القوانين

Single degree of freedom

نحفظ القوانين المكتوب
عليها حفظ فقط لان
الباقى سيكون معطى

[1] Free Undamped vibration :

$F(t) = 0$ & $C = 0$

$$M U'' + K U = 0$$

$$\boxed{\omega = \sqrt{\frac{K}{M}}}$$ حفظ

Where :

$\omega \longrightarrow$ Natural frequancy

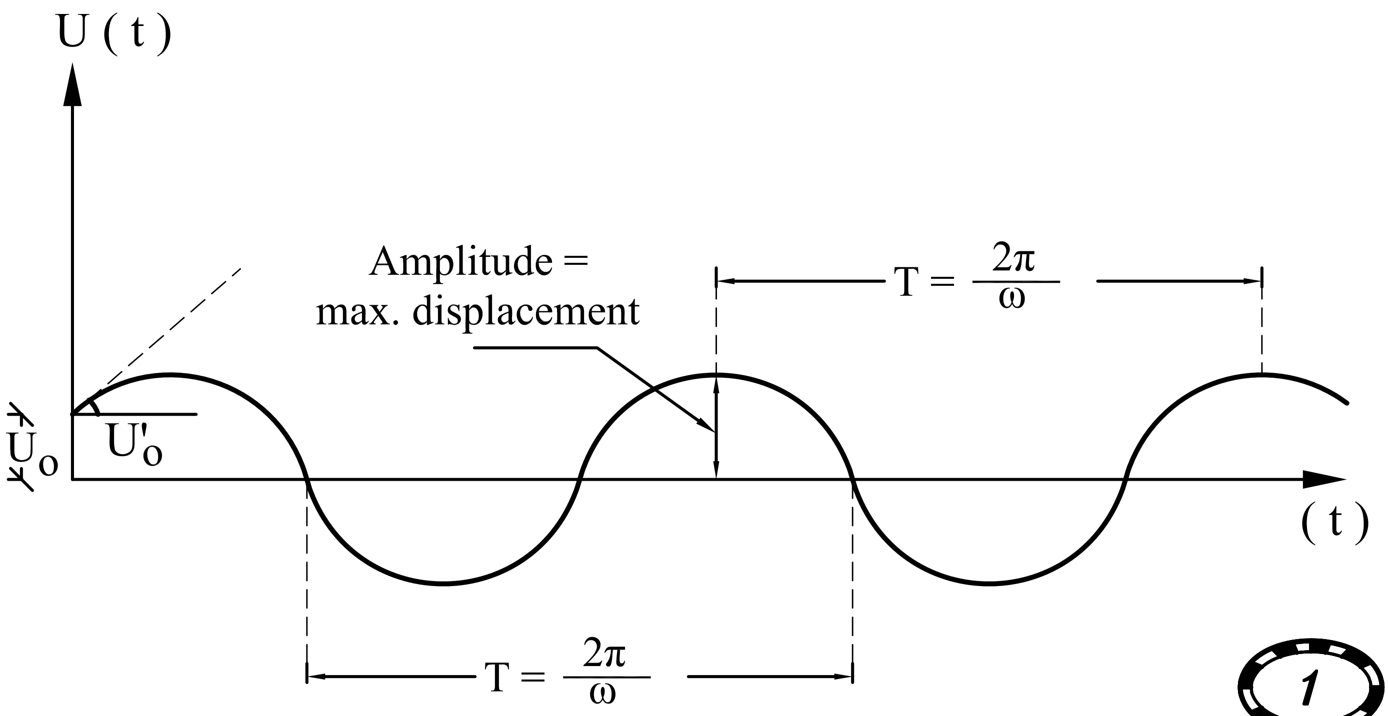
$K \longrightarrow$ Stiffness

$M \longrightarrow$ Mass

$$\boxed{U(t) = U_o \cos \omega t + \frac{U'_o}{\omega} \sin \omega t}$$

$$\boxed{\text{Amplitude} = U_{max.} = \sqrt{(U_o)^2 + \left(\frac{U'_o}{\omega}\right)^2}}$$

Natural Cyclic Frequency $\longrightarrow \frac{1}{T}$



[2] Free damped vibration :

$F(t) = 0$ & $C \neq 0$

$$\underline{M X'' + C X' + K X = 0}$$

Where :

$$\zeta = \frac{C}{2\,\omega_n M} = \frac{C}{C_{cr}}$$

C = Damping constant

ζ = Damping ratio

C_{cr} = Critical Damping constant

$$\boxed{U(t) = e^{-\zeta \omega_n t}\left(U_0 \cos \omega_D t + \frac{U'_0 + \zeta \omega_n U_0}{\omega_D} \sin \omega_D t\right)}$$

