1$$



## Chap. 39 Feedback of the State in a System

### SYSTEMS «SI-MO»

#### OPEN LOOP (Plant Gp)

#### CLOSED LOOP (Glc)

Example of a plant Gp with feedback

Design starting from other poles in Glc

### SYSTEMS «MI-MO»

#### OPEN LOOP (Plant Gp)

#### CLOSED LOOP (Glc)

Design starting from other poles in Glc

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### SYSTEMS «SI-MO»

#### OPEN LOOP (Plant Gp)

We will have the following generalities

$$\begin{cases} \dot{x} = A x + b u \\ y = C x \end{cases}$$

$$G_p = y / u = G_{po} / [(s + s_1)(s + s_2) \dots]$$

$$x / u = b \cdot [G / (1 + G.H)] = b \cdot \{(1/s) / [1 + (A \cdot 1/s)]\} = b / (s + A)$$

A      matrix of speed of the plant Gp [1/seg]

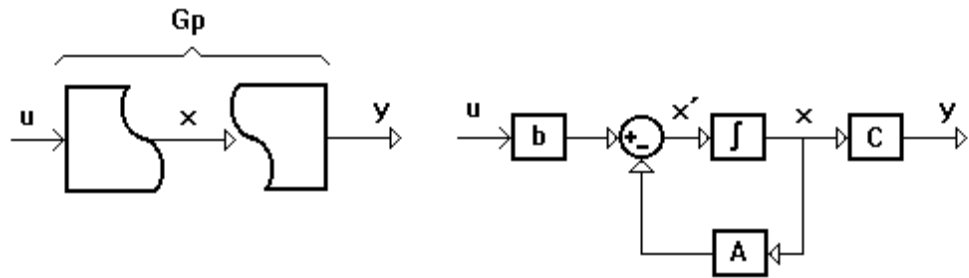
b      vector of control of the input «u» of the plant Gp

C      output matrix «y» of the plant Gp

$$A = \begin{bmatrix} -s_1 & a_{12} \\ a_{21} & -s_2 \end{bmatrix}$$

$$\begin{bmatrix} -s_1 & 0 \end{bmatrix}$$

$$A^* = T^{-1} \cdot A \cdot T = \begin{bmatrix} | & \\ 0 & -s_2 \end{bmatrix}$$



### CLOSED LOOP (Glc)

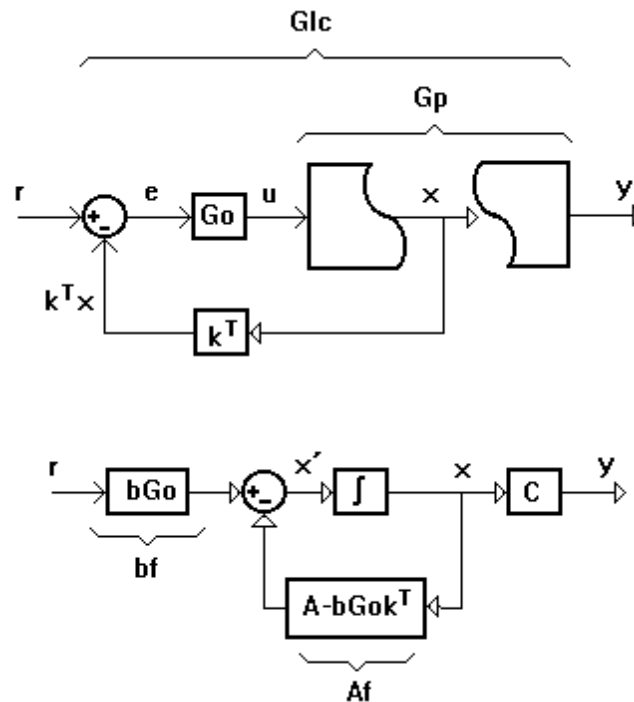
- We will have the following generalities

$$\begin{cases} \dot{x}' = (A - b G_o k^T) x + b G_o u & = A_f x + b_f r \\ y = C x & = C_f x \end{cases}$$

$$G_{lc} = y / r = G_{lco} / [(s + s_1) (s + s_2) \dots]$$

$$A_f = \begin{bmatrix} -sf_1 & a_{f12} \\ a_{f21} & -sf_2 \end{bmatrix}$$

$$A_f^* = T^{-1} \cdot A_f \cdot T = \begin{bmatrix} -sf_1 & 0 \\ 0 & -sf_2 \end{bmatrix}$$



### Example of a plant $G_p$ with feedback

Be the data

$$G = 600 / (1 + 150 s)$$

$$H = 0,015 / (1 + 70 s)$$

$$G_p = G / (1 + GH) = K_1 / [(s + p_1)(s + p_2)]$$

and it is wanted to increase the speed of the plant in approximately 30%; that is to say, in taking to the constant of dominant time —inverse of the dominant pole— to the value

$$\tau_{f1} = 1 / s_{f1} = 0,7 \cdot \tau_1 = 0,7 \cdot 150 = 105 [\text{seg}]$$

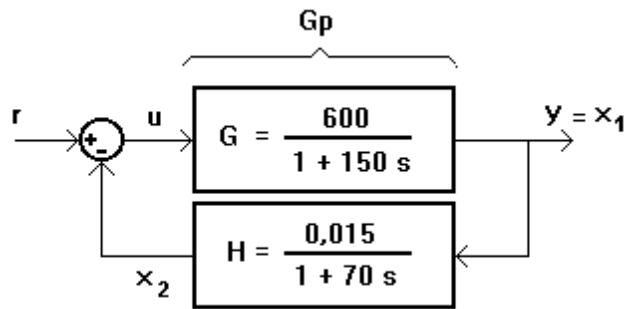
$$G_{lc} = G_{lc0} / [(s + s_{f1})(s + s_{f2})] = K_2 / [(1 + 105 s)(1 + 70 s)]$$

where we should keep in mind that « $G_p = G / (1 + GH)$ » it corresponds to the feedback of the output « $y$ », and « $G_{lc}$ » to that of the state « $x$ ».

Firstly we can find the equations of the system if we choose, for example

$$y = x_1$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



then

$$\begin{aligned}
 x_1 &= 600 (u - x_2) / (1 + 150 s) \\
 \Rightarrow x_1 + x_1 150 s &= 600 u - 600 x_2 \\
 x_1 + x_1' 150 &= 600 u - 600 x_2 \\
 x_1' &= (-1/150) x_1 + (-4) x_2 + 4 u \\
 x_2 &= 0,015 x_1 / (1 + 70 s) \\
 \Rightarrow x_2 + x_2 70 s &= 0,015 x_1 \\
 x_2 + x_2' 70 &= 0,015 x_1 \\
 x_2' &= (0,015/70) x_1 + (-1/70) x_2
 \end{aligned}$$

of where

$$x' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1/150 & -4 \\ 0,015/70 & -1/70 \end{bmatrix} \cdot x + \begin{bmatrix} 4 \\ 0 \end{bmatrix} \cdot u$$

$$y = x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot x$$

Subsequently we find the coefficients of the matrix of speed of the plant without and with feedback

$$A = \begin{bmatrix} -s_1 & a_{12} \\ a_{21} & -s_2 \end{bmatrix} = \begin{bmatrix} -1/150 & -4 \\ 0,015/70 & -1/70 \end{bmatrix}$$

$$A_f = \begin{bmatrix} -sf_1 & a_{f12} \\ a_{f21} & -sf_2 \end{bmatrix} = \begin{bmatrix} -1/105 & a_{f12} \\ a_{f21} & -1/70 \end{bmatrix}$$

$$A_f^* = \begin{bmatrix} -sf_1 & 0 \\ 0 & -sf_2 \end{bmatrix} = \begin{bmatrix} -1/105 & 0 \\ 0 & -1/70 \end{bmatrix}$$

$$\begin{aligned} \text{Det}(sI - A) &= s^2 + a_1s + a_2 = 0 \quad \Rightarrow \quad a_1 = 0,0209, a_2 = 0,00095 \\ \text{Det}(sI - Af^*) &= \text{Det}(sI - Af) = s^2 + a_{f1}s + a_{f2} = 0 \quad \Rightarrow \quad a_{f1} = 0,0238, a_{f2} = 0,000136 \end{aligned}$$

to calculate the feedback vector k

$$k^{*T} = [(a_{f2} - a_2)(a_{f1} - a_1)] = [-0,000814 \quad 0,0029]$$

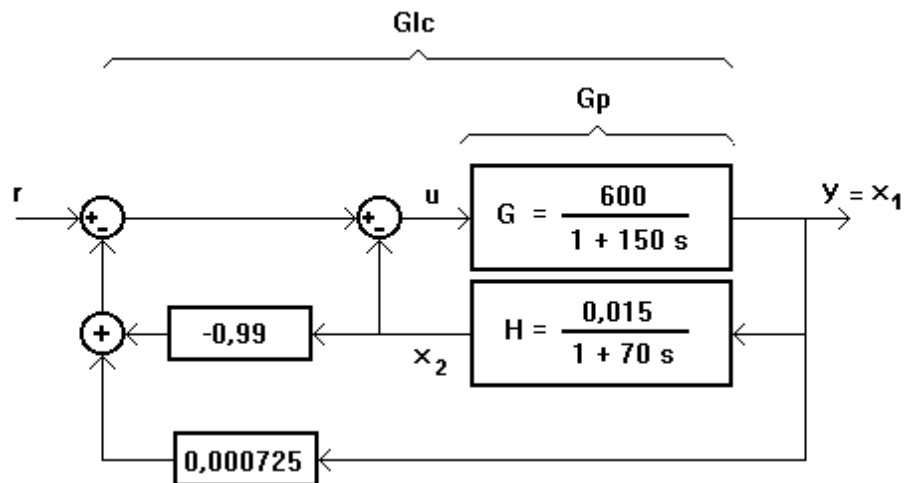
$$q_n = q_2 = b = [4 \quad 0]^T$$

$$q_{n-i} = q_1 = A \cdot q_{n-i+1} + a_i \cdot q_n = A \cdot q_2 + a_1 \cdot q_2 = [-0,057 \quad -0,000857]^T$$