ω_D = Natural frequency of damped system

$$\boxed{\omega_D = \omega_n \times \sqrt{1 - \zeta^2}}$$ حفظ

$$\boxed{\omega_n = \sqrt{\frac{K}{M}}}$$ حفظ

free undamped

T_D = Natural period of damped system

$$\boxed{T_D = \frac{T_n}{\sqrt{1 - \zeta^2}} = \frac{2\pi}{\omega_D}}$$ حفظ

$$\boxed{T_n = \frac{2\pi}{\omega_n}}$$ حفظ

free undamped

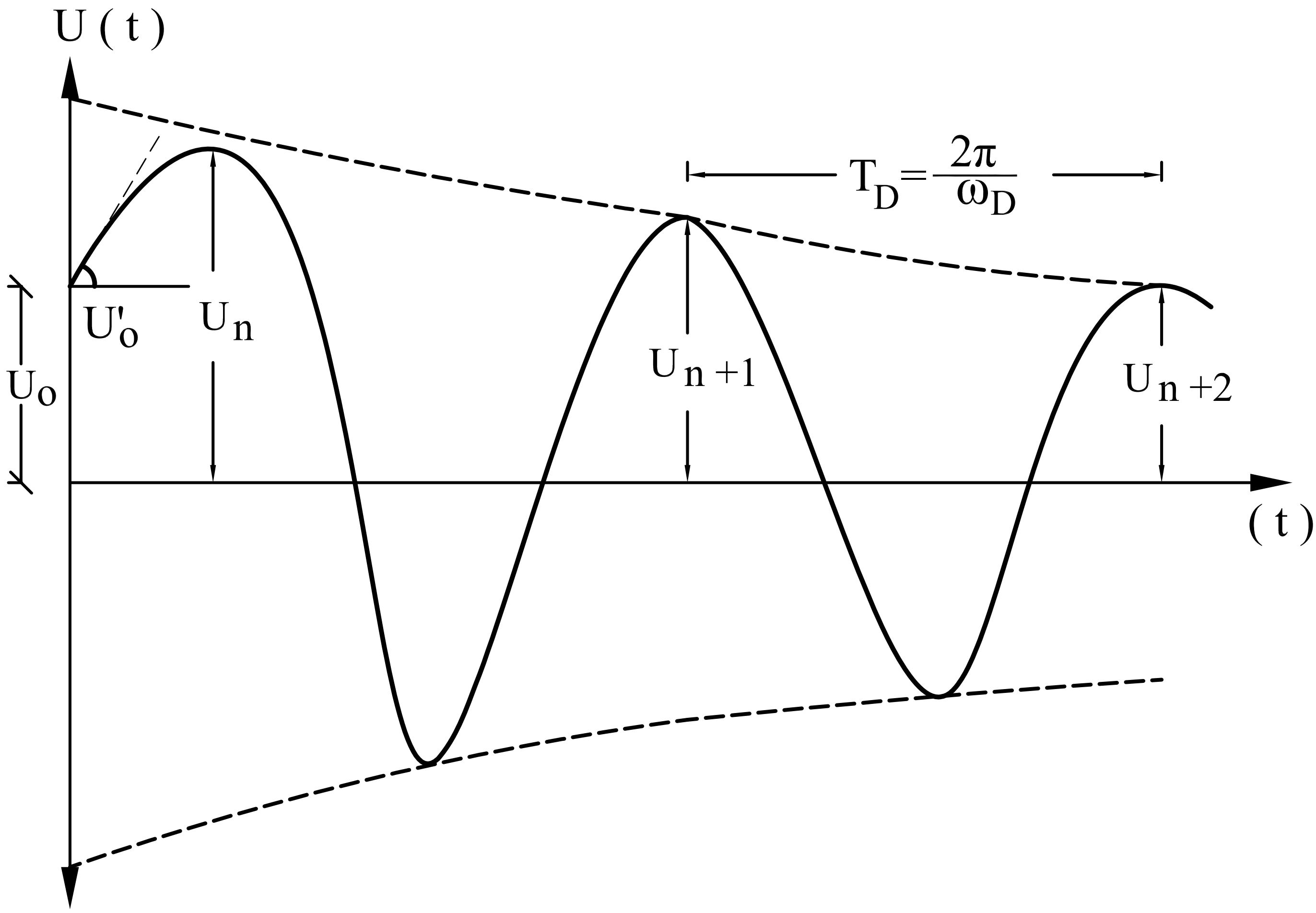
حفظ

$$\frac{U_n}{U_{(n+m)}} = e^{m(2\pi\zeta)} = \left[\frac{U_n}{X_{(n+1)}}\right]^m$$