$$Q = [q_1 \quad q_2 \quad \dots \quad q_n] = [q_1 \quad q_2] = \begin{bmatrix} 0,057 & 4 \\ 0,000857 & 0 \end{bmatrix}$$

$$Q^{-1} = (\text{Adj } Q)^T / \text{Det } Q = \begin{bmatrix} 0 & -4 \\ -0,000857 & 0,057 \end{bmatrix} / (-0,003428) = \begin{bmatrix} 0 & 1166 \\ 0,25 & -16,57 \end{bmatrix}$$

$$k^T = k^{*T} \cdot Q^{-1} = [k_1 \quad k_2] = [0,000725 \quad -0,99]$$



### Design starting from other poles in Glc

It is presupposed to have a plant Gp of three poles (or auto-values)  $[s_1 \ s_2 \ s_3]^T$ . Is then the data

$$A = \dots, b = \dots$$

$$s_1 = \dots, s_2 = \dots, s_3 = \dots$$

The controlability of the plant Gp is verified.

$$U = [b \ A.b \ \dots \ A^{n-1}.b] = [b \ A.b \ A^2.b] = \dots \quad \text{matrix of controlability}$$

Rango U = ...

We propose the poles of the Glc

$$s_{f1} = \dots, s_{f2} = \dots, s_{f3} = \dots$$

We find the coefficients of the Gp

$$\text{Det}(sI - A) = (s + s_1)(s + s_2)(s + s_3) = s^3 + a_1s^2 + a_2s + a_3 = 0$$

$$a_1 = \dots, a_2 = \dots, a_3 = \dots$$

We find the coefficients of the Glc\*

$$\text{Det}(sI - Af^*) = \text{Det}(sI - Af) = (s + s_{f1})(s + s_{f2})(s + s_{f3}) = s^3 + a_{f1}s^2 + a_{f2}s + a_{f3} = 0$$

$$a_{f1} = \dots, a_{f2} = \dots, a_{f3} = \dots$$

We find the vectorial k transformed as k\*

$$k^{*T} = [(a_{f3} - a_3)(a_{f2} - a_2)(a_{f1} - a_1)]^T = \dots$$

We determine the transformation matrix Q

$$q_n = b = \dots$$

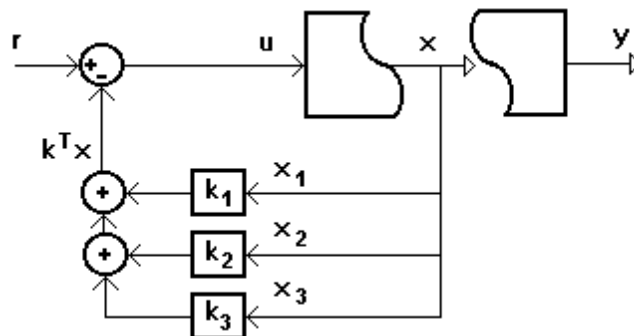
$$q_{n-i} = A \cdot q_{n-i+1} + a_i \cdot q_n = \dots$$

$$Q = [q_1 \ q_2 \ \dots \ q_n] = \dots$$

$$Q^{-1} = (\text{Adj } Q)^T / \text{Det } Q = \dots$$

Finally we calculate the feedback vector k

$$k^T = k^{*T} \cdot Q^{-1} = [k_1 \ k_2 \ k_3] = \dots$$





## SYSTEMS «MI-MO»

### OPEN LOOP (Plant Gp)

We have the following characteristic

$$\begin{cases} \dot{x} = A x + b u \\ y = C x \end{cases}$$

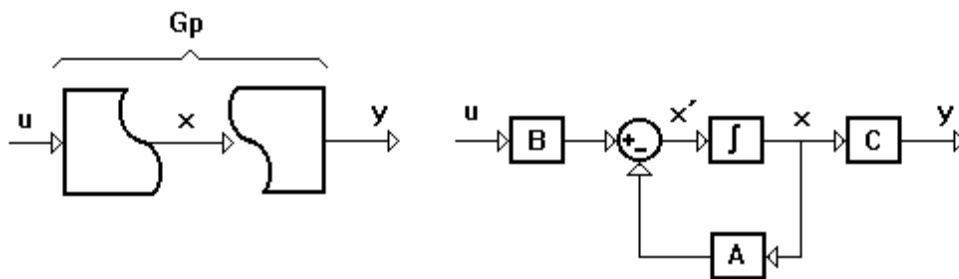
$$G_p = y / u = G_{po} / [(s + s_1) (s + s_2) \dots]$$

$$x / u = b \cdot [G / (1 + G.H)] = b \cdot \{(1/s) / [1 + (A \cdot 1/s)]\} = b / (s + A)$$

- A matrix of speed of the plant Gp [1/seg]  
B matrix of control of the input «u» of the plant Gp  
C output matrix «y» of the plant Gp

$$A = \begin{bmatrix} -s_1 & a_{12} \\ a_{21} & -s_2 \end{bmatrix}$$

$$A^* = \begin{bmatrix} -s_1 & 0 \\ 0 & -s_2 \end{bmatrix} = T^{-1} \cdot A \cdot T$$



### CLOSED LOOP (Glc)

We have the following characteristic

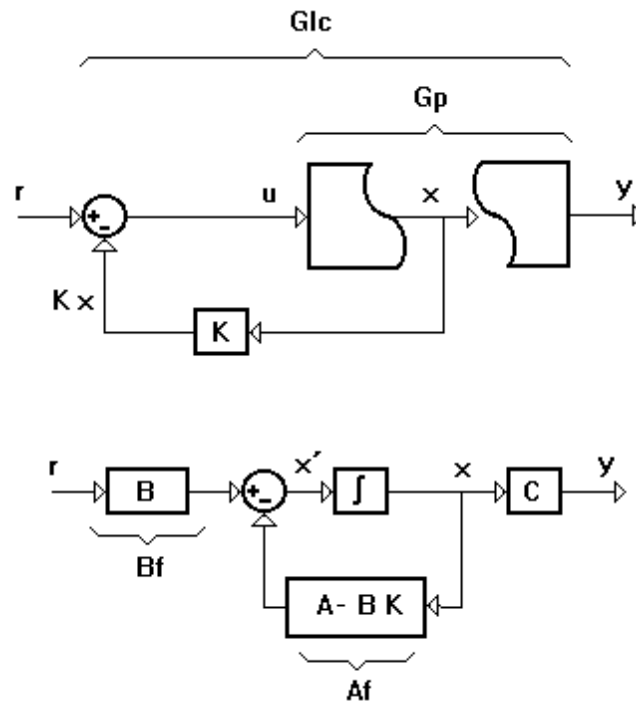
$$\begin{cases} \dot{x} = (A - BK) x + B u = A_f x + B_f r \end{cases}$$

$$y = C x \quad = \quad C_f x$$

$$G_{lc} = y / r = G_{lc0} / [(s + sf_1)(s + sf_2) \dots]$$

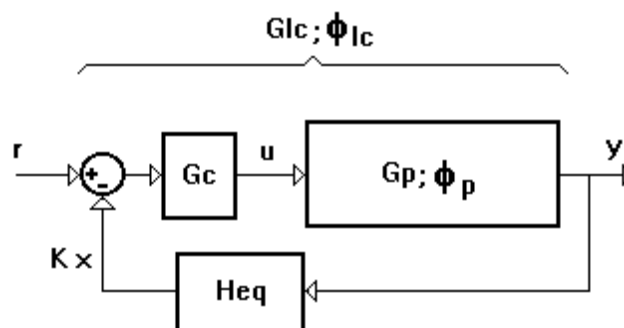
$$A_f = \begin{bmatrix} -sf_1 & a_{f12} \\ a_{f21} & -sf_2 \end{bmatrix}$$

$$A_f^* = \begin{bmatrix} -sf_1 & 0 \\ 0 & -sf_2 \end{bmatrix}$$



We can also think the feedback with an analogy of «equivalent H»

$$G_{lc} = G_c G_p / (1 + G_c G_p H_{eq})$$



and this way, like with conditions null initials and in the transformed field they are

$$G_p = y/u = C \cdot x/u = C \cdot (\Psi_p \cdot B u)/u = C \cdot \Psi_p \cdot B$$

$$G_{lc} = y/r = C \cdot x/r = C \cdot (G_c \cdot \Psi_{lc} \cdot B r)/r = C \cdot G_c \cdot \Psi_{lc} \cdot B$$

It is

$$H_{eq} = K x/y = K x/[C \cdot x] = K (\Psi_p \cdot B u)/[C \cdot (\Psi_p \cdot B u)] = K \Psi_p \cdot B / C \Psi_p \cdot B$$

### Design starting from other poles in Glc

It is presupposed to have a plant  $G_p$  of two poles (or auto-values)  $[s_1 \ s_2]^T$ . Is then the data

$$A = \dots, B = \dots, C = \dots$$

$$s_1 = \dots, s_2 = \dots$$

The controllability of the plant  $G_p$  is verified

$$U = [B \ A \cdot B \ \dots \ A^{n-1} \cdot B] = [B \ A \cdot B] = \dots$$

$$\text{Rango } U = \dots$$

We propose the poles of the Glc

$$sf_1 = \dots, sf_2 = \dots$$

We determine the auto-vector «v» of the matrix A

$$0 = (A - s_1 I) \cdot v_1 = \begin{pmatrix} A - \begin{bmatrix} s_1 & 0 \\ 0 & s_1 \end{bmatrix} \end{pmatrix} \cdot v_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot v_1$$

$$0 = (A - s_2 I) \cdot v_2 = \begin{pmatrix} A - \begin{bmatrix} s_2 & 0 \\ 0 & s_2 \end{bmatrix} \end{pmatrix} \cdot v_2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot v_2$$

and with it

$$a_{11} \cdot v_{11} + a_{12} \cdot v_{12} = 0$$

$$a_{21} \cdot v_{11} + a_{22} \cdot v_{12} = 0 \quad \rightarrow \quad v_{11} = \dots \ v_{12} = \dots$$

$$b_{11} \cdot v_{21} + b_{12} \cdot v_{22} = 0$$

$$b_{21} \cdot v_{21} + b_{22} \cdot v_{22} = 0 \quad \rightarrow \quad v_{21} = \dots \quad v_{22} = \dots$$

We arm the modal matrix T and their inverse T<sup>-1</sup>

$$T = [v_1 \ v_2] = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \dots$$

$$T^{-1} = (\text{Adj } T)^T / (\text{Det } T) = \dots$$

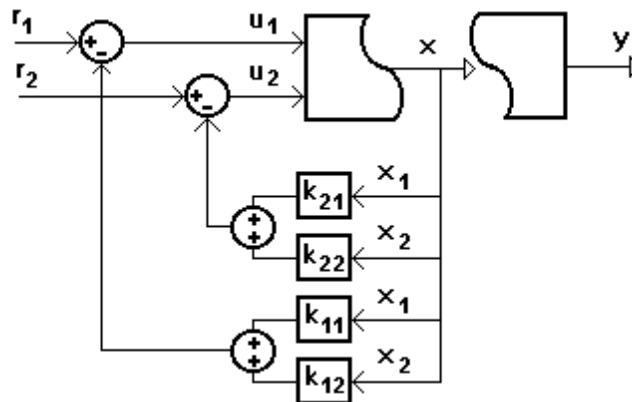
and we calculate

$$A^* = \begin{bmatrix} -s_1 & 0 \\ 0 & -s_2 \end{bmatrix} = \dots$$

$$Af^* = \begin{bmatrix} -sf_1 & 0 \\ 0 & -sf_2 \end{bmatrix} = \dots$$

$$B^{-1} = (\text{Adj } B) / (\text{Det } B) = \dots$$

$$K = B^{-1} \cdot T \cdot (A^* - Af^*) \cdot T^{-1} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \dots$$



## Chap. 40 Estimate of the State in a System

### SYSTEMS «SI-SO» (continuous)

#### ESTIMATORS OF ORDER «n»

##### Estimator Ge fo «d = 0»

##### Design estimator Ge of «order n» and «d = 0»

#### ESTIMATORS OF ORDER «n-1»

##### Design estimator Ge of «order n-1» and «d = 0»

### SYSTEMS «MI-MO» (discreet)

#### EQUATIONS OF STATE

##### Design

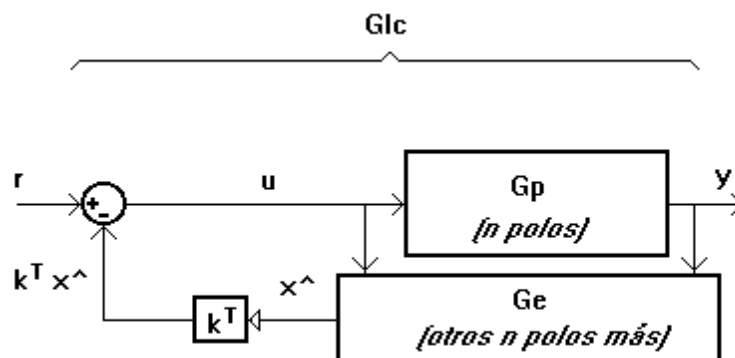
### SYSTEMS «SI-SO» (continuous)

#### ESTIMATORS OF ORDER «n»

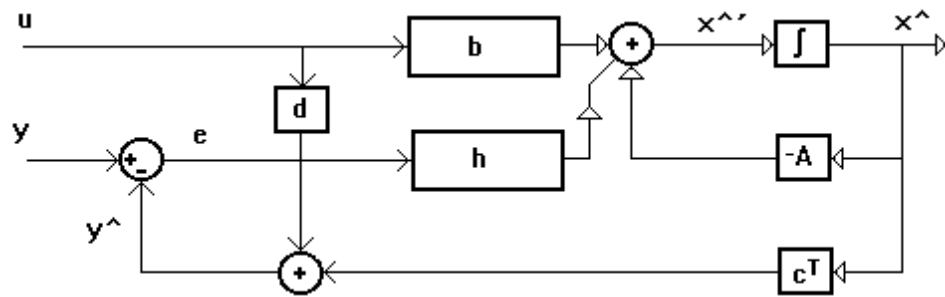
We will work on a plant Gp in the way

$$\dot{x} = A x + b u$$

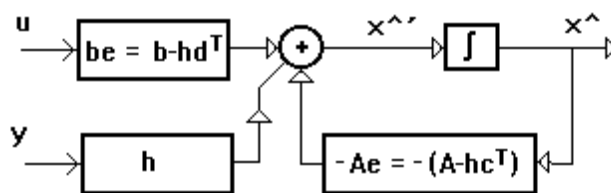
$$y = c^T x + d u$$



with an estimator implemented  $G_e$  in the following way



of where it is deduced applying overlapping the following simplification



$$\begin{aligned} \dot{x}^{\wedge} &= A e x^{\wedge} + h y + b e u \\ y &= c^T x^{\wedge} + d u \end{aligned}$$

$$A e = A - h c^T$$

$$b e = b - h d^T$$

$$A f = A - (b - h d^T) k^T$$

$$A = \begin{bmatrix} -s_1 & a_{12} \\ a_{21} & -s_2 \end{bmatrix}$$

$$A e = \begin{bmatrix} -s e_1 & a_{e12} \\ a_{e21} & -s e_2 \end{bmatrix}$$

$$\text{Det}(s I - A) = (s + s_1)(s + s_2) = s^2 + a_1 s + a_2 = 0$$

### Estimator Ge fo «d = 0»

Be

$$\begin{aligned} \dot{x}^{\wedge} &= A e x^{\wedge} + h y + b u \\ y &= c^T x^{\wedge} \end{aligned}$$

$$A e = A - h c^T$$

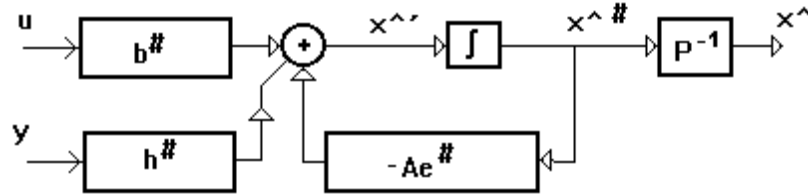
$$b_e = b$$

$$A_f = A - b k^T$$

$$y = x_1 \Rightarrow c^T = [1 \ 0 \ \dots \ 0]$$

$$A^T = \begin{bmatrix} -s_1 & a_{21} \\ a_{12} & -s_2 \end{bmatrix}$$

The lineal transformation P will be used



$$x = P^{-1} \cdot x^{\#}$$

$$x^{\wedge} = P^{-1} \cdot x^{\wedge \#}$$

$$x^{\wedge \#} = A^{\#} x^{\wedge \#} + h^{\#} y + b^{\#} u$$

$$h^T = P^{-1} \cdot h^{\#} = [h_1 \ h_2]^T$$

$$O = [c^T \ A^T c^T \ (A^T)^2 c^T \ \dots \ (A^T)^{n-1} c^T] = [c^T \ A^T c^T] = \begin{bmatrix} o_{11} & o_{12} \\ o_{21} & o_{22} \end{bmatrix}$$

$$O^{-1} = (\text{Adj } O)^T / (\text{Det } O)$$

$$O^{\#} = [c^{\# T} \ A^{\# T} c^{\# T} \ (A^{\# T})^2 c^{\# T} \ \dots \ (A^{\# T})^{n-1} c^{\# T}] = [c^{\# T} \ A^{\# T} c^{\# T}] = \begin{bmatrix} o^{\#}_{11} & o^{\#}_{12} \\ o^{\#}_{21} & o^{\#}_{22} \end{bmatrix}$$

$$P^{-1} = (O^{-1})^T \cdot O^{\# T} = (O^{\#} \cdot O^{-1})^T = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$

$$P = (\text{Adj } P^{-1}) / (\text{Det } P^{-1}) = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$A^{\#} = P \cdot A \cdot P^{-1} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}$$

$$-a_1]$$

$$b^{\#} = P \cdot b \cdot P^{-1} = [\dots b_2^{\#} b_1^{\#}]^T = [b_2^{\#} b_1^{\#}]^T$$

$$c^{\#T} = [0 \ 0 \ \dots \ 1]^T = [0 \ 1]^T$$

$$h^{\#T} = [h_1^{\#} \ h_2^{\#} \ \dots]^T = [h_1^{\#} \ h_2^{\#}]^T$$

$$A^{\#}e = A^{\#} - h^{\#} c^{\#T} = \begin{bmatrix} 0 & 0 & -(a_1+h_1^{\#}) \\ 1 & 0 & -(a_2+h_2^{\#}) \\ 0 & 1 & -(a_3+h_3^{\#}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a^{\#}e_3 \\ 1 & 0 & -a^{\#}e_2 \\ 0 & 1 & -a^{\#}e_1 \end{bmatrix}$$

$$\text{Det}(sI - A^{\#}e) = (s + e_1)(s + e_2) = s^2 + a_{e1}s + a_{e2} = 0$$

and it is observed finally that

$$h^{\#T} = [(a_{e2} - a_2) \ (a_{e1} - a_1)]^T$$

### Design estimator Ge of «order n» and «d = 0»

We propose the poles of the Glc and  $k^T$  is calculated to —go to the chapter of *feedback of the state*. Now, so that it is effective the feedback, we proceed to calculate the estimator Ge. We should for it to have the following data