$$\ln \frac{U_n}{U_{(n+m)}} = m(2\pi\zeta)$$

$$\frac{U''_n}{U''_{(n+m)}} = e^{m(2\pi\zeta)}$$

$$\ln \frac{U''_n}{U''_{(n+m)}} = m(2\pi\zeta)$$



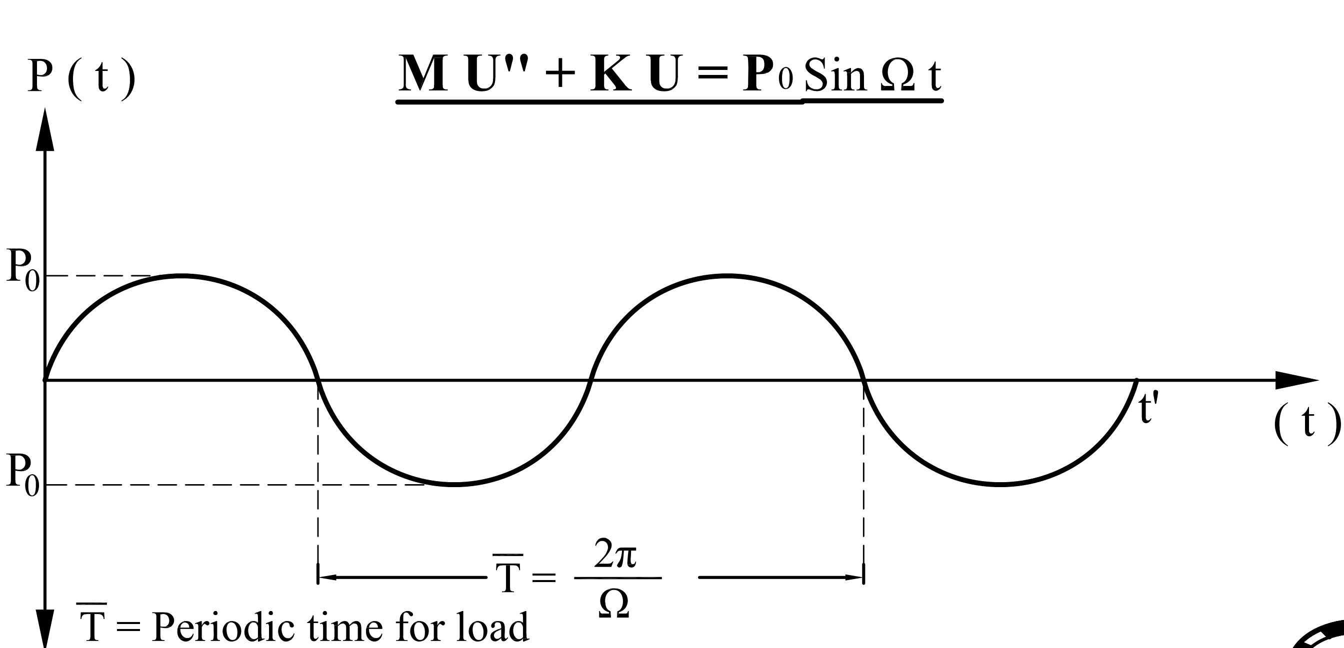
[3] Forced undamped vibration : (harmonic force) :

$F(t) \neq 0$ & $C = 0$

$$\underline{\mathbf{M\,U'' + K\,U = F(t)}}$$

$F(t) = P_0 \sin \Omega t$ & $C = 0$

$$\underline{\mathbf{M\,U'' + K\,U = P_0} \sin \Omega t}$$



$\overline{T}$ = Periodic time for load

Ω = natural frequency for load

$$U(t) = \frac{P_0}{K} \frac{1}{1-(\frac{\Omega}{\omega})^2} \left(\sin \Omega t - \frac{\Omega}{\omega} \sin \omega t \right)$$

$$U(t) = \frac{P_0}{K} \frac{1}{1-(\beta)^2} \left(\sin \Omega t - \beta \sin \omega t \right)$$

Where :

$\omega \longrightarrow$ Natural frequancy for the structure's displacement

$\Omega \longrightarrow$ Natural frequancy for the load

$T \longrightarrow$ Periodic time for the structure's displacement

$\overline{T} \longrightarrow$ Periodic time for the load

$\beta = \frac{\Omega}{\omega} \longrightarrow$ Frequency ratio

$(P_0 / K) \longrightarrow$ Static displacement

$$U_{max.} = \frac{P_0}{K} (D.L.F)_{max.}$$

$$(D.L.F)_{max.} = (Rd)_{max.} = \frac{1}{1-(\frac{\Omega}{\omega})} \quad \text{for total response}$$

$$\boxed{(D.L.F)_{max.} = (Rd)_{max.} = \frac{1}{1-(\frac{\Omega}{\omega})^2} \quad \text{for steady state}}$$