$A = \dots$	$c = \dots$	Dominant $\{s_1; s_2\} = \dots$
$b = \dots$		Dominant $\{sf_1; sf_2\} = \dots$

We will suppose a plant Gp of second order ( $n = 2$ ).

We choose the poles of such very speedy Ge that don't affect those of Gp, that maintain the dominant one in Glc, and that they allow to continue to the real state «x» —minimum error—, that is to say

$$\text{Dominant } \{s_1; s_2\} \ll \text{Dominant } \{e_1; e_1\} \gg \text{Dominant } \{sf_1; sf_2\}$$

$$e_1 = \dots, e_2 = \dots$$

We calculate the coefficients of the Gp

$$\text{Det}(sI - A) = (s + s_1)(s + s_2)(s + s_3) = s^3 + a_1s^2 + a_2s + a_3 = 0$$

$$a_1 = \dots, a_2 = \dots$$

and also the coefficients of the estimator Ge

$$\text{Det}(sI - Ae) = \text{Det}(sI - A^{\#}e) = (s + e_1)(s + e_2) = s^2 + a_{e1}s + a_{e2} = 0$$

$$a_{e1} = \dots, a_{e2} = \dots$$



and we will be able to with it to determine

$$h^{\#T} = [(a_{e2} - a_2) (a_{e1} - a_1)]^T = [h^{\#}_1 \ h^{\#}_2]^T = \dots$$

We find the observability O

$$A^T c^T = \begin{bmatrix} -s_1 & a_{21} \\ a_{12} & -s_2 \end{bmatrix} \cdot [1 \ 0]^T = \dots$$

$$O = [c^T \ A^T c^T \ (A^T)^2 c^T \ \dots \ (A^T)^{n-1} c^T] = [c^T \ A^T c^T] = \begin{bmatrix} o_{11} & o_{12} \\ o_{21} & o_{22} \end{bmatrix} = \dots$$

$$O^{-1} = (\text{Adj } O)^T / (\text{Det } O) = \dots$$

The womb *canonical observability* of the original plant Gp is calculated

$$A^{\#} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 1 & -a_1 \end{bmatrix} = \dots$$

$$c^{\#T} = [0 \ 0 \ \dots \ 1]^T = [0 \ 1]^T$$

$$O^{\#} = [c^{\#T} \ A^{\#T} c^{\#T} \ (A^{\#T})^2 c^{\#T} \ \dots \ (A^{\#T})^{n-1} c^{\#T}] = [c^{\#T} \ A^{\#T} c^{\#T}] = \begin{bmatrix} o^{\#}_{11} & o^{\#}_{12} \\ o^{\#}_{21} & o^{\#}_{22} \end{bmatrix} = \dots$$

$$A_e^{\#} = \begin{bmatrix} 0 & 0 & -(a_1 + h_1^{\#}) \\ 1 & 0 & -(a_2 + h_2^{\#}) \\ 0 & 1 & -(a_3 + h_3^{\#}) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_{e3}^{\#} \\ 1 & 0 & -a_{e2}^{\#} \\ 0 & 1 & -a_{e1}^{\#} \end{bmatrix} = \dots$$

and with it our matrix of lineal transformation P<sup>-1</sup>

$$P^{-1} = (O^{-1})^T \cdot O^{\#T} = (O^{\#} \cdot O^{-1})^T = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = \dots$$

$$P = \dots$$

$$b^{\#} = \dots$$



$$A^{\#} = P . A . P^{-1} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 0 & 1 & -a_1 \\ 1 \end{bmatrix}$$

$$b^{\#} = P . b = [b_n^{\#} \dots b_2^{\#} b_1^{\#}]^T = [b_2^{\#} b_1^{\#}]^T$$

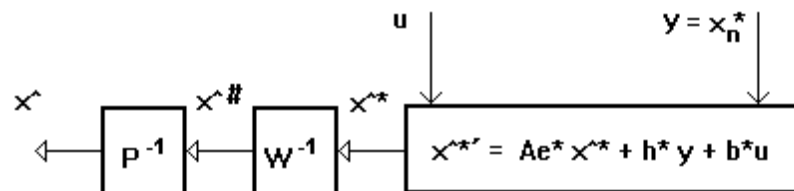
$$c^{\#T} = [0 \ 0 \ \dots \ 1]^T = [0 \ 1]^T$$

we modify again with the following transformation «W»

$$W = \begin{bmatrix} 1 & 0 & -a_{en-1} \\ 0 & 1 & -a_{en-2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$W^{-1} = \begin{bmatrix} 1 & 0 & a_{en-1} \\ 0 & 1 & a_{en-2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$x^* = W . x^{\#}$$



of where

$$\begin{aligned} x^{*'} &= A^* x^* + b^* u \\ y &= c^{*T} x^* \end{aligned}$$

$$A^* = W . A^{\#} . W^{-1} = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_{en-1} & [a_{en-1}(a_1-a_{e1}) & +(0-a_n)] \\ 1 & 0 & \dots & 0 & -a_{en-2} & [a_{en-2}(a_1-a_{e1}) & +(a_{en-1}-a_{n-1})] \\ 0 & 1 & \dots & 0 & -a_{en-3} & [a_{en-3}(a_1-a_{e1}) & +(a_{en-2}-a_{n-2})] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{e1} & [a_{e1} (a_1-a_{e1}) & +(a_{e2}-a_2)] \\ 0 & 0 & \dots & 0 & 1 & [0 & +(a_{e1}-a_1)] \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* \\ 0 \dots 1 & a_{e1}-a_1 \end{bmatrix}$$

$$\begin{bmatrix} (b_n^{\#} - b_1^{\#}) a_{en-1} \end{bmatrix}$$

$$b^* = W \cdot b \cdot W^{-1} = [b_n^* \dots b_2^* b_1^*]^T = \begin{bmatrix} \dots & | & (b_{n-1}^{\#} - b_1^{\#}) a_{en-2} & | \\ & | & (b_{n-2}^{\#} - b_1^{\#}) a_{en-3} & | \\ & | & (b_2^{\#} - b_1^{\#}) a_{e1} & | \\ & | & (b_1^{\#} - 0) & | \end{bmatrix}$$

$$c^{*T} = [0 \ 0 \ \dots \ 1]^T$$

being the equations finally for «n-1» states

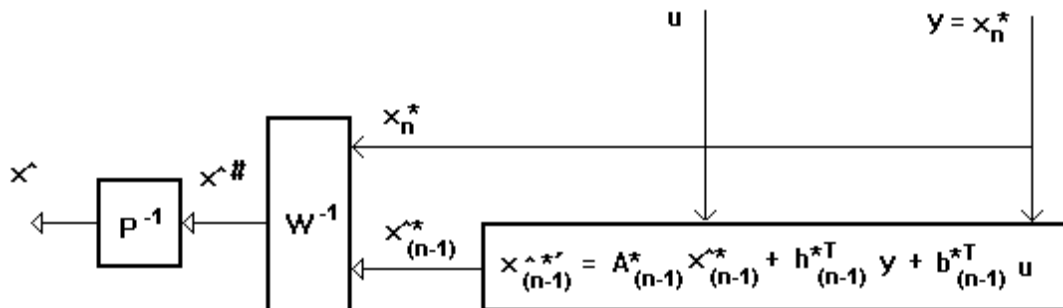
$$x_{(n-1)}^{\wedge^{**}} = A_{(n-1)}^* x_{(n-1)}^{\wedge^*} + h_{(n-1)}^{*T} y + b_{(n-1)}^{*T} u$$

con

$$\begin{aligned} A_{(n-1)}^* &= A_{11}^* \\ h_{(n-1)}^* &= A_{12}^* \\ b_{(n-1)}^* &= [b_n^* \dots b_2^*] \end{aligned}$$

and for all the states

$$x^{\wedge^{**}} = A^* x^{\wedge^*} + h^{*T} y + b^{*T} u$$



### Design estimator Ge of «order n-1» and «d = 0»

We propose the poles of the Glc and  $k^T$  is calculated to —go to the chapter of *feedback of the state*. Now, so that it is effective the feedback, we proceed to calculate the estimator Ge. We should for it to have the following data

$$\begin{aligned} A &= \dots & c &= \dots & \text{Dominant } \{s_1; s_2\} &= \dots \\ b &= \dots & & & \text{Dominant } \{sf_1; sf_2\} &= \dots \end{aligned}$$

We will suppose a plant Gp of second order ( $n = 2$ ).