اذا لم يذكر نشتغل
على أنه Steady State

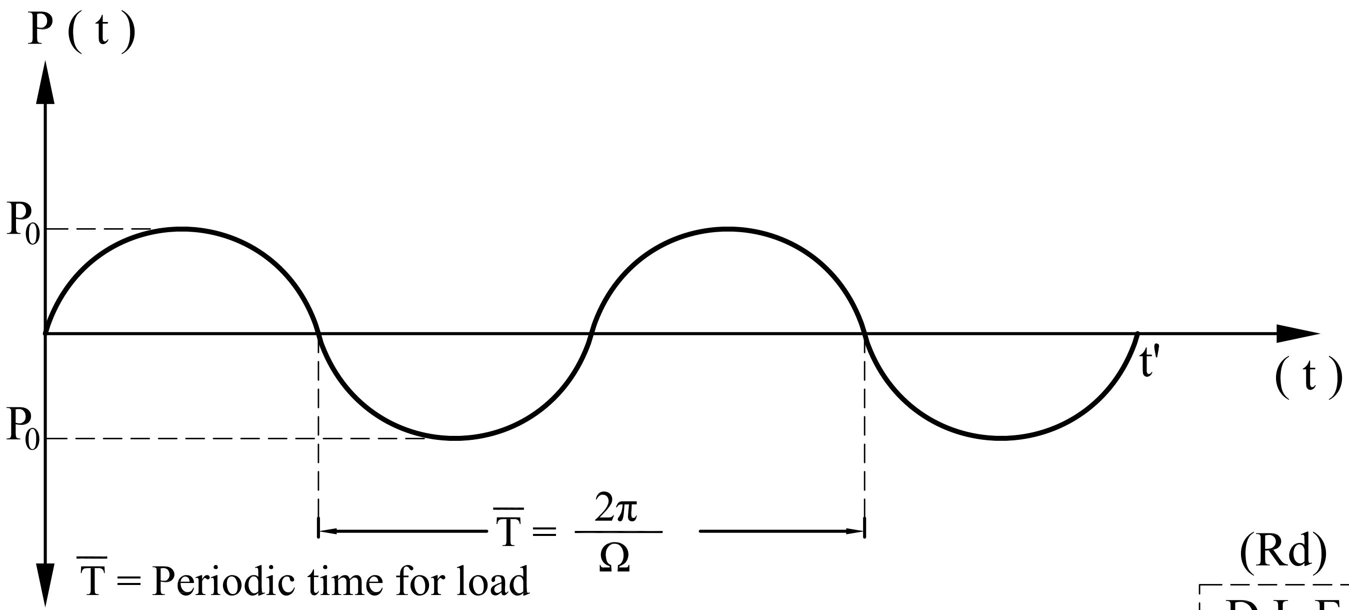
$$\boxed{\text{max. equivalent static force} = K * (U)\ max.}$$

حفظ

[4] Forced damped vibration (harmonic force) :

$F(t) = P \ \text{Sin}\ \Omega t$ & $C \neq 0$

$$\mathbf{M\,U'' + C\,U' + K\,U = P_0}\ \text{Sin}\ \Omega t$$



$\overline{T}$ = Periodic time for load

Ω = natural frequency for load

$$\boxed{U(t) = \frac{P_0}{K}\ \underbrace{\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\ \text{Sin}(\Omega t + \Phi)}_{\substack{(Rd) \\ D.L.F}}}$$

Where :

$\Phi \longrightarrow$ Phase angle $\text{Tan}\ \Phi = \dfrac{2\zeta\beta}{1-\beta^2}$

$\beta = \dfrac{\Omega}{\omega} \longrightarrow$ Frequency ratio (حفظ)

$$U\ max. = \frac{P_0}{K}\ (D.L.F)\ max.$$

$$(D.L.F)\ max. = (Rd)\ max. = \frac{1}{\sqrt{[1-(\frac{\Omega}{\omega})^2]^2 + (2\zeta\frac{\Omega}{\omega})^2}}$$

$$\boxed{\text{At Resonance } (\Omega = \omega) \text{ -------- } (D.L.F)\ max. = Rd\ \ max. = (1/2\zeta)}$$

Maximum displacement

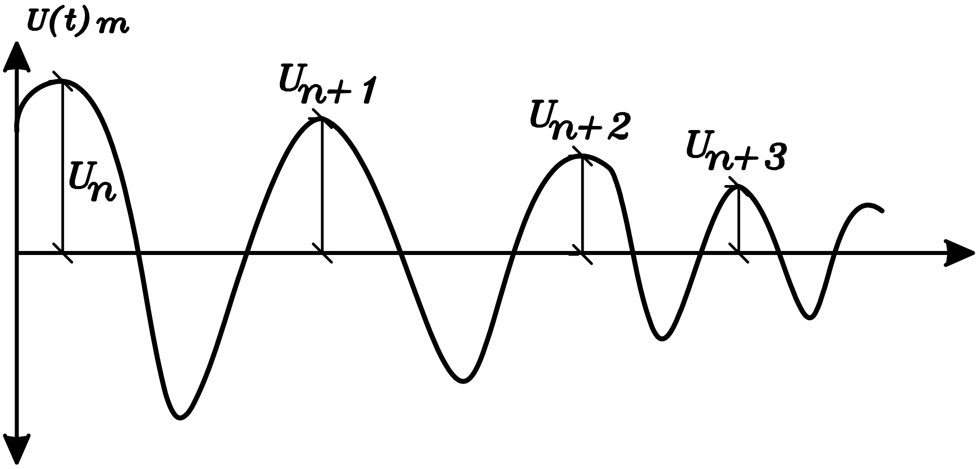
مهمة جدا

ياريت يتحفظ

1- Free undamped $\Longrightarrow$ $\Delta_{max.} = \sqrt{(U_0)^2 + (\frac{\dot{U_0}}{\omega})^2}$

2- Free damped $\Longrightarrow$ ليس لها معادلة و لكن يمكن حسابها
من هذه المعادلة بمعلومية باقى اجزاء المعادلة

$$\frac{U_n}{U_{n+m}} = e^{2\pi m \zeta}$$



3- Forced undamped (Constant force)

$$\Delta_{max.} = \frac{P_0}{K} D.L.F_{Max.} = \frac{P_0}{K} * R_{d\,Max.} = \frac{P_0}{K} * 2$$

4- Forced undamped (Harmonic force) (Steady state)

$$\Delta_{max.} = \frac{P_0}{K} D.L.F_{Max.} = \frac{P_0}{K} * R_{d\,Max.} = \frac{P_0}{K} x \frac{1}{1-\beta^2}$$

$$\beta = \frac{\Omega}{\omega}$$

Forced undamped (Harmonic force) (Total response)

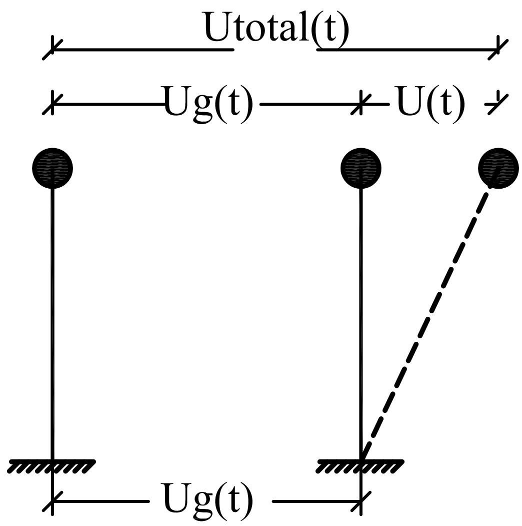
$$\Delta_{max.} = \frac{P_0}{K} D.L.F_{Max.} = \frac{P_0}{K} * R_{d\,Max.} = \frac{P_0}{K} x \frac{1}{1-\beta}$$

5- Forced damped (Harmonic force) (Steady state)

$$\Delta_{max.} = \frac{P_0}{K} D.L.F_{Max.} = \frac{P_0}{K} * R_{d\,Max.}$$

$$= \frac{P_0}{K} x \frac{1}{\sqrt{[1-\beta^2]^2 + (2\zeta\beta)^2}}$$

GROUND ACCELERATION



$$M\,U''\,total(t) + K\,U(t) = 0$$

$$M\,[Ug''(t) + U''(t)] + K\,U(t) = 0$$

$$\boxed{M\,U''(t) + K\,U(t) = -\,M\,U''g(t)}$$

و فى حالة وجود *Damping*

$$\boxed{M\,U''(t) + CU'(t) + K\,U(t) = -\,M\,U''g(t)}$$

ال Units اللى حنشتغل بيها فى أى مسألة

	حفظ	حفظ	حفظ
K	N/m	KN/m	t_f/m (ton/m)
C	N.sec/m	KN.sec/m	t_f.sec/m
M	N.sec²/m (Kg)	KN.sec²/m (t)	t_f.sec²/m
ω	rad/sec	rad/sec	rad/sec
X	m	m	m
X'	m/sec	m/sec	m/sec
X''	m/sec²	m/sec²	m/sec²

للتحويل من ال *weight* بال *kN* الى ال *mass* بال *ton* نقسم على *9.81*

للتحويل من ال *weight* بال *N* الى ال *mass* بال *kg* نقسم على *9.81*

لايجاد و رسم علاقة ال$X \& \dot{X} \& \ddot{X}$ مع الزمن

خطوات المسائل

أ- الخطوات الابتدائية (Initial Calculations)

تحسب فى بداية المسألة

١- نقوم بحساب الاتى

حفظ $T_n = \frac{2\pi}{\omega}$ حفظ $C = 2\ m\,\omega\,\zeta$ حفظ $\omega = \sqrt{\frac{K}{M}}$

و اذا لم تكن ال K معطاه نحسبها مثل ملزمة رقم ٣

٢- نحسب ال $(\ddot{X}_0)$ Initial acceleration

و ال$X_0 \& \dot{X}_0$ تكونا معطيتان

حفظ

$$\ddot{U}_0 = \frac{P_0 - C\,\dot{U}_0 - K\,U_0}{M}$$

٣- نختار ΔT اما ان تكون معطاه فى المسألة من علاقة الForce مع الTime

و اما ناخذها $\Delta t \not> \frac{T_n}{10}$ حفظ

٤- نحسب $\hat{K} = K + \left(\frac{\gamma}{\beta\,\Delta t}\right)C + \left(\frac{1}{\beta(\Delta t)^2}\right)m$

٥- نحسب الثوابت a, b ياريت يتحفظ

$$\boxed{a = \frac{m}{\beta\,\Delta T} + \frac{\gamma\,C}{\beta} \qquad b = \frac{m}{2\,\beta} + \Delta t\left(\frac{\gamma}{2\,\beta} - 1\right)C}$$

ب- حسابات لكل Δt (Calculations for each time step)

١- نحسب $\Delta\hat{P}_i$

$$\Delta\hat{P}_i = \Delta P_i + \left[\frac{m}{\beta\,\Delta T} + \frac{\gamma\,C}{\beta}\right]\dot{X}_i + \left[\frac{m}{2\,\beta} + \Delta t\left(\frac{\gamma}{2\,\beta} - 1\right)C\right]\ddot{X}_i$$

$$\Delta\hat{P}_i = \Delta P_i + a\,\dot{X}_i + b\,\ddot{X}_i$$

٢- نحسب $\Delta U = \frac{\Delta\hat{P}_i}{\hat{K}}$

٣- نحسب

$$\Delta \ddot{U}_i = \frac{\Delta U_i}{\beta (\Delta t)^2} - \frac{\dot{U}_i}{\beta (\Delta t)} - \frac{\ddot{U}_i}{2\beta}$$

$$\Delta \dot{U}_i = \frac{\gamma \Delta U_i}{\beta (\Delta t)} - \frac{\gamma\ \dot{U}_i}{\beta} + \Delta t \left(1 - \frac{\gamma}{2\beta}\right) \ddot{X}_i$$