We choose the poles of such very speedy Ge that don't affect those of Gp that maintain the dominant one in Glc, and that they allow to continue to the real state «x» —minimum error—, that is to say

Dominant  $\{s_1; s_2\} \ll \text{Dominant } \{e_1; e_1\} \gg \text{Dominant } \{sf_1; sf_2\}$

$$e_1 = \dots, e_2 = \dots$$

We calculate the coefficients of the Gp

$$\text{Det } (sI - A) = (s + s_1)(s + s_2)(s + s_3) = s^3 + a_1s^2 + a_2s + a_3 = 0$$

$$a_1 = \dots, a_2 = \dots$$

and also the coefficients of the estimator Ge

$$\text{Det } (sI - Ae) = \text{Det } (sI - Ae^\#) = (s + e_1)(s + e_2) = s^2 + a_{e1}s + a_{e2} = 0$$

$$a_{e1} = \dots, a_{e2} = \dots$$

We find the observability O

$$A^T c^T = \begin{bmatrix} -s_1 & a_{21} \\ a_{12} & -s_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \end{bmatrix}^T = \dots$$

$$O = \begin{bmatrix} c^T & A^T c^T & (A^T)^2 c^T & \dots & (A^T)^{n-1} c^T \end{bmatrix} = \begin{bmatrix} c^T & A^T c^T \end{bmatrix} = \begin{bmatrix} o_{11} & o_{12} \\ o_{21} & o_{22} \end{bmatrix} = \dots$$

$$O^{-1} = (\text{Adj } O)^T / (\text{Det } O) = \dots$$

We calculate the matrix *canonical observability* of the original plant Gp

$$A^\# = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 1 & -a_1 \end{bmatrix} = \dots$$

and with it

$$c^{\#T} = \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

$$O^\# = \begin{bmatrix} c^{\#T} & A^{\#T} c^{\#T} & (A^{\#T})^2 c^{\#T} & \dots & (A^{\#T})^{n-1} c^{\#T} \end{bmatrix} = \begin{bmatrix} c^{\#T} & A^{\#T} c^{\#T} \end{bmatrix} = \begin{bmatrix} o^{\#}_{11} & o^{\#}_{12} \\ o^{\#}_{21} & o^{\#}_{22} \end{bmatrix} = \dots$$

$$P^{-1} = (O^{-1})^T \cdot O^{\#T} = (O^{\#} \cdot O^{-1})^T = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = \dots$$

$$P = (\text{Adj } P^{-1}) / (\text{Det } P^{-1}) = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \dots$$

$$b^{\#} = P \cdot b = [b_n^{\#} \dots b_2^{\#} b_1^{\#}]^T = [b_2^{\#} b_1^{\#}]^T = \dots$$

We are then under conditions of finding to the estimator

$$A_{(n-1)}^* = A_{11}^* = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_{en-1} \\ & 1 & 0 & \dots & 0 & -a_{en-2} \\ & 0 & 1 & \dots & 0 & -a_{en-3} \\ & \dots & & & & \\ 0 & 0 & \dots & 1 & -a_{e1} \end{bmatrix} = \begin{bmatrix} 0 & -a_{en-1} \\ & 1 & -a_{en-2} \end{bmatrix} = \dots$$

$$h_{(n-1)}^* = A_{12}^* = \begin{bmatrix} a_{en-1}(a_1 - a_{e1}) & +(0 - a_n) \\ a_{en-2}(a_1 - a_{e1}) & +(a_{en-1} - a_{n-1}) \\ a_{en-3}(a_1 - a_{e1}) & +(a_{en-2} - a_{n-2}) \\ \dots & \\ a_{e1}(a_1 - a_{e1}) & +(a_{e2} - a_2) \end{bmatrix} = \begin{bmatrix} a_{en-1}(a_1 - a_{e1}) & +(0 - a_n) \\ a_{e1}(a_1 - a_{e1}) & +(a_{e2} - a_2) \end{bmatrix} = \dots$$

$$b_{(n-1)}^* = [b_n^* \dots b_2^*]^T = \begin{bmatrix} (b_n^{\#} - b_1^{\#}) a_{en-1} \\ (b_{n-1}^{\#} - b_1^{\#}) a_{en-2} \\ (b_{n-2}^{\#} - b_1^{\#}) a_{en-3} \\ \dots \\ (b_2^{\#} - b_1^{\#}) a_{e1} \end{bmatrix} = (b_2^{\#} - b_1^{\#}) a_{e1} = \dots$$

$$W^{-1} = \begin{bmatrix} 1 & 0 & a_{en-1} \\ 0 & 1 & a_{en-2} \\ 0 & 0 & 1 \end{bmatrix} = \dots$$

## SYSTEMS «MI-MO» (discreet)

### EQUATIONS OF STATE

The equations of system of the plant Gp is

$$\begin{cases} x_{(k+1)} = A x_{(k)} + B u_{(k)} \end{cases}$$

$$y_{(k)} = C x_{(k)}$$

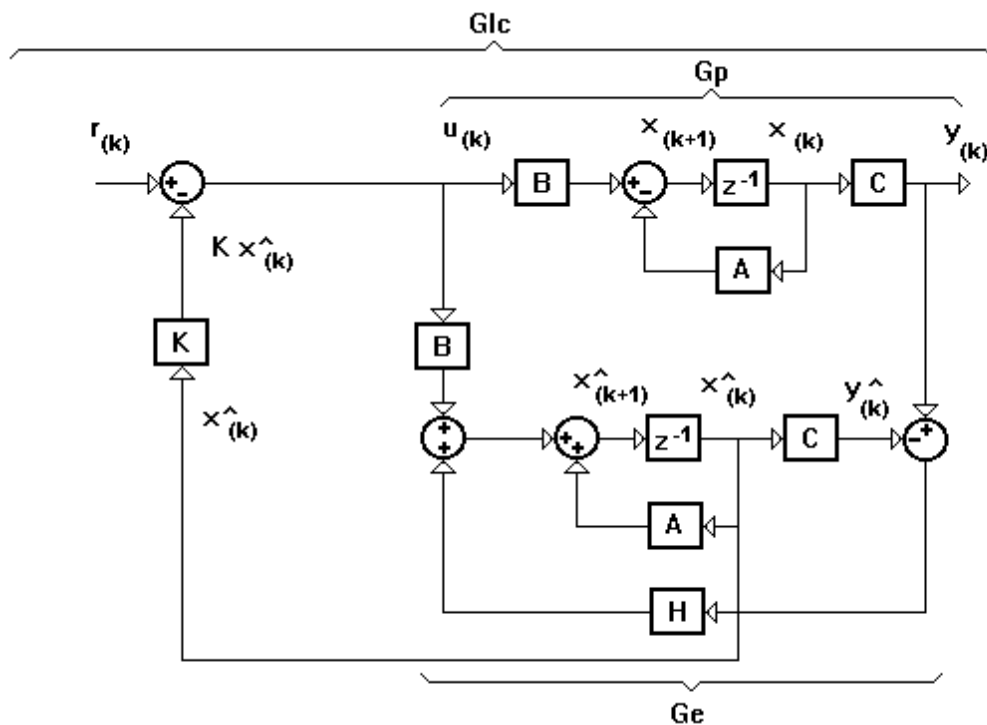
dynamically ( $r = 0$ )

$$u_{(k)} = -K x_{(k)}^{\wedge}$$

$$x_{(k+1)}^{\wedge} = A x_{(k)}^{\wedge} + B u_{(k)} + H [y_{(k)} - y_{(k)}^{\wedge}]$$

the error

$$e_{(k)} = x_{(k)} - x_{(k)}^{\wedge}$$



of where they are deduced

$$x_{(k+1)}^{\wedge} = (A - HC) x_{(k)}^{\wedge} + B u_{(k)} + H y_{(k)}$$

$$e_{(k+1)} = (A - HC) e_{(k)}$$

## Design

Be the transformations

$$x_{(k)} = Q x_{(k)}^*$$

$$x_{(k)}^{\wedge} = Q x_{(k)}^{\wedge*}$$

$$Q = (W O^T)^{-1}$$

$$O = [C^T \quad A^T C^T \quad (A^T)^2 C^T \dots (A^T)^{n-1} C^T] \quad \text{observability matrix}$$

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \dots & a_1 & 1 \\ & a_{n-2} & a_{n-3} & \dots & 1 & 0 \\ & | & \dots & & & | \\ & a_1 & 1 & \dots & 0 & 0 \\ & 1 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$\text{Det}(zI - A) = (z - z_1)(z - z_2) \dots (z - z_n) = z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 0$$

of where they are demonstrated that

$$\begin{cases} x^*_{(k+1)} = Q^{-1} A Q x^*_{(k)} + Q^{-1} B u^*_{(k)} \\ y_{(k)} = C Q x^*_{(k)} \end{cases}$$

$$Q^{-1} A Q = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_n \\ & 1 & 0 & \dots & 0 & -a_{n-1} \\ & | & \dots & & & | \\ & 0 & 0 & \dots & 1 & -a_1 \end{bmatrix}$$

$$C Q = [0 \quad 0 \quad \dots \quad 0 \quad 1]$$

$$\begin{aligned} e_{(k)} &= x^*_{(k)} - \hat{x}^*_{(k)} \\ e_{(k+1)} &= Q^{-1} (A - H C) Q e_{(k)} \end{aligned}$$