٤- نحسب

$$U_{i+1} = U_i + \Delta X_i$$

$$\dot{U}_{i+1} = \dot{U}_i + \Delta \dot{X}_i$$

$$\ddot{U}_{i+1} = \ddot{U}_i + \Delta \ddot{X}_i$$

٥- نكرر الخطوات السابقة

لايجاد الـ $P_{eq.}$ أو حساب الـ $moment$ عند أى زمن

١- نحسب الـ U عند هذ الزمن

٢- نحسب الـ $P_{eq.}$ عند هذ الزمن $P_{eq.} = K\,U$

٣- نحسب الـ $moment$ عند هذ الزمن عند أى نقطة

Linear acceleration method حفظ

$\gamma = 1/2 \qquad \beta = 1/6$

Average acceleration method حفظ

$\gamma = 1/2 \qquad \beta = 1/4$

ملاحظات هامة جدا جدا من المحاضرة

١- عند اعطاء القوانين من الممكن أن يرمز للـ $Time-Step$ بـ (h) و ليس Δt

٢- من الممكن عند اعطاء القوانين أن تعطى بدون قيمة الـ γ و β أى بعد التعويض بهم فى المعادلات فمثلا تكون المعادلات فى حالة $Linear\ acc.\ method$

$$\hat{K} = K + \left(\frac{3}{\Delta t}\right)C \quad + \left(\frac{6}{(\Delta t)^2}\right)m$$

$$\Delta \hat{P}_i = \Delta P_i + \left[\frac{6}{\Delta T}\dot{U}_i + 3\ddot{U}_i\right]m + \left[3\dot{U}_i + \left(\frac{\Delta t}{2}\right)\ddot{U}_i\right]C$$

$$\Delta U = \frac{\Delta \hat{P}_i}{\hat{K}}$$

$$\Delta \dot{U}_i = \frac{3\,\Delta U_i}{(\Delta t)} - 3\dot{U}_i - \left(\frac{\Delta t}{2}\right)\ddot{U}_i$$

٣- فى حالة عدم اعطاء معادلة للـ $\Delta\ddot{U}_i$ فاننا نقوم بحساب الـ $\ddot{U}_i$ عند كل زمن من معادلة الحركة وذلك بمعلومية الـ $\dot{U}_i$ و U_i عند كل زمن .

$$\ddot{U}_i = \frac{P_i - C\dot{U}_i - K U_i}{M}$$

MULTI DEGREE OF FREEDOM SYSTEMS
OR
Genralized Single degree of freedom system

$$(M^*) X'' + (C^*) X' + (K^*) X = P^*(t)$$

Where :

M^* = generalized mass = $\int_0^L m_x \Psi_x^2 \, dx + \Sigma (M \times \Psi^2)$

m = distributed mass & Ψ = shape function & L = length

M = Concentrated mass

K^* = generalized stiffness = $\int_0^L EI_x \Psi_x''^2 \, dx + \Sigma (K \times \Psi^2)$

X'' = acceleration , X' = velocity , X = displacement

E = modulus of elasticity , I = moment of inertia

$$\omega = \sqrt{\frac{K^*}{M^*}} \quad \& \quad T = (2\pi / \omega)$$

P^* = generalized force = $\int_0^L p \;\Psi \; dx + \Sigma (P \times \Psi)$

Where :

p = distributed force , P = Concentrated force

For frames

K^* ***= Generalized Stiffness*** $= \Sigma K_i \, x \, (\Psi_i - \Psi_{i-1})^2$

Umax (at x = ✓) = Zmax x Ψ (at x = ✓) حفظ

Mmax (at x = ✓) = - EI U'' (at x = ✓) = - EI [Ψ''(at x = ✓) Zmax]

حفظ

فى حالة ال ground motion ال Force

$$P^* = -\ddot{U}_g \, [\, L^* \,]$$ حفظ

$$L^* = \Big[\underbrace{\int_0^L m \times \Psi \, dx}_{L_1^*} + \underbrace{\Sigma \, (M \times \Psi)}_{L_2^*} \Big]$$ حفظ

القوانين التى

من المتوقع تعطى فى الامتحان

و أى قانون خارج هذه الصفحات

يجب حفظه

Solution of Free undamped vibration for a SDOF system

General equation of dynamic equilibrium:

$$0 = M\,x''(t) + K\,x(t)$$

Definitions:

Term	Definition	Unit	Formula
M	Mass	kg	
K	Stiffness	N/m	
ω	Natural frequency of system	rad/s	$\sqrt{(K/M)}$
x_0	Initial displacement	m	
x_0'	Initial velocity	m/s	