We look for in the design:

- 1) that  $e_{(k)}$  it is the smallest and quick thing possible
- 2) that  $e_{(k+1)}$  it is stable (denominated as *dynamics of the error* of the system)

for that the poles of the estimator quicker  $G_e$  is adopted that those of the closed loop  $G_{lc}$  (some 4 or 5 times)

$$\begin{array}{lll} \text{Dominant } \{s_1; s_2\} & \ll \text{Dominant } \{e_1; e_2\} & \gg \text{Dominant } \{sf_1; sf_2\} \\ \text{Dominant } \{z_1; z_2\} & \gg \text{Dominant } \{ze_1; ze_2\} & \ll \text{Dominant } \{zf_1; zf_2\} \end{array}$$

$$ze_1 = \dots, ze_2 = \dots$$

We select (or they are data) the coefficients of the plant  $G_p$  and of the closed loop  $G_{lc}$  (here if all the « $\alpha_1, \alpha_2, \dots, \alpha_n$ » they are null then we don't have oscillations in the closed loop)

$$\text{Det}(zI - A) = (z - z_1)(z - z_2) \dots (z - z_n) = z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 0$$



$$a_1 = \dots, a_2 = \dots$$

$$\text{Det}(zI - Af) = (z - zf_1)(z - zf_2) \dots (z - zf_n) = z^n + \alpha_1 z^{n-1} + \dots + \alpha_{n-1} z + \alpha_n = 0$$

$$\alpha_1 = \dots, \alpha_2 = \dots$$

They are calculated finally

$$h^* = Q^{-1}h = \begin{bmatrix} h_1^* \\ h_2^* \\ \vdots \\ h_n^* \end{bmatrix} = \begin{bmatrix} \alpha_n - a_n \\ \vdots \\ \alpha_2 - a_2 \\ \alpha_1 - a_1 \end{bmatrix} = \dots$$

$$h = Q h^* = (W O^T)^{-1} h^* = \dots$$

## Chap. 41 Controllers of the State in a System

### CONTROLLERS TYPE «P.I.D.»

#### INTRODUCTION

#### Optimization for Ziegler-Nichols

#### First form

#### Second form

### COMPENSATORS TYPE «DEAD-BEAT» AND «DAHLIN»

#### Compensator «dead-beat»

#### Compensator «Dahlin»

### CALCULATION OF A CONTROLLER COMPESATOR

#### DATA

#### PHYSICAL IMPLEMENTATION

#### GENERALITIES

#### CALCULATION

#### DIAGRAM OF FLOW

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### CONTROLLERS TYPE «P.I.D.»

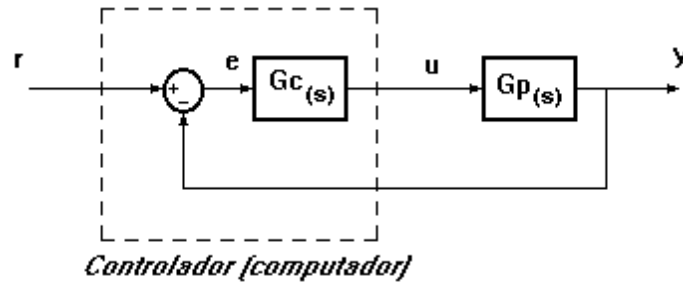
#### INTRODUCTION

The transfer of the compensator  $G_c$  is given in the following way

$$G_{c(s)} = K_p [ 1 + T_d s + 1/T_i s ] = K_p + K_d s + K_i/s$$

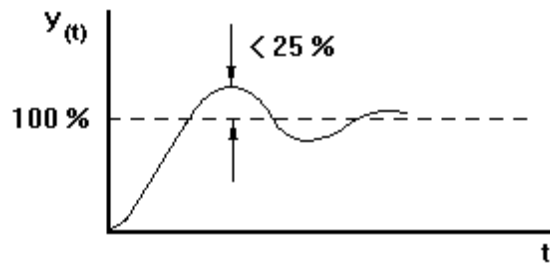
where

- $K_p$  = proportion gain
- $K_d$  = derivation gain
- $K_i$  = integration gain
- $T_d$  = constant of derivative time
- $T_i$  = constant of integration time



### Optimization for Ziegler-Nichols

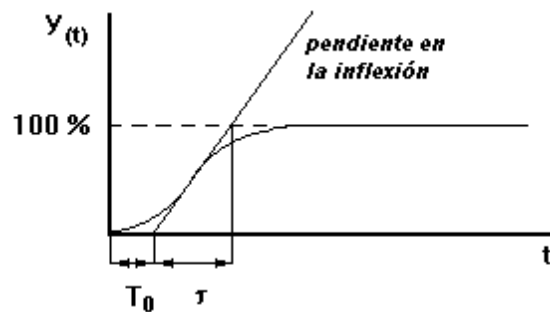
It consists on two methods to calculate the  $G_c$  in such a way that the over-impulse in  $y(t)$  it doesn't overcome 25% for an input  $r(t)$  in pedestal.



### First form

We apply a pedestal in  $u(t)$  and they are experimentally for the plant  $G_p$

$T_0$  = ... Delay time  
 $\tau$  = ... Constant of time



and if approximately for the graph a plant transfer

$$G_p(s) = K \cdot e^{-sT_0} / (1 + s\tau)$$

some values are suggested for the design

	P	PI	PID
$K_p$	$\tau / T_0$	$0,9 \cdot \tau / T_0$	$1,2 \cdot \tau / T_0$
$T_d$	0	0	$0,5 \cdot T_0$
$T_i$	$\infty$	$3,33 \cdot T_0$	$2 \cdot T_0$

that is to say that finally is for our case of PID

$$G_{C(s)} = 0,6 \cdot \tau (s + 1/T_0)^2 / s$$

### Second form

We apply this method to those plants  $G_p$  that have harmonic oscillations in closed loop  $G_{lc}$  when it experiences them to him with a proportional compensator  $G_{C(s)} = K_p$ .

We find the critical value that makes oscillate the plant and their period

$$\begin{aligned} K_{pc} &= \dots K_p \text{ critical} \\ T_0 &= \dots \text{period of critical oscillation} \end{aligned}$$

and it is suggested to use the values

	P	PI	PID
$K_p$	$0,5 \cdot K_{pc}$	$0,45 \cdot K_{pc}$	$0,6 \cdot K_{pc}$
$T_d$	0	0	$0,125 \cdot T_0$
$T_i$	$\infty$	$0,83 \cdot T_0$	$0,5 \cdot T_0$

that is to say that finally is for our case of PID

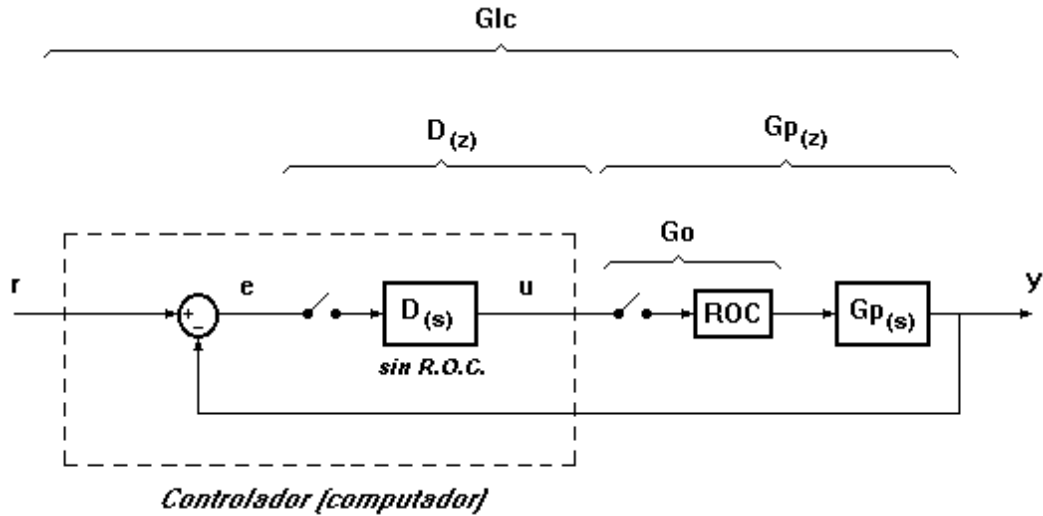
$$G_{C(s)} = 0,075 \cdot K_{pc} \cdot T_0 (s + 4/T_0)^2 / s$$