Solution:

$$x(t) = \left[x_0 \cos \omega t + \left(\frac{x_0'}{\omega}\right) \sin \omega t \right]$$

$$= x_{max} \cos(\omega t + \theta) \quad \text{where } \theta \text{ is a phase angle}$$

and $x_{max} = \sqrt{x_0^2 + (x_0'/\omega)^2}$

Support movement of structures

Definitions:

Term	Definition	Unit	Term	Definition	Unit
$x(t)$	Displacement of system	m	$x_s(t)$	Displacement of support	m
$x'(t)$	Velocity of system	m/s	$x_s'(t)$	Velocity of support	m/s
$x''(t)$	Acceleration of system	m/s^2	$x_s''(t)$	Acceleration of support	m/s^2

Term	Definition	Unit	Formula
$u(t)$	Displacement of system relative to support	m	$x(t) - x_s(t)$
$u'(t)$	Velocity of system relative to support	m/s	$x'(t) - x_s'(t)$
$u''(t)$	Acceleration of system relative to support	m/s^2	$x''(t) - x_s''(t)$

Undamped system:

$$M\,x''(t) + K\,x(t) = K\,x_s(t)$$

$$M\,u''(t) + K\,u(t) = -M\,x_s''(t)$$

Damped system:

$$M\,x''(t) + C\,x'(t) + K\,x(t) = K\,x_s(t) + C\,x_s'(t)$$

$$M\,u''(t) + C\,u'(t) + K\,u(t) = -M\,x_s''(t)$$

Solution of Free damped vibration for a SDOF system

General equation of dynamic equilibrium:

$$0 = M\,x''(t) + C\,x'(t) + K\,x(t)$$

Definitions:

Term	Definition	Unit	Formula
C	Damping coefficient	Ns/m	
C_{cr}	Critical damping coefficient	Ns/m	$2M\omega$ **Or** $2\sqrt{(KM)}$
ξ	Damping ratio		C / C_{cr}
ω_D	Damped natural frequency		$\omega\sqrt{(1-\xi^2)}$

Case: 1 $\xi < 1$ $\quad C < C_{cr}$ **Under Damped System**

$$x(t) = e^{-\xi\omega t}\left[x_0 \cos \omega_D t + \left(\frac{x_0' + x_0\xi\omega}{\omega_D}\right) \sin \omega_D t \right]$$

Ratio of two successive peaks: $x_{max1} / x_{max2} = e^{2\pi\xi}$

Case: 2 $\xi = 1$ $\quad C = C_{cr}$ **Critically Damped System**

$$x(t) = e^{-\omega t}\left[x_0(1 + \omega t) + x_0'\,t \right]$$

Solution of Forced undamped vibration for a SDOF system

General equation of dynamic equilibrium:

$$F(t) = M\,x''(t) + K\,x(t)$$

Case: 1 **Constant force function**

$$F(t) = M\,x''(t) + K\,x(t) = P_0$$

Solution

$$x(t) = \frac{P_0}{K}\left(1 - \cos \omega t\right)$$

Case: 2 **Sinusoidal (harmonic) force function**

$$F(t) = M\,x''(t) + K\,x(t) = P_0 \sin \Omega t$$

Solution

$$x(t) = \frac{P_0}{K}\left(\frac{1}{1-(\Omega/\omega)^2}\right)\left[\underbrace{\sin \Omega t}_{\text{Steady state}} - \underbrace{\left(\frac{\Omega}{\omega}\right)\sin \omega t}_{\text{Transient sol.}}\right]$$

$$\text{DLF max.} = \left(\frac{1}{1-(\Omega/\omega)}\right) \quad \text{for total response}$$

$$\text{DLF max.} = \left(\frac{1}{1-(\Omega/\omega)^2}\right) \quad \text{for steady state only}$$

Solution of Forced damped vibration for a SDOF system

General equation of dynamic equilibrium:

$$F(t) = M\,x''(t) + C\,x'(t) + K\,x(t)$$

Case: 1 **Constant force function**

$$F(t) = M\,x''(t) + C\,x'(t) + K\,x(t) = P_0$$

Solution

$$x(t) = \frac{P_0}{K}\left[1 - e^{-\xi\omega t}\left(\cos \omega t + \xi \sin \omega t\right)\right]$$

Case: 2 **Sinusoidal (harmonic) force function**

$$F(t) = M\,x''(t) + C\,x'(t) + K\,x(t) = P_0 \sin \Omega t$$

Solution

$$x(t) = \frac{P_0}{K}\left(\frac{1}{\sqrt{[1-(\Omega/\omega)^2]^2 + [2\xi\Omega/\omega]^2}}\right)\left[\sin(\Omega t+\theta)\right]$$

where θ is a phase angle — Steady State only

$$\text{DLF max.} = \left(\frac{1}{\sqrt{[1-(\Omega/\omega)^2]^2 + [2\xi\Omega/\omega]^2}}\right)$$

At resonance, $\omega = \Omega$ DLF max. $= 1 / 2\xi$

Genralized Single degree of freedom system

$(M^*) U'' + (C^*) U' + (K^*) U = P^*(t)$

Where :