## COMPENSATORS TYPE «DEAD-BEAT» AND «DAHLIN»

### Compensator «dead-beat»

Given the figure of closed loop  $G_{lc}$ , we look for to design the compensator  $D_{(z)}$  such that the output follows the most possible to the input; that is to say, that diminishes the error «e»

$$y(k) = r_{(k-1)}$$



With the purpose of simplifying the nomenclature we will use the following expressions:

$$G_{p(z)} = y_{(z)} / u_{(z)} = Z [G_o(s) \cdot G_p(s)] = Z (1 - e^{-sT}) \cdot Z [G_p(s) / s]$$

$$D_{(z)} = u_{(z)} / e_{(z)} = Z [D_{(s)}]$$

Then, first we outline the transfer of closed loop

$$G_{lc(z)} = y_{(z)} / r_{(z)} = D_{(z)} \cdot G_{p(z)} / [1 + D_{(z)} \cdot G_{p(z)}]$$

and we clear

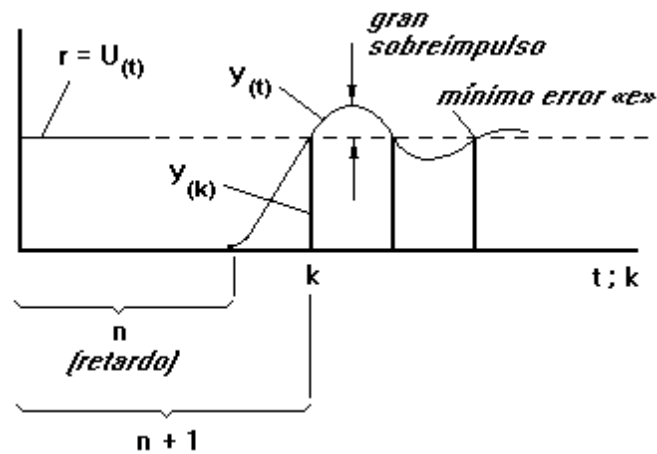
$$D_{(z)} = [1 / G_{p(z)}] \cdot \{ G_{lc(z)} / [1 - G_{lc(z)}] \}$$

If now we keep in mind the transformation of the impulse of Kronecker in the sample moment «k»

$$Z [\delta_{(k)}] = z^{-k}$$

and that for the equation  $y_{(k)} = r_{(k-1)}$  it is in the practice that for a delay (retard of  $G_{lc}$ ) of «n» pulses (the sample «k» a sample will be for above for the maxim possible response of  $G_{lc}$ )

$$\begin{aligned} k &= n + 1 \\ y_{(k)} &= r_{((n+1)-1)} = r_{(n)} \end{aligned}$$



of where we obtain

$$y(z) = r(z) \cdot z^{-1} \rightarrow \text{para } n = 0$$

$$y(z) = r(z) \cdot z^{-(n+1)} \rightarrow \text{para } n \neq 0$$

$$G_{lc(z)} = y(z) / r(z) = z^{-(n+1)} = z^{-n-1}$$

what determines finally

$$D(z) = [1 / G_p(z)] \cdot \{ z^{-n-1} / [1 - z^{-n-1}] \}$$

### Compensator «Dahlin»

Here they will diminish the undesirable over-impulses in exchange for allowing a worsening in the error «e».

For it Dahlin proposes the following algorithm

$$y(k) = q \cdot y(k-1) + (1 - q) \cdot r(k - (n+1))$$

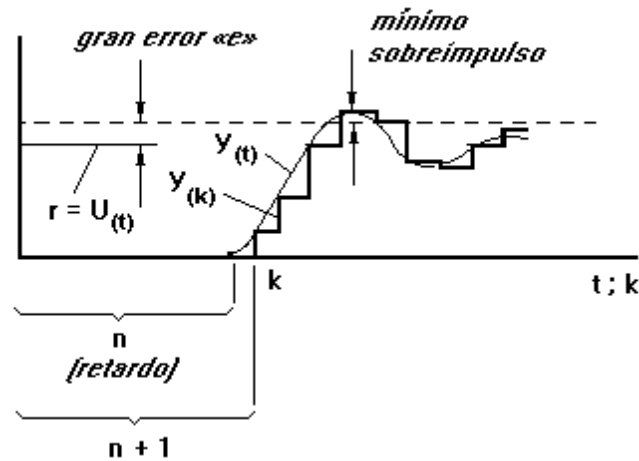
where «q» it is defined as a «syntony factor»

$$0 < q = e^{-\lambda T} < 1$$

and what allows to determine (to observe that the expression coincides with that found in «dead-beat» for  $\lambda \rightarrow \infty$ )

$$G_{lc(z)} = y(z) / r(z) = (1 - q) \cdot z^{-n-1} / (1 - q \cdot z^{-1})$$

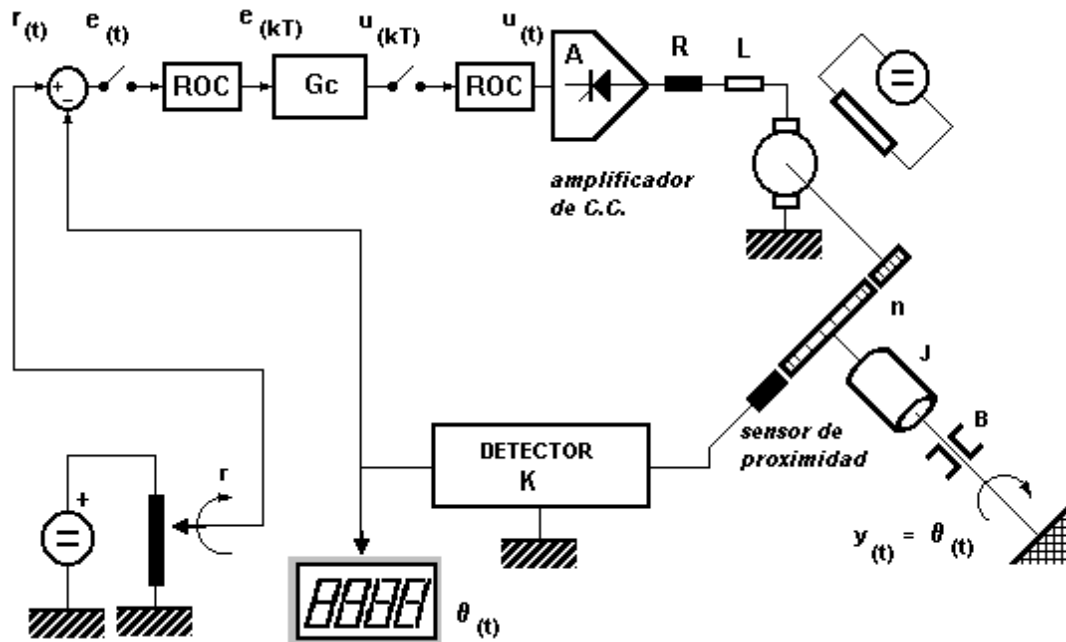
$$D(z) = [1 / G_p(z)] \cdot \{ (1 - q) \cdot z^{-n-1} / [1 - q \cdot z^{-1} - (1 - q) \cdot z^{-n-1}] \}$$

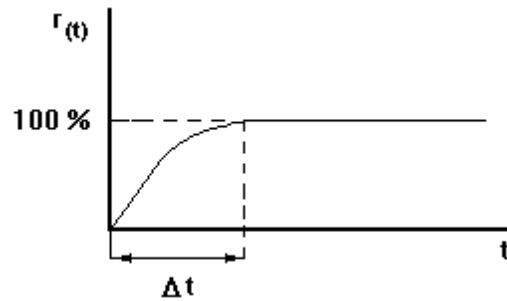


## CALCULATION OF A CONTROLLER COMPESATOR

### DATA

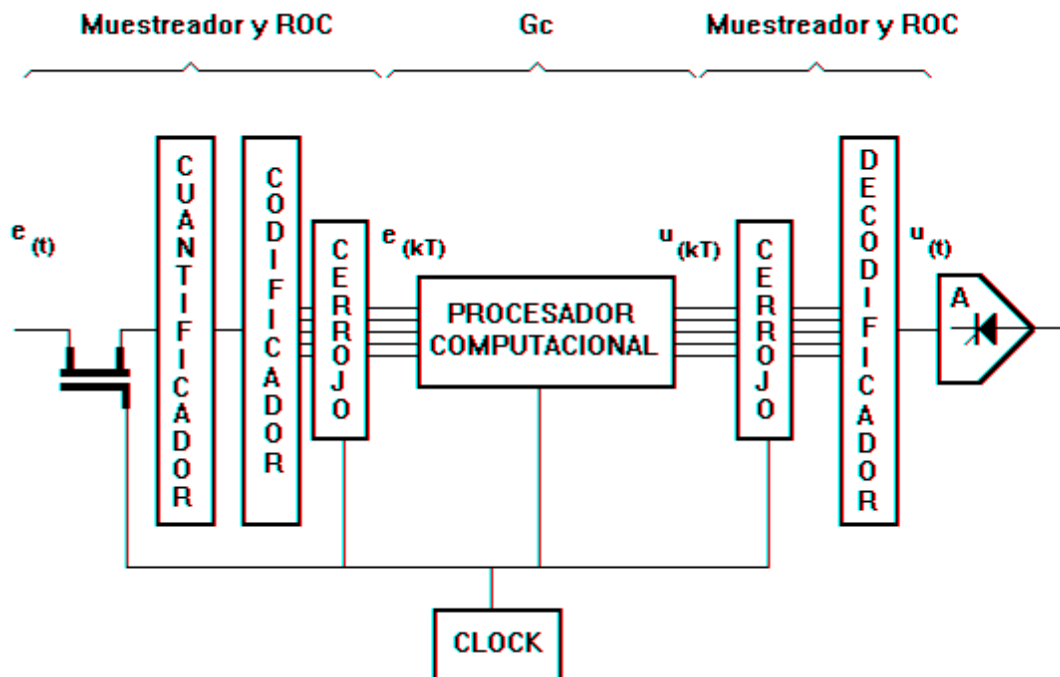
We have the following servomechanism position controller, and it is wanted him not to possess oscillations to their output. It is asked to use a digital processor of control  $G_c$  to their input that avoids the effect.