M^* = generalized mass = $\int_0^L m_x \Psi_x^2 \, dx + \Sigma (M \times \Psi^2)$

m = distributed mass & Ψ = shape function & L = length

M = Concentrated mass

K^* = generalized stiffness = $\int_0^L EI_x \, {\Psi''_x}^2 \, dx + \Sigma (K \times \Psi^2)$

X'' = acceleration , X' = velocity , X = displacement

E = modulus of elasticity , I = moment of inertia

$$\omega = \sqrt{\frac{K^*}{M^*}} \quad \& \quad T = (2\pi / \omega)$$

P^* = generalized force = $\int_0^L p \;\Psi \; dx + \Sigma (P \times \Psi)$

Where :

p = distributed force , P = Concentrated force

For frames

K^* = *Generalized Stiffness* = $\Sigma K_i \; x \; (\Psi_i - \Psi_{i-1})^2$

Newmark acceleration method

$$\hat{K} = K + \left(\frac{\gamma}{\beta\,\Delta t}\right)C + \left(\frac{1}{\beta(\Delta t)^2}\right)m$$

$$\Delta\hat{P}_i = \Delta P_i + \left[\frac{m}{\beta\,\Delta T} + \frac{\gamma\,C}{\beta}\right]\dot{U}_i + \left[\frac{m}{2\,\beta} + \Delta t\left(\frac{\gamma}{2\,\beta} - 1\right)C\right]\ddot{U}_i$$

$$\Delta U = \frac{\Delta\hat{P}_i}{\hat{K}}$$

$$\Delta\ddot{U}_i = \frac{\Delta U_i}{\beta(\Delta t)^2} - \frac{\dot{U}_i}{\beta(\Delta t)} - \frac{\ddot{U}_i}{2\,\beta}$$

$$\Delta\dot{U}_i = \frac{\gamma\,\Delta U_i}{\beta(\Delta t)} - \frac{\gamma\,\dot{U}_i}{\beta} + \Delta t\left(1 - \frac{\gamma}{2\,\beta}\right)\ddot{U}_i$$

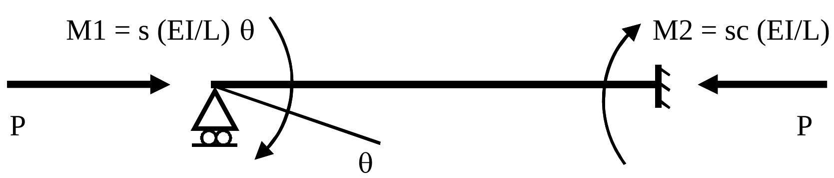
FORMULAS

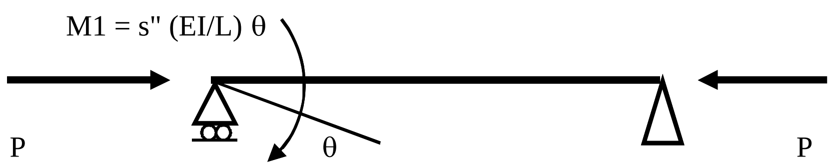
Buckling load of member hinged at both ends (Euler load)

Term	Definition	Unit
P_{Euler}	Euler buckling load.	is the load at which a member hinged at its two ends buckles
E	Young's modulus	Pa
I	Moment of inertia	m^4
L	Length of member	m

$$\mathbf{P_{Euler} = \pi^2 \; EI / L^2}$$

Slope-deflection coefficients for members subjected to axial forces:





Stability Coefficients

ρ P/P Euler	s	c	s"
0.0	4.00	0.50	3.00
0.1	3.87	0.53	2.80
0.2	3.73	0.56	2.58
0.3	3.59	0.59	2.35
0.4	3.44	0.62	2.10
0.5	3.29	0.67	1.83
0.6	3.14	0.71	1.54
0.7	2.98	0.77	1.22
0.8	2.82	0.83	0.86
0.9	2.64	0.91	0.46
1.0	2.47	1.00	0.00

ρ P/P Euler	s	c	s"
1.1	2.28	1.11	-0.54
1.2	2.09	1.25	-1.18
1.3	1.89	1.43	-1.95
1.4	1.68	1.66	-2.93
1.5	1.45	1.98	-4.23
1.6	1.22	2.44	-6.05
1.7	0.98	3.18	-8.86
1.8	0.71	4.52	-13.85
1.9	0.44	7.71	-25.55
2.0	0.14	25.24	-88.86

ρ P/P Euler	s	c	s"
2.1	-0.18	-20.69	
2.2	-0.52	-7.46	
2.3	-0.90	-4.61	
2.4	-1.30	-3.36	
2.5	-1.75	-2.67	
2.6	-2.25	-2.23	
2.7	-2.82	-1.93	
2.8	-3.45	-1.71	
2.9	-4.18	-1.54	
3.0	-5.04	-1.41	

ρ P/P Euler	s	c	s"
3.1	-6.06	-1.31	
3.2	-7.31	-1.24	
3.3	-8.88	-1.17	
3.4	-10.93	-1.12	
3.5	-13.75	-1.08	
3.6	-17.92	-1.05	
3.7	-24.78	-1.03	
3.8	-38.38	-1.01	
3.9	-79.14	-1.00	
4.0	-798.48	-1.00	