$B = 0,15 \text{ [Nms/r]}$   
 $J = 0,15 \text{ [Nms}^2\text{/r]}$   
 $R = 1 \text{ } [\Omega]$   
 $L = 0,03 \text{ [H]}$   
 $N_{\text{nom}} = 1500 \text{ [RPM]}$   
 $P_{\text{nom}} = 3/4 \text{ [Hp]}$   
 $U_{\text{nom}} = 200 \text{ [V]}$   
 $I_{\text{nom}} = 3 \text{ [A]}$   
 $A = 10$   
 $K = 1 \text{ [V/}^\circ\text{]}$   
 $n = 1/10$   
 $\Delta t \leq 0,1 \text{ [s]}$   
 Oscillations died in  $\theta_{(t)}$

## PHYSICAL IMPLEMENTATION





## GENERALITIES

We determine the characteristics of the motor

$$\begin{aligned}\omega_{\text{nom}} &= 2\pi \cdot N / 60 = 157 \text{ [r/s]} \\ P_{\text{nom}} &= 3/4 \text{ [Hp]} / 740 = 555 \text{ [W]} \\ k_g &\sim U_{\text{nom}} / \omega_{\text{nom}} = 1,27 \text{ [Vs/r]}\end{aligned}$$

and finding the poles of the plant

$$\begin{aligned}\tau_{\text{elec}} &= L / R = 0,03 \text{ [s]} \\ \tau_{\text{mec}} &= J / B = 1 \text{ [s]} \quad \rightarrow \quad \text{dominant}\end{aligned}$$

we make their transfer

$$\begin{aligned}G_p(s) &= y(s) / u(s) \sim A \cdot (\omega / U) / s (1 + s \tau_{\text{mec}}) = \\ &= A \cdot k_g^{-1} / s (1 + s \tau_{\text{mec}}) \sim 15,7 \text{ [r/V]} / s (1 + s)\end{aligned}$$

The sampling frequency obtains it of the *Theorem of the Sampling*

$$T \gg \Delta t$$

for what is adopted for example

$$T = 1 \text{ [s]}$$

## CALCULATION

Given the gain of the R.O.C.

$$G_o(s) = 1 - e^{-sT} / s$$

it is the plant  $G_p$

$$\begin{aligned}G_p(z) &= \mathcal{Z} [ G_o(s) G_p(s) ] = \mathcal{Z} (1 - e^{-sT}) \cdot \mathcal{Z} [ 15,7 / s^2 (1 + s) ] = \\ &= 5,65 (1+0,71z^{-1}) z^{-1} / (1-z^{-1}) (1-0,36z^{-1})\end{aligned}$$

If we outline an input generic type pedestal

$$\begin{aligned}r(t) &= U \\ r(z) &= 1 / (1 - z^{-1})\end{aligned}$$

the signal of control  $u_{(z)}$  it is

$$u_{(z)} = \text{Glc} \cdot r / G_p = \text{Glc} \cdot (1-0,36z^{-1}) / 5,65z^{-1}(1+0,71z^{-1})$$

and like we know that the order of Glc will be smaller than 3 to avoid oscillations, and that in turn it will be same or bigger than that of  $G_p$ , we can conclude here that it is correct that it has two poles

$$\text{Glc}_{(z)} = a z^{-1} + b z^{-2}$$

and in turn if to simplify calculations we also make

$$\text{Glc}_{(z)} = K z^{-1}(1+0,71z^{-1})$$

they are

$$K = (a z^{-1} + b z^{-2}) / z^{-1}(1+0,71z^{-1}) = a + R_K [(b-0,71a)z^{-1}]$$

$$u_{(z)} = 0,18 K (1-0,36z^{-1})$$

If now we outline the closed loop Glc again

$$\text{Glc}_{(z)} = y / r = (r - e) / r = G_c G_p / (1 + G_c G_p)$$

and we clear the error  $e_{(z)}$

$$e_{(z)} = (1 - \text{Glc}) r = (1 - \text{Glc}) / (1 - z^{-1})$$

that we know it will be a polynomial  $N_{(z)}$

$$e_{(z)} = N_{(z)}$$

we can deduce

$$\begin{aligned} N_{(z)} &= (1 - \text{Glc}) / (1 - z^{-1}) = [1 - (a z^{-1} + b z^{-2})] / (1 - z^{-1}) = \\ &= 1 + (1-a)z^{-1} + R_N [(1-a-b)z^{-2}] \end{aligned}$$

If now of the two equations of K and N we adopt null the remains

$$R_K = R_N = 0$$

we can calculate

$$\begin{aligned} a &= 0,58 \\ b &= 0,41 \\ K &= 0,58 \end{aligned}$$

$$\begin{aligned} \text{Glc}_{(z)} &= 0,58 z^{-1} + 0,41 z^{-2} \\ \text{N}_{(z)} &= 1 + 0,41 z^{-1} \end{aligned}$$

as well as if we outline again

$$\begin{aligned} \text{Glc}_{(z)} &= \text{GcGp} / (1 + \text{GcGp}) \\ e_{(z)} &= \text{N}_{(z)} = (1 - \text{Glc}) r = (1 - \text{Glc}) / (1 - z^{-1}) \end{aligned}$$

we clear the filter Gc controller finally

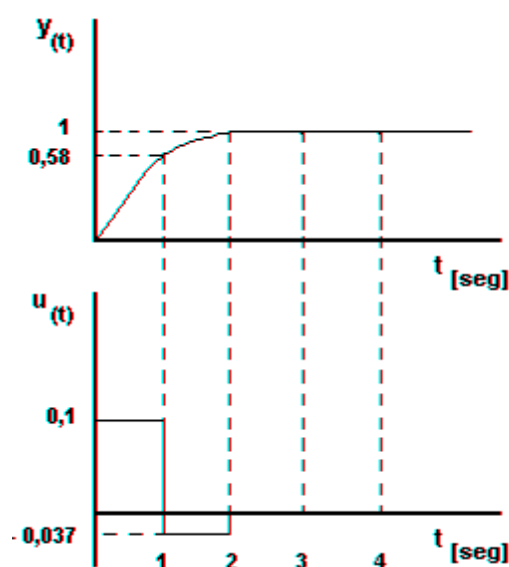
$$\begin{aligned} \text{Gc}_{(z)} &= \text{Glc} / \text{Gp} (1 - \text{Glc}) = \text{Glc} / \text{Gp} \text{N}_{(z)} (1 - z^{-1}) \sim \\ &\sim 0,11 (z - 0,36) / (z + 0,41) \end{aligned}$$

what will give us an output and an error

$$\begin{aligned} y_{(z)} &= \text{Glc} \cdot r = (0,58 z^{-1} + 0,41 z^{-1}) / (1 - z^{-1}) = 0,58 z^{-1} + z^{-2} + z^{-3} + \dots \\ u_{(z)} &= 0,18 \cdot 0,58 (1 - 0,36 z^{-1}) = 0,1 - 0,037 z^{-1} \end{aligned}$$

and in the time

$y_{(k=0)} = 0$	$y_{(k=1)} = 0,58$	$y_{(k=2)} = 1$	$y_{(k=3)} = 1$	$y_{(k=4)} = 1$
$u_{(k=0)} = 0,1$	$u_{(k=1)} = -0,037$	$u_{(k=2)} = 0$	$u_{(k=3)} = 0$	$u_{(k=4)} = 0$



To implement the filter digital controller we outline their transfer first

$$\text{Gc}_{(z)} = u / e = 0,11 (z - 0,36) / (z + 0,41)$$

and we proceed

$$z u + 0,41 u = 0,11 z e - 0,04 e$$

that is to say that is for a k-generic instant

$$u_{(k+1)} + 0,41 u_{(k)} = 0,11 e_{(k+1)} - 0,04 e_{(k)}$$

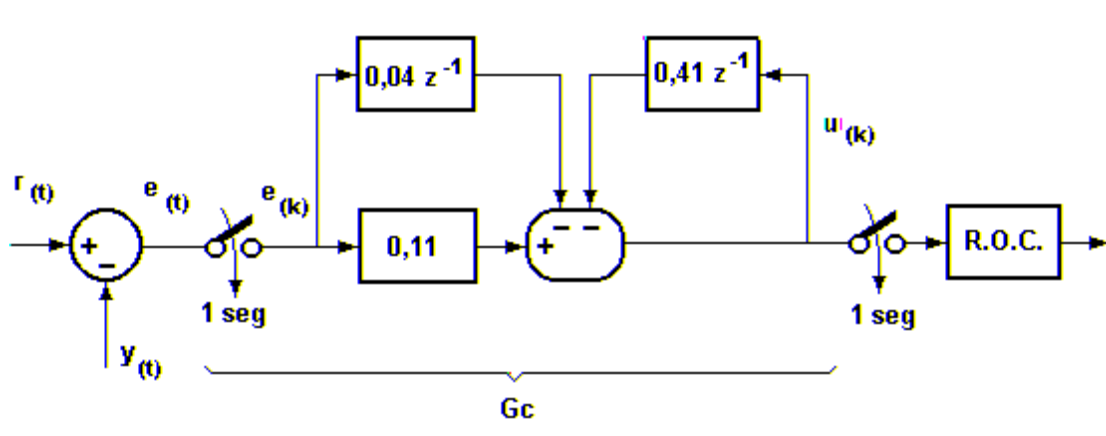
or

$$u_{(k)} + 0,41 u_{(k-1)} = 0,11 e_{(k)} - 0,04 e_{(k-1)}$$

what will determine us a control  $u_{(k)}$  to implement in the following way

$$u_{(k)} = 0,11 e_{(k)} - 0,04 e_{(k-1)} - 0,41 u_{(k-1)}$$

### DIAGRAM OF FLOW



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